AN INTRODUCTORY LOGIC

THE MACMILLAN COMPANY
NEW YORK · BOSTON · CHICAGO
SAN FRANCISCO

MACMILLAN & CO., LIMITED
LONDON · BOMBAY · CALCUTTA
MELBOURNE

THE MACMILLAN CO. OF CANADA, LTD.
TORONTO

AN

INTRODUCTORY LOGIC

BY

JAMES EDWIN CREIGHTON

SAGE PROFESSOR OF LOGIC AND METAPHYSICS IN CORNELL UNIVERSITY

FOURTH EDITION, REVISED WITH THE ADDITION OF NEW PROBLEMS AND EXAMPLES

New York
THE MACMILLAN COMPANY
LONDON: MACMILLAN & CO., LTD.
1921

Set up and electrotyped. Published September, 1898.
Fourth edition, October, 1920.

Norwood Press
J. S. Cushing Co. — Berwick & Smith Co.
Norwood, Mass., U.S.A.

PREFACE

This volume is intended primarily as a text-book for college students, and grew out of my lectures on Logic to undergraduate classes in Cornell University. It aims at being both practical and theoretical. In spite of the obvious deficiencies of formal Logic as a theory of the nature of thought, I am convinced that it is one of the most valuable instruments in modern education for promoting clear thinking, and for developing critical habits of mind. J. S. Mill, speaking in the *Autobiography* of the discipline which he received from working logical exercises, expresses the following opinion: "I am persuaded that nothing, in modern education, tends so much, when properly used, to form exact thinkers, who attach a precise meaning to words and propositions, and are not imposed on by vague, loose, or ambiguous terms." Although in treating the syllogistic Logic I have followed to a large extent the ordinary mode of presentation, I have both here, and when dealing with the Inductive Methods, endeavoured to interpret the traditional doctrines in a philosophical way, and to prepare for the theoretical discussions of the third part of the book.

The advisability of attempting to include a theory of thought, or philosophy of knowledge, even in outline, in an elementary course in Logic, may at first sight appear doubtful. It seems to me, however, that this inclusion is not only justifiable, but even necessary at the present time. Psychology is no longer a 'philosophy of mind'; but, under the influence of experimental methods, has differentiated itself almost entirely from philosophy, and

become a 'natural' science. As a natural science, it is interested in the structure of the mental life, — the characteristics of the elementary processes, and the laws of their combination, — and not primarily in the function which ideas play in giving us knowledge. It is clear that psychology does not undertake to give a final account of all that mind is and does. It belongs to Logic to investigate intelligence as a knowing function, just as it is the task of Ethics to deal with the practical or active mental functions.

The practical question still remains as to whether this side of Logic can be made profitable to students who have had no previous philosophical training. I am well aware of the difficulty of the subject, but my own experience leads me to believe that the main conceptions of modern logical theory can be rendered intelligible even to elementary classes. Of the incompleteness and shortcomings of my treatment I am quite conscious; but I have endeavoured to make the matter as simple and concrete as possible, and to illustrate it by means of familiar facts of experience.

*   *   *   *   *   *   *   *

J. E. C.

Cornell University,
August, 1898.

PREFACE TO THIRD EDITION

THE present edition represents a somewhat thorough revision of this book, which had remained substantially unchanged since its first publication, eleven years ago. Changes of more or less importance have been introduced into every chapter; new paragraphs have been added to many of the sections; and, especially in the Second Part, many of the sections have been entirely rewritten. Chapter XIII. of the old text, on the "Problem of Induction," has been expanded into two chapters; and, throughout this Part, an attempt has been made to bring the treatment of the various inductive methods into closer relation with a general philosophical theory. The chapter with which the text formerly closed, "Rational and Empirical Theories," has been replaced by one entitled "The Unification of Knowledge." It has seemed important to conclude the discussion of the nature of thought with some statement of the meaning and function of the main categories which experience involves, and, in this connection, to indicate in a general way the necessity of a philosophical interpretation of the results of the special sciences. The number of problems and examples of reasoning to be analyzed has been more than doubled in the belief that fresh material of this nature will prove welcome to teachers of the subject.

The two purposes of an introductory course in logic which were emphasized in the preface to the first edition — to afford discipline in thinking and to furnish an introduction to philosophical studies — have thus been kept in mind

in the present revision. The Third Part of the book presents an elementary account of knowledge from the developmental standpoint. The conceptions there treated in a somewhat systematic way are, however, introduced from time to time in the earlier chapters to modify and interpret the results of the older logical theories. It will be found that the more theoretical considerations have generally been printed as separate paragraphs in smaller type, and may therefore conveniently be omitted, if thought desirable, when the time devoted to the subject does not allow a consideration of all the topics dealt with in the book. These paragraphs are usually intended merely to suggest further problems to the student, or to furnish a text to the teacher for explanation and elaboration.

I am indebted to many of my colleagues who have used the book in the classroom for helpful criticisms and suggestions regarding its revision. In particular, I wish to acknowledge my obligations to Dr. Edmund H. Hollands for many suggestions and much valuable assistance, especially in the collection and arrangement of the examples. My thanks are also due to Dr. Hollands and to Mr. C. H. Williams for aid in proof-reading.

* * * * * * * *

J. E. C.

CORNELL UNIVERSITY,
August, 1909.

PREFACE TO FOURTH EDITION

In the present edition no fundamental changes have been made in the text. I have, however, sought to render the meaning clearer and more precise by means of slight verbal alterations here and there throughout the book. The section containing Questions and Exercises has been carefully revised, many of the old questions being omitted and new problems and illustrative material added. I have drawn freely from various sources, from the published examination papers of colleges and universities, and especially from scientific books and the histories of science. For assistance in collecting and arranging this material I am indebted to Dr. Katherine E. Gilbert.

Many of the exercises are intended to stimulate thought and discussion, rather than to call out an answer in set terms. This is particularly true of the examples of inductive reasoning. The hope is that the student may learn to appreciate, through these examples, not only the abstract form and method of thought, but something of its living richness as well.

J. E. C.

May, 1920.

TABLE OF CONTENTS

INTRODUCTION

CHAPTER I

THE STANDPOINT AND PROBLEM OF LOGIC

PAGE

§ 1. Definition of the Subject 1
§ 2. Relation to Psychology 5
§ 3. Logic as a Science and an Art 9
§ 4. The Material of Logic 14

CHAPTER II

IMPORTANT STAGES IN THE DEVELOPMENT OF LOGIC

§ 5. Socrates and the Concept 19
§ 6. Aristotle and the Syllogism 23
§ 7. Bacon and the Inductive Method 28
§ 8. Logic from the Evolutionary Standpoint 32

PART I.—THE SYLLOGISM

CHAPTER III

THE SYLLOGISM AND ITS PARTS

§ 9. The Nature of the Syllogism 36
§ 10. The Parts of the Syllogism 39
§ 11. Perception, Conception, and Judgment 44

CHAPTER IV

THE VARIOUS KINDS OF TERMS

§ 12. Singular, General, and Collective Terms 49
§ 13. Abstract and Concrete Terms 51
§ 14. Positive and Negative Terms 55
§ 15. Absolute and Relative Terms 57
§ 16. Extension and Intension of Terms 58

CHAPTER V

Definition and Division

PAGE
§ 17. Fixing the Meaning of Terms 64
§ 18. Definition 66
§ 19. Division 77

CHAPTER VI

Propositions

§ 20. The Nature of a Proposition 84
§ 21. The Quality and Quantity of Propositions 86
§ 22. Difficulties in Classification 89
§ 23. Formal Relation of Subject and Predicate 90

CHAPTER VII

The Interpretation of Propositions

§ 24. The So-called Process of Immediate Inference 97
§ 25. The Opposition of Propositions 99
§ 26. The Obversion of Propositions 103
§ 27. The Conversion of Propositions 105
§ 28. Contraposition and Inversion 108

CHAPTER VIII

The Syllogism

§ 29. The Nature of Syllogistic Reasoning 112
§ 30. The Rules of the Syllogism 115
§ 31. The Figures of the Syllogism 120

CHAPTER IX

The Valid Moods and the Reduction of Figures

§ 32. The Moods of the Syllogism 122
§ 33. The Special Canons of the Four Figures 123
§ 34. The Determination of the Valid Moods in Each of the Figures . 127
§ 35. The Mnemonic Lines 129

CHAPTER X

Abbreviated and Irregular Forms of Argument

§ 36. Enthymemes 133
§ 37. Prosyllogisms and Episyllogisms 134

PAGE

§ 38. Sorites, or Chains of Reasoning 136
§ 39. Irregular Arguments 139

CHAPTER XI

HYPOTHETICAL AND DISJUNCTIVE ARGUMENTS

§ 40. The Hypothetical Syllogism 144
§ 41. Relation of Categorical and Hypothetical Arguments . . . 148
§ 42. Disjunctive Arguments 154
§ 43. The Dilemma 156

CHAPTER XII

FALLACIES OF DEDUCTIVE REASONING

§ 44. Classification of Fallacies 164
§ 45. Errors in Interpretation 166
§ 46. Formal Fallacies 170
§ 47. Material Fallacies 171

PART II. — INDUCTIVE METHODS

CHAPTER XIII

THE PROBLEM OF INDUCTION

§ 48. The Problem of Induction 190
§ 49. The Enumeration of Instances 192
§ 50. Induction through Analysis 196

CHAPTER XIV

THE ASSUMPTIONS OF INDUCTION — STAGES IN THE INDUCTIVE PROCEDURE

§ 51. The Assumptions of Induction 202
§ 52. Stages in the Inductive Process 205
§ 53. Observation and Explanation 207

CHAPTER XV

ENUMERATION AND STATISTICS

§ 54. Enumeration or Simple Counting 216
§ 55. Statistics and Statistical Methods 220
§ 56. The Calculation of Chances 228

CHAPTER XVI

Determination of Causal Relations

PAGE
§ 57. Causal Connection 234
§ 58. Mill's Experimental Methods 237
§ 59. The Method of Agreement 239
§ 60. The Method of Difference 244

CHAPTER XVII

Determination of Causal Relations (*continued*)

§ 61. The Joint Method of Agreement and Difference 249
§ 62. The Method of Concomitant Variations 255
§ 63. The Method of Residues 260

CHAPTER XVIII

Analogy

§ 64. Explanation by Analogy 266
§ 65. Analogy as Suggestive of Explanatory Hypotheses . . . 271
§ 66. The Incompleteness of Analogical Reasoning 274

CHAPTER XIX

The Use of Hypotheses

§ 67. Reasoning from an Hypothesis 278
§ 68. Formation of Hypotheses 282
§ 69. The Proof of an Hypothesis 285
§ 70. Requirements of a Good Hypothesis 293

CHAPTER XX

Fallacies of Induction

§ 71. The Source of Fallacy 298
§ 72. Fallacies due to the Careless Use of Language 299
§ 73. Errors of Observation 303
§ 74. Mistakes in Reasoning 309
§ 75. Fallacies due to Individual Prepossessions 312

PART III. — THE NATURE OF THOUGHT

CHAPTER XXI

JUDGMENT AS THE ELEMENTARY PROCESS OF THOUGHT

PAGE
§ 76. Thinking the Process by which Knowledge grows or develops . 316
§ 77. The Law of Evolution and its Application to Logic . . . 317
§ 78. Judgment as the Starting-point 322
§ 79. Concepts and Judgment 324

CHAPTER XXII

THE MAIN CHARACTERISTICS OF JUDGMENT

§ 80. The Universality of Judgments 329
§ 81. The Necessity of Judgments 331
§ 82. Judgment involves both Analysis and Synthesis 334
§ 83. Judgment as constructing a System of Knowledge . . . 339

CHAPTER XXIII

THE LAWS OF THOUGHT

§ 84. The Law of Identity 343
§ 85. The Law of Contradiction 350
§ 86. The Law of Excluded Middle 352

CHAPTER XXIV

TYPES OF JUDGMENT

§ 87. Judgments of Quality 355
§ 88. Judgments of Quantity 358
§ 89. Judgments of Causal Connection 362
§ 90. Judgments of Individuality 370

CHAPTER XXV

THE NATURE OF INFERENCE. — INDUCTION AND DEDUCTION

§ 91. Judgment and Inference 373
§ 92. The Nature of Inference 378
§ 93. Induction and Deduction 384

CHAPTER XXVI

The Unification of Knowledge

PAGE

§ 94. Science and Philosophy 390
§ 95. Science as Philosophy 395
§ 96. The Assumptions of the Sciences 399
§ 97. Philosophy as the Interpretation of the Sciences 405

Questions and Exercises 409
Miscellaneous Exercises in Propositions 422
Miscellaneous Examples of Deductive Arguments 433
Miscellaneous Examples of Inductive Arguments 450

Index 499

INTRODUCTION

CHAPTER I

THE STANDPOINT AND PROBLEM OF LOGIC

§ 1. **Definition of the Subject.** — Logic may be defined as the science of thought, or as the science which investigates the process of thinking. Every one knows, in a general way at least, what is meant by thinking, and has noticed more or less consciously some of its characteristics. Thinking is the intellectual act by means of which knowledge is obtained. We do not really *know* any fact until we *think* it; that is, until the mind sets it in its proper relation to the other parts of its experience, and thus comes to understand its true meaning. We make a distinction, for example, between what has come to us through report or hearsay, and conclusions which we have reached by our own thinking. 'I have *heard*,' we say, 'that A is dishonest, but I do not *know* it.' That is, this fact has not been reached as a result of our own thinking, and cannot therefore claim the title of knowledge. On the other hand, that the earth is round, is not a mere matter of hearsay for an educated man. It is a piece of *knowledge*, because it is a conclusion which he has reached by thinking, or by putting together various facts for himself.

Logic, then, is the science which treats of the operations of the human mind in its search for truth. Logic must always

assume that the thinking which it investigates has, as its aim and object, the attainment of truth. Thinking is thus an expression of the will as well as of the intelligence. Again, as seeking truth, thinking is not a mere arrangement of ideas in our heads, but is a dealing with the nature of objects. Thought cannot exist in itself or by itself as something merely in our minds, but it is its very nature to refer to real things, existing in an objective world. This follows directly from our definition of thought as concerned with truth. Truth is no private state of the subjective mind, but something objective that is, in a sense, independent of the individual thinker and his ideas.

In defining Logic as a *science*, we mean that it seeks to substitute exact and systematic knowledge regarding the nature of thought for the popular notions to be found in everyday life. Like all the sciences, logic has to correct and supplement ordinary knowledge. It is its mission to help us to understand more exactly and completely the way in which thinking goes on, and to enumerate and describe, as fully and precisely as possible, the various modes and types of thought which are employed in gaining knowledge.

But it is also the business of a science to systematize facts. Logic, then, cannot content itself with a mere description of this or that kind of thinking, in isolation from other ways in which we think. It must go on to explain how the various forms of thinking are related. For example, we apply such terms as 'conception,' 'judgment,' 'induction,' and 'deduction' to different intellectual operations, and give the distinguishing characteristic in each case. But it is necessary as well to understand how these processes are related. Since all thinking has one end, the discovery of

truth, the various intellectual operations must mutually coöperate and assist in this result. All of the logical processes, then, stand in relation to one another. They are all parts of the one intelligence, though they may well represent different stages or steps in its work of obtaining knowledge. It is therefore the business of logic to show us the total movement of thought. In other words, Logic must furnish a comprehensive view of the way in which intelligence acts, and the part which processes like 'conception,' 'judgment,' 'induction,' etc., play.

(1) The word 'logic' is derived from the adjective corresponding to the Greek noun λόγος, which signifies either a complete thought, or a word as the expression of that thought. The singular form of the adjective λογική, from which the English word is derived, was supposed to qualify either ἐπιστήμη, as applying to the theoretical *science* of logic, or τέχνη, as referring to the practical application of its rules and as affording guidance in the *art* of correct reasoning. We shall have to raise the question in a subsequent section how far it is possible to regard logic as an art, or a system of rules which teach us how to reason correctly.

The use of the same term (λόγος) by the Greeks to denote both 'thought,' and 'word' or 'discourse,' emphasizes the close and vital relation between thought and its expression in language. Whether thinking can go on without language is a psychological question that we cannot here decide. But it is certain that in adult human thinking the thought and its verbal expression are inseparably connected, just as the principle of life is connected with the functions and activities of the physical organism. The *word* is no arbitrary or external mark attached to a ready-made *thought* which exists independently. The verbal expression is rather the means in which and through which the thought completes itself. It is that which gives to the thought, not only a name, but an

abiding reality as a permanent possession. To introduce a new term into a science is not indeed always a great intellectual achievement. New names may be coined for facts and conceptions that are already familiar. But, on the other hand, new thoughts and discoveries must find expression either in the employment of new terms, or in the use of old terms in a new and more definite sense. It is thus possible to appreciate the remark that a science is only '*une langue bien faite.*'

What has been said will suffice to make clear the close relation between Logic and Rhetoric. Logic finds the products of thinking expressed in language, and to a considerable extent may be said to be concerned with the meaning of words, sentences, and spoken or written arguments. It is impossible to make any sharp division between the thoughts and their relations, on the one hand, and the form of the words and sentences with which rhetoric concerns itself, on the other. We may say, then, that definiteness of thought is a condition of clearness and accuracy in the use of language, and also that the effort to express oneself with clearness and precision demands and involves logical pains and exactness. Indeed, clear thinking and accurate verbal expression are one and inseparable, as are also careless or indolent ways of thinking and slipshod and slovenly use of language. By taking the trouble to express oneself with precision one forms the habit of thinking rightly.

(2) We have defined logic as the science of the operations and processes of thought, or as the science of thinking. It is evident, however, that this definition does not carry us very far unless we know what thinking means. And to gain a clearer idea of this common term may be said to be the problem of logic. This is, however, by no means as easy a task as may at first appear. Familiar words and phrases often conceal difficulties just because they are so pat. It is only when we become discontented with our knowledge regarding any subject, when doubts arise whether we really understand the meaning of the words which we use, that we

attempt to make our knowledge scientific, *i.e.*, to gain clear, definite, and systematic ideas. This can perhaps be made clearer by considering the main differences between an educated and an uneducated man. The educated man has, of course, a great deal more information than the other, and his knowledge is more definite and systematic. But a second and more important distinction is found in *the attitude of mind* which education begets. The educated man is desirous of knowing more, because he is sensible of his own ignorance. The uneducated man, on the other hand, supposes that he knows all about things whose names are familiar to him. He can settle puzzling theological or political problems off-hand in a way which is perfectly satisfactory to himself, without study, and almost without reflection.

§ 2. **Relation to Psychology.** — It may aid us in obtaining a clearer view of what thinking is, if we compare the general standpoint of logic with that of psychology. Both of these sciences deal with what goes on in mind or consciousness, and are thus opposed to the so-called objective sciences, which are all concerned with some group or field of external facts. But, in spite of this agreement, there is an important distinction between logic and psychology. In the first place, psychology deals with all that there is in mind. It describes pleasures and pains, acts of will, and the association of ideas, as well as what is usually called logical thinking. But logic does not differ from psychology simply by being less inclusive than the latter. It is true that, from the standpoint of psychology, the thought-process is merely a part of the mental content, which has to be analyzed and described like anything else which goes on in consciousness. Thinking has doubtless for

psychology peculiar marks or characteristics which distinguish it from other related processes like those of association; but when these have been found, and the psychological description of thinking is complete, the question with which logic deals has not yet been raised. For logic, as we shall see presently, adopts a different standpoint, and investigates with a different end in view.

The important difference is this: In psychology we are interested in the content of consciousness *for its own sake*, and just as it stands. We try to find out what actually goes on in our minds, and to describe it just as we should any event which occurs in the external world. But in logic the question is not: What are mental processes? but rather: What knowledge do they give us, and is this knowledge true or false? Logic, in other words, does not regard the way in which ideas exist, and is not interested in them *for what they are*, but rather in the purpose which they subserve in affording us knowledge of something beyond themselves. Psychology, in its description of conscious states, inquires regarding their quality, intensity, duration, etc., and the ways in which they combine with each other to form complex ideas. The problem with which logic is concerned, on the other hand, has reference to the *value* of ideas when they are taken to represent facts in the real world. As we have already seen, thinking is the pursuit of truth; and, in dealing with thoughts, logic has to describe and evaluate them in relation to this end. Hence for logic thoughts are true or false, *i.e.*, they are in harmony or not in harmony with truth, which is the *standard or norm* that thought sets up as its purpose or end. Psychology, on the other hand, does not ask at all whether the

ideas are true or false, good or bad. It does not seek to evaluate ideas in the light of some standard, but confines itself to describing their actual mode of existence.

Consider a little further the nature of the ideas with which logic deals. Every idea, as we have seen, not only exists in some definite fashion in some particular consciousness, connected with certain other ideas, and with a definite quality, intensity, etc., but it has a meaning or significance as a piece of knowledge. It not only *is* something, but it also stands for or *signifies* something. Now it is not with the existence, but with the meaning side of ideas that logic has to do. A logical idea, or piece of knowledge, is not merely a modification of consciousness which exists in the mind of some individual at a particular time. For example, the proposition: 'The three angles of a triangle are equal to two right angles,' will give rise to a number of definite psychological processes (probably auditory or visual in character) in the mind of any individual. These processes would also probably differ in character in the case of two persons. The *meaning* of the proposition, however, is distinct from the definite processes which arise in particular minds. The proposition has a significance as an objective fact, or piece of knowledge, outside my individual mind; the psychological images or processes may differ for different persons, but the fact expressed is the same for all minds and at all times.

(1) The relation between logic and psychology may perhaps be illustrated by referring to that which exists between morphology and physiology. Morphology deals with the form and structure of living organisms, and physiology with the various acts and functions which these organisms discharge in fulfilling the ends of life.

Thus we speak of the former as the science of form or structure, and of the latter as the science of function. In the same way, psychology may be said to deal with the actual structure of mental processes, and logic with the part which they play in giving us knowledge.

It must be noticed, however, that this is a distinction made for purposes of investigation, and does not denote that structure and function have nothing to do with each other. On the contrary, some knowledge of the function is often necessary in order to understand the structure of an organ; and, on the other hand, it is usually true that the nature of a function only becomes completely intelligible when the character of the mechanism with which it works is known. And the same holds true, I think, of the relations between psychology and logic. Although it has been found profitable when dealing with consciousness, as in the biological realm, to investigate the nature of structure and function separately, yet here, as there, the two lines of inquiry cross each other; for it is beyond question that the knowledge we obtain by thinking is largely dependent upon the character (quality, intensity, etc.) of the actual processes in consciousness. To understand the nature of a logical idea, then, it is often necessary to refer to the psychological facts and their actual mode of behaviour. And it is equally true that one cannot carry on a psychological investigation into the nature of mental processes without taking account, to some extent, of the part which they play in giving us knowledge. No psychology is able to take ideas simply as existing conscious processes to which no further meaning or importance attaches; it is only with reference to the function they perform as *knowing* states that their own peculiar character can be understood. In other words, the intellectual activities and purposes of mind must be presupposed in psychology, though this science, for the most part, goes its way as if the ideas were not cognitive at all. At least this seems to be true of the 'new' or experimental psychology, as opposed to the older philosophies of mind.

(2) It would of course be presumptuous, as well as utterly useless, for any writer to draw a hard and fast line between logic and psychology, and to forbid others to overstep it. In attempting to discover the dividing line between two closely related sciences, one must be guided by the procedure of those who are working in the fields which it is proposed to divide. Now, it must be admitted that by no means all of the recent writers in psychology limit the sphere of their science in the way above described; that is, there are certain psychologists who do not confine their attention to the mere mental processes as such, but include in their investigations the further problem regarding the function which these processes play in giving us knowledge. Thus in Professor James's *Principles of Psychology* there is an excellent chapter on 'Reasoning,' which certainly contains as much logical as psychological matter. In general, one may say that at the present time psychologists are tending to deal with mind more from a 'functional' than a 'structural' point of view. That is, the tendency is now to emphasize the activities of conscious life, and thus to interpret mind in the light of the results it achieves, rather than to explain it solely in terms of the elements of which it is composed. But this functional psychology is not identical with logic. For, in the first place, it does not limit itself, as the latter does, to the cognitive functions of mind. And, secondly, it tends to interpret even ideas and judgments in their relation to the life of the psychophysical organism in general, rather than as elements in the life of reason or truth. It is only logic which looks at mental life definitely and exclusively from this point of view. For logic, the thinking process is not a mere aspect of living, but something to be investigated and understood solely in its relation to truth, or the rational consistency which is its end and goal.

§ 3. **Logic as a Science and an Art.** — We have defined logic as the science of thought, but it has often been pointed

out that there are equally strong reasons for considering it to be an art. The purpose of logical study, it is often said, is to help us to think correctly, to prevent us from falling into errors in our own reasoning, and from being misled by the fallacious arguments of others. The difference between a science and an art in general is that a science is interested in the discovery of facts and laws without any thought of what use may be made of this knowledge; an art, on the contrary, gives practical guidance and direction for some course of action. The question before us, then, is this: Does logic merely give us knowledge about the ways in which we think, or does it also help us to think rightly?

Before we attempt to answer this question, we must note that practical rules of action are based upon scientific knowledge. An art, in other words, depends upon science, and grows in perfection with the advance of scientific knowledge. Thus medicine, as the art of healing, is founded upon the sciences of chemistry, physiology, and anatomy, and it is because of the great discoveries which have been made in these fields within recent years that it has been able to advance with such gigantic strides. Again, the art of singing, in so far as it is an art which can be taught and learned, depends upon a knowledge of the physical and physiological laws of the vocal organs. An art, then, always presupposes a certain amount of science, or knowledge, and is simply the application of this knowledge to some practical purpose. In some cases, the application is very obvious and direct; in others, it is much more difficult to determine; but, in general, there is always this relation between theory and practice, between knowledge and action.

From what has been already said, it will be evident that

logic must first be a science before it can become an art. Its first business must be to investigate the nature of thought, and to attempt to discover the different forms which the latter assumes in its work of attaining knowledge. So that we were right in defining it as primarily a science. But the further question remains: How far is it possible to apply the laws of logic, after they have been discovered, in such a way as to obtain directions for reasoning correctly in every case? Can we not apply our knowledge of the laws of thought in such a way as to get a complete art of reasoning, just as the laws of chemistry and biology are applied in medicine?

It is no doubt true in logic, as everywhere, that scientific knowledge is capable of practical application. But I do not think that logic can be regarded as an art, in the sense that it furnishes a definite set of rules for thinking correctly. There is an important distinction in this case which must not be left out of account. The physical, and even the biological sciences, deal with things whose way of acting is perfectly definite and uniform. The character of any of the physiological functions, as, *e.g.*, digestion, may be comparatively complex and difficult to determine, but it normally attains its end through the use of the same means. When once its laws are understood, it is not difficult to prescribe just how the proper means may always be secured for the attainment of the desired end. But thinking has much more flexibility in its way of acting. We cannot say with the same definiteness, as in the cases we have been considering, that in order to reach a certain end we must use a definite set of means. It is not possible, that is, to say: If you would learn what is true about any

particular subject, you must follow this rule and that in your thinking. Logic, it seems to me, cannot be regarded as an art like photography, or even like medicine; for it is not possible to lay down definite rules for the guidance of thinking in every case. What we can do, is to show the method by which new truths have been discovered, and the general conditions which must always be fulfilled in reasoning correctly. And it is also possible to point out the more common errors which arise when these conditions are violated. But it is beyond the power of logic to formulate any definite set of rules for the guidance of thinking that can be learned and applied as a prescription for every case; and students whose only interest in the subject is the practical one of finding some rules that may be directly applied to make them infallible reasoners are likely to be disappointed.

The necessity of devoting oneself to a science quite unselfishly cannot be too strongly enjoined, nor the evils which arise when one begins a study 'greedy for quick returns of profit,' too often emphasized. Nevertheless, when this is understood it is quite legitimate to raise the question regarding the practical results to be expected from a study of logic. As we have seen, we cannot hope to become infallible reasoners by its aid. It is just as true here as in any other field, however, that knowledge is power, and ignorance synonymous with weakness. For even if one resolves never to look inside a logic book, one must nevertheless have some theory, or act upon some principle — it may be quite unconsciously — in deciding what is true and what is false. For instance, a man may act upon the principle that those things are likely to be true which are favourable to his own interests, or which agree with his own preju-

dices, or with the articles of his church or political party. Or again, he may regard his senses as the standards of truth. Mr. Bradley says that *if* dogs reason, they proceed upon the principle, 'what smells, exists, and what does not smell does not exist.' It is not uncommon to hear it announced: What can be perceived through the senses is true; what cannot be sensed, or is contrary to the testimony of the senses, is an absurdity. This was the standard of truth adopted, for example, by those who attempted to overthrow the Copernican theory by declaring it to be in plain contradiction to the testimony of the senses.

It seems evident, therefore, that intellectual beings cannot escape some kind of logical theory, whether they hold it consciously or unconsciously. It is clear, too, that the character of this theory will determine to a great extent their thoughts and opinions. The only question which remains is whether it is better to leave this matter entirely to chance, or to attempt to gain some clear ideas regarding the nature of thinking, and the conditions under which knowledge arises. It can scarcely be doubted that, even from a practical point of view, a true theory is better than a false one. A man who has reflected upon the nature of proof, and the principles of reasoning, is much less likely to be deceived than one who is guided unconsciously by assumptions which he has never examined. It is always an advantage to know exactly the nature of the result at which we are aiming, and to be perfectly clear as to our own purposes. And this is just what a study of logic aids us in attaining. It helps us to understand the structure of knowledge and the conditions of proof. Moreover, it engenders

the habit of criticising propositions, and examining the evidence upon which they rest. Further, the importance of this study for a theory of education may well be emphasized. For education, at least in so far as it undertakes to train the knowing powers of the individual, must be based upon a knowledge of the necessary laws of intelligence, and of the steps or stages which it passes through in its process of development.

§ 4. **The Material of Logic.** — The business of logic, as we have seen, is to discover the laws of thought and to show the differences which exist between real and imaginary knowledge. Where now shall we find the materials for this study? Where are the facts which are to be taken as a starting-point? It is, of course, impossible to learn directly from one's own consciousness *all* that thinking is, or everything of which it is capable. For, quite apart from the difficulty of observing the process of thought while it is actually going on, no one can suppose that his own mind furnishes an example of all that thinking has done, or can do. It is necessary to take a broader view, and learn how other men think. Of course, we cannot look into the consciousness of other men, but we can study the products and results of their thoughts. The history of the way in which truth has been discovered is of the greatest importance for logic. We have already spoken of thinking as having truth as its standard or norm. It is for this reason that logic is sometimes called a *normative* science, since like ethics and æsthetics it looks at the experience it studies as realizing an end. But where does logic find its norm? It has no *a priori* method of deciding what is true and what is false, what is knowledge and what is

not. But in the various sciences of nature and of man, we have a body of accepted truth that has been verified by the experience of a great many individuals. Now, it is to this we must look if we would know what knowledge is, and it is in the processes through which it has been built up that we find the norm of correct thinking. The history of the various sciences furnishes a record of the steps by means of which thought has built up knowledge. And, in this record, we have also a revelation of the nature of the thinking process itself, and of the stages through which it has passed in the course of its development.

It is by a reflection, then, upon the nature of propositions which are universally regarded as true that the laws of logic are obtained. There is always a permanent body of knowledge which no one thinks of calling in question. Both in everyday knowledge; and in the sciences, there are a great number of propositions which are found true by everybody who takes the trouble to verify them. And it is here that logic finds its material. Taking the facts and propositions which are recognized as certain by every one, logic examines their structure in order to learn about the nature of the intellectual processes by which they have been discovered. What principles, it asks, are involved in these bodies of knowledge, and what particular acts of thought were necessary to discover them? It is only by examining various pieces of knowledge in this way, and attempting to trace out the conditions of their discovery, that one can learn anything new regarding the laws and character of thought. The best way of getting information about what thought can do, is to study what it has already accomplished. In other words, there is no

way of learning about thinking except by studying what it has done.

Every piece of knowledge, as the product of thinking, is to some extent a revelation of the nature of intelligence. But scientific knowledge — by this I mean the results of the philosophical and historical sciences as well as of the so-called natural sciences — exhibits perhaps most clearly the nature of thought. For the history of these sciences enables us to see the process of knowledge, as it were, in the making. In tracing the history of philosophical and scientific ideas, we are at the same time following the laws of the development of thought. It is this fact which makes the history of philosophy and of the various sciences so instructive. It was with this object in view, to take but a single example, that Whewell wrote his famous *History of the Inductive Sciences*. He was interested, that is, not so much in the mere facts and names with which he dealt, as in showing the nature of thinking and the methods which had been employed in gaining a knowledge of the world. This is made very clear in the introduction to another work of Whewell from which I quote: "We may best hope to understand the nature and conditions of real knowledge by studying the nature and conditions of the most certain knowledge which we possess; and we are most likely to learn the best methods of discovering truth by examining how truths, now universally recognized, have really been discovered. Now there do exist among us doctrines of solid and acknowledged merit certainly, and truths of which the discovery has been received with universal applause. These constitute what we commonly term *sciences;* and of these bodies of exact and enduring knowledge we have within our reach so large a collection that we may hope to examine them and the history of their formation with a good prospect of deriving from the study such instruction as we need seek."[1]

[1] Whewell, *History of Scientific Ideas*, 3d ed., Vol. I., p. 4.

We have been insisting that the materials for the study of logic are to be found mainly in the records which we possess of what thinking has actually accomplished. Our own consciousness, it was said, can supply but a very small quantity of material. To learn what thinking is, one must have as broad a survey as possible of its achievements.

But there is another side to the matter. It must never be forgotten that it is the actual operations of thought with which logic is concerned. The words and propositions which express the results of thinking must never be allowed to take the place of the thoughts themselves. Now, we cannot directly study the thoughts of any other individual. It is only in so far as we interpret, through our own consciousness, the records of what thinking has done, that these records are able to throw any light upon the problem of logic. So in this study, as elsewhere, we must find the key to the material in our own consciousness. If we are to gain any real ideas of the character of the thinking processes by means of which the sciences have been built up, we must reproduce these in our own minds. One's own consciousness must, after all, furnish the key which makes intelligible the account of the various steps which the thought of mankind has taken in building up science or knowledge. The materials of logic which history furnishes become significant only when translated into acts and operations which may be observed in our own minds.

REFERENCES

The following references may be given in connection with §§ 1 and 2:—

C. Sigwart, *Logic*, Vol. I., General Introduction.

F. H. Bradley, *The Principles of Logic*, pp. 1–10.

B. Bosanquet, *Logic*, Vol. I., Introduction.

H. L. Mansel, *Prolegomena Logica*, Chap. I.

R. Adamson, The first part of the article 'Logic' in the *Encyclopædia Britannica.*

D. G. Ritchie, The Relation of Logic to Psychology, *Philos. Review*, Vol. V., pp. 585–600; Vol. VI., pp. 1–17.

CHAPTER II

IMPORTANT STAGES IN THE DEVELOPMENT OF LOGIC

§ 5. **Socrates and the Concept.** — Logic was founded as a separate and independent branch of inquiry by Aristotle (387–322 B.C.). Almost from the first beginning of philosophical speculation, — which took its rise in the sixth century in the Greek cities on the coast of Asia Minor, and in Sicily and southern Italy, — questions had, however, been raised regarding the nature of knowledge and the proper value to be assigned to different forms of experience. More particularly, these early thinkers emphasized the distinction between the knowledge given by sense-perception and that obtained by thinking or reasoning. The latter kind of knowledge, it was generally agreed, is alone trustworthy and genuine; while the senses, on the other hand, are bad witnesses and do not show us the true nature of things. In these early schools, however, logical questions about truth and knowledge were largely incidental, the fundamental interest being to explain the nature of the physical universe. It was not until after the Persian wars, when Athens had become the intellectual and commercial centre of Greece, that the inner world of human experience — man's knowledge, moral beliefs, and practices, customs, laws, and religions — came to be of primary interest and importance to philosophical inquirers.

The political prominence and wealth that came to Athens as a result of her leadership in the wars with Persia, led to the rapid transformation of the outward appearance of the city and also of the life and thought of its inhabitants. The new times and the wider circle of political and social activities which were thus opened up to citizens of Athens, demanded that the older system of education — the traditional music and gymnastic — should be supplemented by some more advanced instruction. And, in response to this demand, there arose a class of teachers called Sophists, who made it their business to instruct young men in all the practical affairs of life, and especially in the use of words and the art of public speaking, or rhetoric, as it was called. The Sophists do not seem to have made it their object to teach truth to their pupils, or to inculcate in them a love and reverence for truth; they sought rather to make those whom they taught clever men of the world. In teaching the art of argumentation or public speaking they did not confine themselves to pointing out the methods by which true conclusions could be reached, but went on to teach the arts by which the judges could be persuaded, and tricks for the discomfiture of one's adversary. The rhetoric of the Sophists, in other words, was not a science of reasoning, but an art of persuasion and of controversy. It was not essential to have any real knowledge of the subject under discussion in order to argue well, from their point of view, but only to be well versed in all the arts of persuasion, and quick to take advantage of an opponent's errors.

The theory on which the teaching of the Sophists was based is usually known as Scepticism. The Sophists, that is, had come to the conclusion that it is impossible to find

any fixed standard of truth. Looking at the diversity of individual opinions and of individual feelings, they declared that knowledge or truth as something objective, or the same for all, is an illusion. Only individual opinions exist; there is no standard by reference to which these opinions may be measured. Indeed, the words 'truth' and 'falsehood' can have only a practical meaning; each individual must be the measure of truth for himself. They lacked the scientific spirit that aims at truth which is objective and real; like men everywhere whose interest is exclusively practical, they thought truth in this sense abstract and unmeaning, and aimed only at knowledge which has some direct application.

Moreover, in the opinion of the Sophists, the same state of things exists with regard to our moral ideas. There is no standard of right and wrong, just as there is no standard of truth and falsehood. Each man has the right to choose what he regards as most advantageous for himself. The traditional rules of morality have no authority over the individual, nor is it possible to discover any rules of morality which are binding on all men. It is the part of wisdom to consult one's own interest in acting, and to seek to secure one's own advantage. Moral distinctions, like logical distinctions, are purely relative and individual.

Socrates was the great opponent of this doctrine of Scepticism and Relativity as taught by the Sophists. They had concluded, from the diversity of individual opinion on moral questions, that there is no real or absolute distinction between right and wrong, false or true. Socrates, however, was convinced that if one examined more carefully the nature of the judgments which are passed by different

individuals, one would find common elements or ideas. It is possible, he believed, to find a definite standard, both in matters of theory and in matters of practice. This common element, however, is not to be discovered in sensation, or in feelings of pleasure and pain; these experiences are purely individual, and can never serve as a universal standard. But beneath the diversity of sensation and feelings there is the thought, or *concept*, which is common to all men. When rational beings come to understand one another, they must agree as to the nature of the fundamental virtues, — justice, temperance, courage, etc. It is true that few men have thought about these matters, and are able to express their meaning clearly, but every man, as a rational being, carries these fundamental notions in his mind. Now, in order to refute the moral scepticism of the Sophists (and it was this side of their teaching which Socrates especially opposed), it is necessary that the ethical notions, or concepts, which are implicit in the minds of men shall be drawn out and carefully defined. How is this to be accomplished? Socrates did not undertake to teach men what ideas they should hold regarding the nature of any of the virtues; he rather made them partners in an investigation, and by means of skilful questions tried to assist them in discovering the real nature of goodness for themselves. Another point to be noticed is that the definition of the various virtues was reached as a result of comparing the views of a number of individuals. In this way, by comparing the opinions of many men of different professions and of different grades of society, he was able to separate what was merely individual and relative in these opinions from what was unchanging and absolute.

Plato, the disciple of Socrates, continued the work of his master. He did not confine his attention wholly to the moral conceptions, but showed that the Socratic method could also be used to refute the intellectual scepticism of the Sophists. In other words, he proved that in the concept, or thought, as opposed to sensation, a standard of truth is to be found, as well as a standard of morality. Knowledge arises from thinking, and it is possible to compare our thoughts, and thus to reach what is objective and real in itself, however impossible it may be to find any basis of comparison in our sensations. In Plato's *Dialogues* a great many logical questions come up for discussion, and in these discussions we can often see some of the fundamental distinctions of present day thought and language, as it were, in the making. But Plato made no attempt to organize and arrange these results into a single science.

§ 6. **Aristotle and the Syllogism.** — This work of organization was accomplished by Plato's disciple, Aristotle. He undertook a thorough investigation of the process of reasoning, and sought to show what conditions and principles are necessarily involved in reaching certainty. Aristotle was thus the founder of logic, as well as of psychology, zoölogy, and most of the other sciences which have come down to us from the ancient world. His most important logical works are the *Categories*, *De Interpretatione*, *Prior Analytics*, *Posterior Analytics*, *Topics*, and the *Sophistical Elenchus*, a treatise on Fallacies. These writings came afterwards to be known as the *Organon* (or scientific instrument) of Aristotle. They contained, in the first place, what we call theory of knowledge (a discussion of the structure of knowledge, and of the scientific principles upon which it rests), which formed

an essential part of Aristotle's philosophical system. But they also furnished the practical application of these principles. In his doctrine of the syllogism, which is found mainly in the *Prior Analytics*, he showed what are the only valid forms of reasoning from general propositions, and thus furnished the pattern or type to which all such proofs must conform. He also classified, in his work on Fallacies, the various species of false reasoning, and showed how false arguments could be refuted and exposed by the principles which he had discovered. The form to which Aristotle maintained that all true reasoning can be reduced was as follows: —

> All men are mortal,
> Socrates is a man,
> Therefore Socrates is mortal.

This is called a **Syllogism**, and it is made up of three propositions. The first two propositions are called **Premises**, and the last the **Conclusion**. All reasoning from premises, all proof, can be reduced to this form. Of course, the propositions which make up the syllogism do not always stand in this order, and sometimes one of them may be omitted. Thus in the argument: 'he ought to be supported by the state, for he is an old soldier,' the conclusion stands first, and one premise is wanting entirely. It is easy to see, however, that the real argument when properly arranged is equivalent to this: —

> All old soldiers ought to be supported by the state,
> He is an old soldier,
> Therefore he ought to be supported by the state.

Now the part of Aristotle's logic which was best worked out was a theory of proof or demonstration by means

of the syllogism. Here he showed clearly the various ways in which different kinds of propositions could be combined as premises to yield valid conclusions, and proved that no conclusion could be drawn from other combinations. This part of the Aristotelian logic has come down to us almost unchanged, and is the subject of Part I. of the present volume.

It will be noticed that, in the doctrine of the syllogism, Aristotle was dealing with that kind of reasoning which undertakes to *demonstrate* the truth of some fact, by showing its relation to a general principle which every one admits. In other words, this part of his work may be called the logic of proof or demonstration. Aristotle was at one time of his life a teacher of rhetoric, and he seemed always to have aimed at putting this art of reasoning on a scientific basis. That is, for the rules of thumb and questionable artifices of the Sophists, he wished to substitute general laws and methods of procedure which were based upon a study of the principles and operations of reason. By complying with the rules which he laid down, an argument will necessarily gain the assent of every rational being.

But we do not employ our reason merely in order to demonstrate to ourselves or to others what we already know. We seek to discover new facts and truths by its aid. In other words, we not only wish to prove what is already known, but also to discover new facts, and we need a logic of Discovery, as well as a logic of Proof. This distinction between proof and discovery corresponds in general to that between Deduction and Induction. It is not an absolute distinction, as will appear later, for both processes are constantly employed in conjunction. But, for the present, it may be said that deduction is the process of show-

ing how particular facts follow from some general principle which everybody admits, while Induction shows the methods by which general laws are obtained from an observation of particular facts. Now Aristotle, as we have seen, furnished a very complete theory of Deduction, or method of proof. But he did not treat of Induction, or the method of passing from particular facts to general laws, with anything like the same completeness. Moreover, what he did write on this subject received no attention for many centuries. Aristotle was himself a great scientific observer, and may well be regarded as the father of many of our modern sciences. But, in his logical writings, his main object seems to have been to present a true theory of argumentation, as opposed to the false theories of the Sophists. Science, too, was only in its beginning when Aristotle wrote, and it was impossible for him to foretell the methods of discovery which it has actually employed.

After Aristotle's death (322 B.C.), and after the loss of Athenian independence, there was a great decline of interest in matters of mere theory which had no direct application to the practical affairs of life. The Stoic school did make some slight additions to logical theory, but like their opponents, the Epicureans, they regarded practice, the art of living well, as the supreme wisdom of life. The Romans, who derived their knowledge of Greek philosophy largely from the Stoics, were also interested in the practical advantages of logic, rather than in its theoretical side. It was the possibility of applying the laws of logic to rhetoric and public speaking which especially interested Cicero, who was the first to make Latin paraphrases and adaptations of Greek logic in his rhetorical works.

For more than seven hundred years, during the Middle Ages, the Greek language and literature was almost unknown in Western Europe. During this time, almost the only sources of information regarding logic were Latin translations of Aristotle's *Categories*, and of an Introduction to the same work by Porphyry, who lived 232–303 A.D. Both of these translations were made by Boethius (470–525), who is best known as the author of *The Consolations of Philosophy*. Even when scholars again became acquainted with the original works of Aristotle, in the latter part of the Middle Ages, they did not really understand their true significance. They took the husk, one may say, and neglected the kernel. They adopted the Aristotelian logic as an external and arbitrary set of rules for the guidance of thinking, and neglected entirely the scientific theory upon which these rules were based. A great deal of ingenuity was also shown in subdividing and analyzing all possible kinds of argument, and giving the particular rule for each case. This process of making distinctions was carried so far that scholastic logic became extremely cumbersome and artificial. Its pretensions, however, rapidly increased; it claimed to furnish a complete instrument of knowledge, and a sure standard for discriminating between truth and falsehood.

It is not very difficult to understand why this set of logical rules seemed so satisfactory to the age of Scholasticism. The men of this period were not greatly interested in new discoveries; they supposed that they were already in possession of everything which was worth knowing. Their only object was to weave this knowledge into a system, to show the connection and interdependence of all its parts, and thus to put it beyond the possibility of attack. And for this purpose the school logic was admirably

adapted; it was always possible to bring every case which could arise under one or other of its rules.

There is no doubt that the Aristotelian logic had a real value of its own, and that it exercised a very important influence upon Western civilization, even in the form in which it was taught by the Schoolmen; but there is, of course, nothing complete or final about it. Its main purpose, as we have already seen, was to furnish a method by means of which the knowledge we already possess may be so arranged as to be absolutely convincing. But the centre of intellectual interest has changed since mediæval times. We are not content merely to exhibit the certainty and demonstrative character of the knowledge which we already have, but we feel that there is a great deal of importance still to be discovered. So that, in modern times, one may say the desire to make discoveries, and so add to the general stock of knowledge, has taken the place of the mediæval ideal of showing that the traditional doctrines taught by the church are absolutely certain and convincing. And when men became conscious of the importance of gaining new knowledge, and especially knowledge about nature, they at once saw the necessity for a new logic, or doctrine of method, to aid them in the undertaking.

§ 7. **Bacon and the Inductive Method.** — All the great thinkers of the sixteenth and seventeenth centuries saw clearly that the school logic is simply a method of showing the certainty of the knowledge we already possess, and does not aid us at all in making new discoveries. A new method, they all declared, was an absolute necessity. The new point of view was put most clearly and eloquently by the famous Francis Bacon (1561–1626), at one time Lord Chancellor of England. Bacon called his work on logic the *Novum Organum*, thus contrasting it with the

Organon, or logical treatises of Aristotle. An alternative title of the work is, *True Suggestions for the Interpretation of Nature.* Bacon begins this work by showing the advantages to be gained from a knowledge of nature. It is man's true business, he tells us, to be the minister and interpreter of nature, for it is only by becoming acquainted with the laws of nature that we are ever able to take advantage of them for our own ends. "Knowledge and human power are synonymous, since ignorance of the cause prevents us from taking advantage of the effect." The discovery of the laws of nature, which is therefore of so great practical importance, cannot be left to chance, but must be guided by a scientific method. And it is such a method which Bacon endeavours to supply in the *Novum Organum.*

The method which Bacon proposed seems to us very simple. If we would gain new knowledge regarding nature, he says, and regarding natural laws, we must go to nature herself and observe her ways of acting. Facts about nature cannot be discovered from logical propositions, or from syllogisms; if we would know the law of any class of phenomena, we must observe the particular facts carefully and systematically. It will often be necessary, also, to put pointed questions to nature by such experiments as will force her to give us the information we want. Knowledge, then, must begin with observation of particular facts; and only after we have made a great number of particular observations, and have carefully classified and arranged them, taking account of all the negative cases, are we able to discover in them the general law. No hypotheses or guesses are to be made; but we must wait until the

tabulations of the particular phenomena reveal the general 'form' or principle which belongs to them all.

It will be frequently necessary to refer to Bacon's work in what follows. At present, it is sufficient to note that Bacon showed that a knowledge of nature cannot be attained through general propositions and logical arguments, but that it is necessary to begin with the observation of particular facts. He emphasized, also, the importance of systematic observation and carefully planned experiments, and showed that knowledge must begin with facts of perception. This is the method of induction, and Bacon is usually said to have been the founder of the inductive sciences of nature.

Another and quite different method of extending knowledge was proposed by the great Frenchman, Descartes (1596–1650), who took mathematics as the type to which all knowledge should conform. That is, he supposed that the true method of extending knowledge was to begin with general principles, whose truth could not be doubted, and to reason from them to the necessary character of particular facts. Descartes and his followers thought that it was possible to discover certain universal propositions from which all truth could be derived through reason. They thus emphasized Deduction rather than Induction, and reasoning rather than observation and experiment. The spirit of Bacon's teaching was, however, continued in England by John Locke, in the *Essay Concerning Human Understanding* (1690). During the next centuries, philosophical thinkers were divided into two great schools: Rationalists, or those who agreed in the main with Descartes; and Empiricists, or Sensationalists, who followed the teachings of Bacon and Locke.

Although the natural sciences made great advances during the seventeenth and eighteenth centuries, there seems to have been no effort made to analyze and describe the methods which were actually being employed. In England, at least, it seems to have been assumed that all discoveries were made by the use of the rules and methods of Bacon. One of the first writers to attempt to explain the method used by the natural sciences was Sir John Herschel (1792–1871). His work, *Discourse on the Study of Natural Philosophy*, was published in 1832. A little later, and with the same object in view, William Whewell (1794–1866), afterwards Master of Trinity College, Cambridge, undertook his *History of the Inductive Sciences*, which was followed some time after by the *Philosophy of the Inductive Sciences*. The man, however, who did most towards putting the study of logic on a new basis was John Stuart Mill (1806–1873), the first edition of whose *Logic* appeared in 1843. We shall have frequent occasion to refer to this work in future discussions. It is sufficient to say here that Mill continues the empirical tradition of the earlier English writers in his general philosophical position. Mill's book gave a great impulse to the study of logic. Before it was published, writers on the subject had confined their attention almost exclusively to the syllogistic or deductive reasoning. Mill, however, emphasized strongly the importance of induction; indeed, he regarded induction as the only means of arriving at new truth, the syllogism being merely a means of systematizing and arranging what we already know. Though few logicians of the present day adopt this extreme view, the importance of inductive methods of reasoning, and the necessity of studying them, have

now become generally recognized. Most modern writers on logic devote a considerable amount of attention to induction. The reader will find that Part II. of the present volume deals with this subject.

§ 8. **Logic from the Evolutionary Standpoint.**—There is still another side of logic which has been developed in the English-speaking world since the time of Mill, though it is a direct continuation of the movement started in Germany by Kant more than a hundred years ago. The so-called 'modern' logic has laid aside the formalism and paradoxical mode of expression adopted by Hegel, but the fundamental conception with which it works—that of development—is essentially the same as that employed by the latter in his *Wissenschaft der Logik* (1816–1818). It is, of course, true that the work of Darwin in biology and the rapid extension of the evolutionary method tended to make the older idea of development more concrete and render it more attractive. Moreover, evolutionary studies, particularly in psychology and anthropology, have contributed directly to genetic logic. For logic, from this standpoint, seeks to describe and explain intelligence in terms of its own development. It looks at the logical mind as a system of functions or activities that have a work to do and that progressively develop in the capacity to perform that work.

The Aristotelian doctrine of the syllogism is a purely formal science. In the form in which it is represented in ordinary text-books, it might perhaps be more properly described as the art of arranging our knowledge in such a way as to compel assent. The 'matter' with which thought is supposed to work is supplied to it in form of

concepts and judgments. The problem which formal logic has to solve is to define and classify the various kinds of concepts with which thought operates, and to determine the various relations in which these stand when combined into judgments. Similarly, it has to show what combinations of judgments can be employed as premises leading to valid conclusions in the syllogism. The criterion of truth employed in these investigations is the principle of non-contradiction or *consistency*. Inconsistent combinations of concepts, that is, are ruled out; but so far as the doctrine of the syllogism goes, anything is true which is not self-contradictory.

Now, without questioning the practical value of its canons, it is obvious that formal or syllogistic logic does not take any account of many of the processes of everyday thought, and that its rules go but a little way in helping us to distinguish the true from the false. For, in the first place, to think is not merely to combine and arrange ideas already in our possession. This might enable us to render clearer and more definite what we already know, but would never enable us to gain new knowledge. The real movement of thought — as opposed to its merely formal procedure — consists in the formation of new ideas and new knowledge through actual contact with the world of experience. A complete account of the intellectual process, then, must deal with the relation of the mind to objects; it must investigate the various activities by means of which thought interprets the world and builds up the various sciences of nature and of man.

The recognition of the importance of induction, and of the necessity of studying the methods of the inductive

sciences, which was brought about by Whewell, Mill, and others, was a step in the right direction, for it called attention to a kind of thinking which occupies a large place in our intellectual life, and also gave rise to a truer conception of the nature of thought itself. But even Mill did not reach the idea which guides modern logicians, namely, that thought or intelligence, as the function of interpreting reality, is one from beginning to end; and that the various logical operations are all parts of one whole, or rather, are ways in which intelligence operates in different circumstances, or at different stages of its development. He still tended to treat of logical processes, like conception, judgment, and reasoning, as if they were separate and distinct processes, each existing, as it were, on its own account. In short, we may say that Mill was still influenced by an atomistic and static view of mind: he does not think of knowledge as essentially all of a piece, or of its movement or history as that which reveals its nature.

As opposed to the conception of mind as made up of separate ideas, the thought by which modern logic is dominated is that of the unity and continuity of all intellectual life. Thought is regarded as an organic, living function or activity, which remains identical with itself throughout all its developing forms and phases. The problem, accordingly, which logic must set before itself is to show the unity and interrelation of all of the intellectual processes. No one of the steps or stages in this process can be completely understood when viewed by itself: each is what it is only in and through its connection with the whole of which it forms a part. No hard-and-fast boundary lines are to be drawn between the different stages of the reasoning process, but

it must be shown that the whole nature of intelligence is involved more or less explicitly at each step. So far only the broad outlines of this theory have been filled in; but the conception of an organism whose parts are developing in mutual relation and interdependence promises to be as fruitful when applied to logic as it has already shown itself to be in the other sciences.

Besides the ordinary histories of philosophy the reader may consult for the history of logic: Prantl, *Geschichte der Logik im Abendlande*, 4 vols., Leipsic, 1855–1870; which extends, however, only to the close of the mediæval period. Harms, *Geschichte der Logik*, Berlin, 1881. Ueberweg, *System der Logik*, 4th ed., 1874; Eng. trans. of 3d ed., London, 1874. Adamson, article 'Logic,' in the *Encyl. Brit.*, 9th ed. Sir William Hamilton's *Lectures on Logic*, also containing much historical information.

Among modern works on logic, the following may be mentioned: J. S. Mill, *A System of Logic*, London, 1st ed., 1843; 9th ed., 1875. W. S. Jevons, *The Principles of Science*, London, 1874; 2d ed., 1877. Also by the same author, *Studies in Deductive Logic*, 1880; and *Pure Logic*, 1890. H. Lotze, *Logik*, 1874; Eng. trans., London, 1881 and 1888. W. Wundt, *Logik*, 3d ed., 1906–1907. C. Sigwart, *Logik*, 2d ed., 1889–1893; Eng. trans., London and New York, 1895.

The newer development of logic is well represented by F. H. Bradley, *The Principles of Logic*, London, 1886. B. Bosanquet, *Logic, or the Morphology of Knowledge*, London, 1888; and *The Essentials of Logic*, London and New York, 1895. L. T. Hobhouse, *The Theory of Knowledge*, London, 1896, may also be mentioned in the same group of writers, although he has been, perhaps, more influenced by Mill than by any other writer. J. M. Baldwin, *Thought and Things, or Genetic Logic*, New York, 1906–1907, has emphasized especially the genetic processes through which logical thinking is built up.

PART I. — THE SYLLOGISM

CHAPTER III

THE SYLLOGISM AND ITS PARTS

§ 9. **The Nature of the Syllogism.** — The theory of the syllogism, as has been already stated (§ 5), was first worked out by Aristotle. And it stands to-day in almost the same form in which he left it. A few additions have been made at different points, but these do not affect materially the main doctrine. In dealing with the nature of the syllogism, we shall first try to understand its general aim and purpose, or the results which it seeks to bring about. We shall then have to analyze it into the parts of which it is composed, and to examine and classify the nature of these elements. Finally, it will be necessary to discover what rules must be observed in order to obtain valid conclusions, and to point out the conditions which most commonly give rise to error or fallacy.

In the first place, it is to be noticed that syllogistic logic deals with the results of thinking, rather than with the nature of the thought-process. Its object is less to give an account of the way in which thinking goes on, than to show how the ideas and thoughts which we already possess may be combined, so as to lead to conclusions which are certain, and which will compel assent. The ideas which the syllogism uses as material are fixed by having been

expressed in language. Indeed, it is largely with words, as the expression of thoughts, that syllogistic logic deals. Many of the discussions with which it is occupied have reference to the proper interpretation of words and propositions; and the rules which it furnishes may be taken as directions for putting together propositions in such a way as to lead to a valid conclusion. Nevertheless, it is important to remember that these rules are not arbitrary and external, but find their justification in the nature of thought. Indeed, the theory of the syllogism, when rightly understood, may be said to reveal the fundamental characteristics of the process of intelligence. For it brings together facts in such a way as to make evident their interrelation and dependence. It connects a judgment with the grounds or reasons which support it, and is thus a process of systematization. In order to understand the significance of the rules of syllogistic logic, then, it will generally be necessary to look beyond words and propositions to the act of thought whose results they express.

A great deal has been written regarding the principles or Laws of Thought, which are employed in all logical reasoning. It seems better, however, to postpone the definite consideration of this subject until the student has learned more about the various operations of thought, and has had some practice in working examples. In dealing with the nature and principles of thought, in the third part of this book, it will be necessary to discuss this question at length. Even at the present stage of our inquiry, however, it is important to notice that syllogistic reasoning presupposes certain simple and fundamental principles of thought as the basis of its valid procedure. In particular, the regular

syllogism is founded on a principle which we may call the law of Identity, or the law of Contradiction, according as it is stated affirmatively or negatively. Stated affirmatively, this so-called 'law' simply expresses the fact that every term and idea which we use in our reasonings must remain what it is. A is A, or has the same value and meaning wherever employed. The law of Contradiction expresses the same thing in negative language. A cannot be both B and not B. If any term is taken to be the same as another in one connection, it must always be taken to be so; if it is different, this relation must everywhere be maintained. The data or materials which are employed in the syllogism are ideas whose meanings are supposed to be permanently fixed and expressed in words which have been carefully defined. It would be impossible to reason, or to determine the relation of our ideas, if their meaning were to change without notice, or if the words by means of which they are expressed were used now in one sense and now in another. It is true, of course, that our ideas regarding the nature of things change from time to time. And, as is evident from one's own experience, as well as from the history of language, a corresponding change takes place in the meaning of words. But the assumption upon which syllogistic reasoning proceeds is that the ideas which are to be compared are fixed for the meantime, and that the words by which they are expressed are used in the same sense throughout the course of the argument. The laws of Identity and Contradiction are, then, simply the expression, in positive and negative form respectively, *of the principle of consistency*. The one fundamental postulate of all thought is that it must be consistent with itself.

We may, however, have formal consistency without having real truth. It is quite possible that all the requirements of the syllogism may be met without its conclusions being true of reality. In other words, an argument may be *formally* true, but really false. It is not difficult to understand why this may happen. The syllogism accepts without criticism the ideas and judgments which it compares. These data are, of course, the product of previous acts of thinking. But in proceeding to arrange them in syllogistic form, we do not inquire whether or not they are true, *i.e.* adequate to express the nature of the things for which they stand. For the formal purposes of the syllogism it is only essential that their meanings be clearly understood, and that these meanings be regarded as fixed and permanent.

§ 10. **The Parts of a Syllogism.** — The syllogism may be said to express a single comprehensive act of thought. We may define the reasoning expressed in a syllogism as a judgment which has been expanded so as to exhibit the reasons by which it is supported. In the syllogism,

> The geranium has five pointed sepals,
> This plant has not five pointed sepals,
> Therefore it is not a geranium,

we may say that we have the judgment, 'this plant is not a geranium,' supported by the propositions which precede it, and that the whole syllogism taken together expresses a single thought, which is complete and self-sufficient. It is possible, however, even when one is dealing directly with the process of thinking, to distinguish in it different subordinate steps, various stages which serve as resting-

places, in the course of its passage to the complete and comprehensive form represented by the syllogism. But it is usual, in dealing with the syllogism, to take a more external view of its nature, and to regard it primarily as made up of words and propositions.

In this sense, a syllogism can, of course, be divided into parts. In the first place, it is composed of three statements, or propositions. In the example given above the two propositions which stand first are called the **Premises,** since they furnish the grounds or reasons for the proposition which stands last, and which is known as the **Conclusion.** However, it is not true that we always find the two premises and the conclusion arranged in this regular order in syllogistic arguments. Oftentimes the conclusion is given first. Frequently, too, one of the premises is not expressed, and has to be supplied in order to complete the argument. Thus the statement, 'he must be more than sixteen years of age, for he attends the university,' is an incomplete syllogism. The conclusion, as will be readily seen, stands first. There is also only one premise expressed. To put this statement in the regular syllogistic form we have to supply the missing premise and arrange it as follows:—

All students of the university are more than sixteen years of age,
He is a student of the university,
Therefore he is more than sixteen years of age.

When one of the premises or the conclusion is not expressed, the argument is called an **Enthymeme.** Such an argument is defective only in form: the missing premise or conclusion is really present and operative in thought.

It is of great importance to form the habit of making clear to oneself the premises by which any conclusion claims to be supported. In this way groundless assumptions are often brought to light, and the weakness of an argument exposed. Whenever words like 'therefore,' 'for,' 'because,' 'it follows,' etc., are used in their proper signification, it is possible to find an argument composed of two premises and a conclusion. But one must not allow oneself to be imposed upon by the mere words, but must insist on understanding exactly what are the premises in the case, and how the conclusion follows from them. Not only may some part of the argument be taken for granted, as a kind of tacit agreement, but oftentimes, in arguments as actually used, there is a considerable amount of repetition and illustration of the principles employed, without any attempt to bring these various statements into relation in a formal way as premises of a syllogism. To reduce such arguments to syllogistic form requires, accordingly, a certain amount of interpretation of the statements they contain, involving oftentimes both condensation and rearrangement. Such reduction of the usual extended form of arguments is usually necessary in order to bring out clearly their essential structure — the premises which are actually employed to carry the conclusion — and to estimate accurately their logical force and value. Take, for example, the following passage from Jonathan Edwards: —

Why should we be afraid to let persons who are in an infinitely miserable condition know the truth, or bring them into the light for fear it should terrify them? It is light that must convert them if they are ever to be converted. The ease, peace, and comfort which natural men enjoy have their foundation in

darkness and blindness ; therefore as that darkness vanishes and light comes in their peace vanishes and they are terrified. But that is no good argument why we should endeavor to hold their darkness that we may uphold their comfort.

This may be reduced to the form of two syllogisms somewhat as follows: —

(1)

The terror of sinners is what dispels their blindness,
Light is a terror to sinners,
Therefore light is what dispels their blindness.

(2)

What dispels blindness is really a benefit to sinners,
Light is what dispels their blindness,
Therefore light is a real benefit to sinners.

It is necessary to carry the division of a syllogism still farther. Every logical proposition may be divided into two **Terms**, and a **Copula** or connecting link. The terms, which are the extremes of the proposition, are named the subject and the predicate. Thus in the proposition, 'the fields are covered with snow,' 'the fields' is the subject, 'are,' the copula, and 'covered with snow,' the predicate. To reduce a proposition to the logical form in which it is most conveniently treated, it is necessary to express it in such a way that the two terms are united by some part of the verb 'to be,' preferably 'is' or 'are.' Thus the sentence, 'No plant can grow without light and heat,' would be expressed as a logical proposition in the following, or some similar, form: 'No plant is an organism which can grow without light and heat.' 'Men have strong passions' may be written, 'Men are beings having strong passions.' It is always well to reduce a sentence to some such form, by

substituting for the verb of predication some part of the verb 'to be.'

The analysis of the syllogism gives us the divisions under which it is convenient to treat this part of logic. We shall accordingly deal (1) with Terms, (2) with Propositions, and (3) with the Syllogism as a whole.

These divisions, however, are only made for the sake of convenience in treatment. It must not be forgotten that a term is a part of a proposition. To understand the nature of a term, it is necessary to consider the part which it plays in the judgment which the proposition expresses. In other words, the *function* of the term, rather than the form of the word or words employed, must be considered. It is, of course, true that we naturally and commonly use certain word forms to express certain kinds of ideas, just as in the grammatical sentence the different 'parts of speech'—nouns, verbs, etc.—have each a definite and comparatively permanent function. But even in the sentence it is the part which the word in its grammatical function plays, rather than its form, which determines whether it is to be classified as a noun or an adjective, a
preposition or a conjunction. In dealing separate through
terms, as we propose tc do in the next ch the particular
be occupied to a large extent with the f. A concept, on the
certain kinds of ideas are usua or idea. It does not refer
same word or group of wordense. It is not an individual
purposes, it will be necessaring, but is a thought-construc-
meaning of terms, to refer a of a general nature or mean-
in which they are used in a number of individuals. Thus,
The same difficulty exi individual tree at which I am
sidered by themselves, the neral idea of tree which I use

of which they form a part being thus ignored. In this case, however, the results of the isolation are not so apparent; for a proposition forms, in a certain sense, a whole by itself. It is the expression of a judgment which, as we shall see later, is the unitary process of thought. It has thus a significance of its own, as expressing a more or less complete and independent act of thought. Nevertheless, it must not be forgotten that its independence and completeness are only partial and relative. To interpret a proposition correctly and fully we must know its context. In order to make it intelligible, it must be brought into relation with the other propositions which state the grounds or reasons upon which it rests, or the conclusion which it helps to support. The logical meaning of a proposition, therefore, depends upon its function in an argument, and in treating of propositions this fact must not be forgotten. To understand, is to appreciate the context.

§ 11. **Perception, Conception, and Judgment.** — Before beginning our examination of the elements of the syllogism, it is necessary to define some terms that describe certain phases or modes of our knowledge. These are Perception,
To reduce.. and Judgment. Judgment is both the ele-
most convenient.. universal form of knowing. It includes
such a way that the t.. them as a means to its own end of
the verb 'to be,' preferably '..rhaps, be best described as the
'No plant can grow withou mind. At all the stages of
expressed as a logical proposit..uing things in terms of ideas
similar, form: 'No plant is an ideas in the light of new
without light and heat.' 'Men lore definite and more con-
be written, 'Men are beings haorm of the general intellec-
is always well to reduce a senteig is to express it in terms of

ideas, to qualify it in our thought as this or that, as belonging to a certain class of things, or perhaps as differing in some respect from another class of things. But it must not be supposed that judgment — or any form of thinking — is concerned only with *our own ideas*. Judgment is the interpreting, idealizing response of the mind to the real world, with which it is always in relation. To think is not to play with our own ideas: real thinking deals, more or less directly, with a world of real objects and persons. In the process of judgment, then, reality is interpreted and its meaning expressed in terms of ideas. The expression of such an act of thought is a proposition, which, as we have already seen, is composed of a subject and a predicate term related by means of a copula.

Now the terms of which a proposition is composed may be either Percepts or Concepts, *i.e.* the result of a perceptive act or of a conception. A percept is the result of the mind's direct mode of apprehending real things as distinct individuals. Hence a percept always refers to 'this' or 'that,' some distinct individual thing having its own place in space or in time. Thus, I perceive, or have a percept of, the objects in this room, and of the tree which I see through the window. Similarly, one may perceive the particular states of consciousness in one's mind. A concept, on the other hand, is a general meaning or idea. It does not refer directly to some one object of sense. It is not an individual embodiment of a particular thing, but is a thought-construction, carrying with it the idea of a general nature or meaning which may apply to a number of individuals. Thus, my direct experience of the individual tree at which I am looking is a percept, the general idea of tree which I use

when I say 'trees are either deciduous or evergreens' is a concept. I may have a percept of the statue of Liberty at the entrance to New York harbour; 'liberty,' on the other hand, is a concept made up of a more or less definite group of meanings, which are unified and held together by the word in which it is expressed.

What, now, is the relation between the percepts and concepts which are expressed in the terms of a proposition, and the judgment which is represented by the proposition as a whole? In the first place, it is to be noted that percepts and concepts are the results of previous acts of judgment. Ideas are formed only through the mind's own act of interpretation; they never pass over into the mind from some external source as ready-made objects. Even in the case of perception, where the object seems to be thrust upon us, a little reflection will show that the judging activity of attention is involved, selecting and arranging the various sensation elements, and interpreting them as the parts of a single concrete object, in accordance with past experience. A concept like 'man' or 'justice' is still more obviously a thought or judgment construction. As expressed in words, it may be said to be an embodiment of a judgment or a group of judgments.

And, in the second place, it is from these percepts and concepts that new judgments proceed. In other words, the basis of our thought in going on to the discovery of new facts and relations is *what we already know.* But what we already know at any time is summed up in the ideas we possess, that is, in the percepts and concepts which have been formed by previous acts of judgment, and embodied in names. In the development of our knowledge,

however, we are constantly discovering that our knowledge on this or that point is unsatisfactory. The old way of thinking is perhaps too vague and indefinite to furnish us with a satisfactory rule of action, or it may be perceived to be inconsistent with new facts that have arrested our attention. Indeed, the inadequacy of the habitual, accepted point of view may be forced upon us in a variety of ways. Frequently, no doubt, the occasion is furnished by some practical necessity of action. Necessity is oftentimes the mother of invention, and the spur to the discovery of new theories and conceptions. In other cases the stimulus to criticise our old conceptions may come from social intercourse; the conflict of our views with those of people with whom we converse, or whose opinions we read, first arouses us from our dogmatic slumber. More rarely, perhaps, in the case of ordinary minds, theoretical interest may be aroused without any external occasion, and the desire for truth and consistency may itself be sufficient to lead one to reëxamine and transform one's old ideas. Whatever the stimulus, thinking is, on one side, a process in which old conceptions are recast, and accepted truths transformed, a constant process of change in which the old conceptions are superseded and destroyed. The old terms, both percepts and concepts, which form the starting-point are reconstituted through a new act of judgment. From one point of view, then, it may be said that, like Kronos, thought exists by devouring its own children.

But there is another side. Thinking is a process of conservation as well as of transformation. The old ideas are not so much destroyed and displaced by the new judgment, as further developed and defined. The partial truth

which the old formulas contain is taken up and preserved in the later judgment or series of judgments. Moreover, as we have seen, the results of these judgments are again laid down as new thought-contents embodied in language, and these, in their turn, form the starting-point for further judgments. These two aspects or moments of thought, then, — what we have called the transforming and the conserving functions, — mutually presuppose and imply each other. They are not distinct and independent mental operations, but organically related moments or phases in the life of thought. Perceptions and conceptions can arise only through judgments, while judgments presuppose perceptions and conceptions as their necessary basis and starting-point. Thus the total movement of the whole thought-process is rightly described as Judgment, since the growing insight of mind is its beginning and end.

CHAPTER IV

THE VARIOUS KINDS OF TERMS

§ 12. **Singular, General, and Collective Terms.**—A logical term, as we have already seen, is any word or group of words which can be used as the subject or predicate of a proposition. It is only in propositions, and as elements of propositions, that terms have any assignable meaning. It will be impossible, therefore, to fix the meanings of isolated terms without reference to the way in which they are used in propositions. In dealing with terms apart from propositions, we shall be concerned mainly with different *classes of words* and the meanings which they usually express.

The first division which we have to notice is that into Singular or Individual, General, and Collective terms.

(1) A **Singular** or **Individual** term is one which can be applied in the same sense to but a single thing. The main purpose of Singular terms is to refer to, or identify, some thing or experience which can be regarded as a single existence. Proper names are all singular. It is true that proper names are sometimes used to denote a class of objects, as *e.g.*, 'a Daniel,' 'a Mephistopheles.' But, when thus employed, they lose their real character as proper names. That is, their function is no longer merely to identify certain individuals by *naming* them, but to *describe* them by mentioning certain qualities or characteristics which they are supposed to possess. But the ordinary purpose in using a proper

name is to indicate some individual to whom the name belongs. In this sense, then, proper names are Singular.

In addition, any word or group of words which is applied to a single thing may be regarded as singular. And by 'single thing,' we mean anything which is thought of as one, as well as objects which are perceived through the senses. Thus, 'the waterfall just below the bridge,' 'the thought of the present moment,' are singular terms, and so, also, are words like 'justice,' 'goodness,' 'the chief end of man.' It is perhaps more doubtful whether we should call terms such as 'whiteness,' 'sweetness,' singular, since we speak of different degrees and kinds of whiteness and sweetness. The question would have to be decided in every case by reference to the way in which the terms are employed in propositions.

(2) A **General** term is a name which is capable of being applied to a whole group of objects. It is not limited, like the singular term, to a single thing, but can be used in the same sense of an indefinite number of units. All class names, like 'metal,' 'man,' 'works on logic,' are of this character. Thus a general name is one that refers to a group which may be divided into smaller groups, or into individual units. Thus iron, gold, silver, etc., are 'metals,' and A, B, and C, 'men.'

A **Collective** term, on the other hand, is a name applied to a number of individual things when taken together and treated as a whole, as 'an army,' 'an audience.' It is important to distinguish carefully between general and collective terms. A general term is a name which applies equally to each individual of the group; or, in other words, it is used of the individuals *distributively*. A collective

name belongs to the whole, but not to the separate parts of the whole. Thus we say that 'soldier' is a general name, and is used distributively of each man in a regiment. 'Regiment,' however, is a collective name, for it applies only to the whole group, and not to the individual soldiers.

Ambiguity sometimes arises from the fact that the English word 'all' is used in both of these senses. That is, it may mean 'all taken together' or 'each and every.' Thus we can say: 'All the angles of a triangle are less than two right angles,' and 'All the angles of a triangle are equal to two right angles.' In the former sentence, the word 'all' is used distributively, in the latter collectively. In Latin two different words are used: *cuncti* expresses the collective sense of 'all,' and *omnes* its distributive signification.

It is worth noticing in this connection that it is the *use* which is made of terms, rather than the *form* of the words composing them, which determines their logical character. Thus terms which are collective in one connection may be general in another. 'Regiment,' for example, is a collective term with reference to the soldiers which compose it, but general when used as a common term for a number of similar divisions of an army. The same is also true of terms like 'grove,' 'mob,' 'class,' etc. Again, collective terms may be very properly regarded as singular when the proposition in which they are used emphasizes the unity and solidarity of the group. A proper name is sometimes applied to a collection of individuals that are permanently united or that have acted together on some historic occasion, as, for example, 'The Fifth Cavalry Regiment,' 'The Charge of the Six Hundred.'

§ 13. **Abstract and Concrete Terms.** — Terms are further divided into *abstract* and *concrete* terms. The word 'abstract' is often used popularly to describe anything

which is difficult to understand. Etymologically, it signifies drawn off, separated (*abstraho*, to draw off, take away). We may distinguish two senses in which the word is used, both, however, being derived from its etymological signification.

(1) A term is called *abstract* when it refers to some thing which cannot be directly perceived through the senses, or otherwise directly experienced as an individual object or state, and *concrete* when such form of experience is possible. Thus 'a beech tree,' 'a tall man,' 'a sweet taste,' being names of things which can be perceived, are concrete. Words like 'sweetness,' 'hardness,' etc., have no objects of immediate experience corresponding to them, and are for this reason called abstract. The same is true of terms like 'individuality,' 'equality,' 'justice,' etc. These words represent objects of thought, rather than objects that are directly experienced. There may be cases or instances of 'equality,' 'justice,' etc., which fall under our perception, but the real object to which these words correspond is not a thing which can be perceived through the senses at all. Their reality is conceptual, or for thought, not something directly revealed through the senses.

It is important to notice that there are degrees of abstractness in terms, according as the objects for which they stand are nearer to, or farther removed from, ordinary sense-perception. All general or class names are abstract. One cannot point to a single object to which the term 'metal,' for example, or the term 'man' corresponds. But although such terms have no direct sensuous object, yet we feel that they stand nearer to sense-perception, and are therefore less abstract than words like 'animal,' 'inorganic substance.' These terms, again, are perhaps less abstract than 'energy,' or 'spirit,'

or even than singular terms like 'justice,' 'the ground of the universe,' etc.

(2) Again, the word 'abstract' is applied to any object which is treated apart from the whole to which it belongs. Thus it would be an abstraction to attempt to explain the nature of a leaf in complete isolation from the plant to which it belongs, or to consider the nature of a man without regard to the social institutions — family, church, state, etc. — of which he is a member. Of course, it is essential when dealing with a complex whole to analyze it into its parts, and to understand just what is the nature of each part when taken by itself. But, in order to comprehend fully the nature of the parts, it is necessary to restore them to their proper setting, and to see their relation to the concrete whole. In this sense of the word, then, 'abstract' applies to what is taken out of its proper setting, broken off, and considered apart from the things to which it is organically related. Concrete, on the other hand, means what is whole and complete, a system of things which mutually support and explain one another.

Since science has to analyze things into their elements, and to investigate and describe these elements in detail, it is impossible entirely to avoid abstraction. But it is necessary, in order completely to understand the nature of a complex object, that the abstractions of analysis shall be corrected. In other words, the concrete relations in which things stand must not be ignored in investigating them. The conception of evolution in recent times has done much to render the biological sciences more concrete in the sense in which we are now using the term. For it has substituted for the old method of treating each species

of plant and animal as distinct and separate, 'cut off from each other as if by a hatchet,' the view that all organic beings are members of one family, and can be properly understood only in their relations to one another (cf. pp. 74–75).

It is interesting to notice that, from this point of view, sense-perception is more abstract than thought. For the senses represent things in isolation from each other. Each thing is known in sense-perception as a separate individual, occupying its own space and time, and, in this way, cut off from its fellows. It is the business of thought, on the other hand, to discover the relations between things, and the principles according to which they are united. Thinking thus overcomes the abstract point of view of sense-perception by showing that what appear to the latter as separate objects are really closely and necessarily connected as members of a common unity or system. Each science takes as its province certain facts which resemble one another, but which nevertheless appear to sense-perception to be quite independent. It attempts by thinking to bring these facts into relation, to show that they are all cases of some law, that there is a common principle which unites them as parts of a whole or system. The law of gravitation, for example, expresses the unity which thought has discovered in things which appear to sense-perception as different as the falling of an apple, the movements of the heavenly bodies, and the ebb and flow of the tides. Scientific knowledge, then, is more concrete than the facts which we learn from ordinary sense-perception, because it brings to light real unity and connection in facts which appear to be entirely isolated and independent from the latter point of view.

In employing the terms 'abstract' and 'concrete' it is of the utmost importance to distinguish the two significations of the words. From one point of view, as we have seen, all thought terms are abstract, as opposed to words

which refer directly to objects of sense-perception. In another sense, 'abstract' denotes what is partial and incomplete, what is taken by itself and out of relation to the system of things to which it belongs. And, since the real connection and relations of things are not given by perception, but have to be discovered by thought, the knowledge which the latter yields is more concrete, in this latter sense of the term, than that afforded by the former.

§ 14. **Positive and Negative Terms.** — The distinction between Positive and Negative terms is very obvious. **Positive** terms express the existence of some quality, or group of qualities, in the objects which they denote; as, *e.g.*, 'happy,' 'good,' 'equality,' 'organism,' etc. A **Negative** term, on the other hand, indicates the absence of qualities or properties in some object; 'bad,' 'unhappy,' 'inorganic,' 'injustice,' for example, are negative terms. Negative terms are often formed from positive by means of the affix *less*, as in 'hopeless,' or by means of certain prefixes, of which the more common are *un*, *in*, *dis*, *a*, *anti*. Words which are positive in form are, however, often negative in meaning, and are used as the contradictories of other terms. Thus 'ignorant' is generally regarded as the negative of 'learned,' 'darkness' is the negative of 'light,' etc. It is not always possible, however, to find a separate word to express the exact opposite of every positive term. Words are used primarily to express the presence of qualities, and the negative idea may not be referred to so frequently as to require a separate word to express it. Thus there is no independent term to express the opposite of 'transferable,' but by employing 'non' as a negative prefix we obtain 'non-transferable.'

It is always advisable when we wish to limit a term strictly to its negative application to employ *not* or *non* as a prefix. Words which are negative in form frequently have a more or less definite positive signification. Jevons points out that words like 'unloosed' and 'invaluable,' though negative in form, have a positive meaning. But, in addition, terms like 'unhappy,' 'immoral,' do not merely indicate the absence of positive qualities, but also express some positive properties of the objects to which they are applied. We speak of a person 'being positively unhappy'; and we employ 'non-moral' to express the simple negative relation rather than 'immoral.'

On the other hand, there are certain terms which are positive in form that express the absence of qualities or attributes. Words like 'blind,' 'dumb,' 'maimed,' orphaned,' may be given as examples. These are often called Privative terms, rather than Negative, the distinction being that they refer to qualities or attributes which the objects to which they are applied naturally and usually have, but of which they have been deprived, or which they have never possessed. Thus 'blind,' as applied to a man, implies that he has lost, or is destitute of, the ability to see which naturally belongs to a human being.

Again, other terms seem to be positive and negative solely in relation to each other. 'Element' and 'compound' are related as negatives or contradictories. It is difficult, however, to say which term is in itself negative or positive.

It is important to notice the distinction between the relation in which positive and negative terms stand to each other, and that expressed by words which have to do with opposite extremes of something which possesses quality or degree. Positive and negative terms are mutually **Contradictory**. An element is what is *not* a compound, 'dishonest' is the contradictory of 'honest,' and as contradictories there is no middle ground between them. What

is not an element is a non-element or a compound. **Contrary** terms, on the other hand, express a great difference of degree in the objects to which they refer. Thus 'foolish' is the opposite of 'wise,' 'cold' the opposite of 'hot,' and 'bitter' of 'sweet.' But there is always the possibility of a middle ground between opposites. We cannot say that a man must be either wise or foolish, a taste either sweet or bitter. The logical contradictory of 'wise' is 'not-wise,' of 'bitter' is 'not-bitter,' etc. Contrary terms, then, must be carefully distinguished from contradictories, and we cannot conclude because one contrary term is false in a given case that the other is necessarily true (cf. § 25).

§ 15. **Absolute and Relative Terms.** — Another classification of terms, which is usually given by logicians, is that into absolute and relative terms. An **Absolute term** is one which refers to an object which exists by itself, and has an intelligible meaning when taken alone. Thus 'tree,' 'house,' 'the State of New York,' are examples of absolute terms. A **Relative term**, on the contrary, is a name which only derives a meaning from its relation to something else. The term 'parent,' for example, cannot be thought of except in relation to 'child.' Similarly, 'teacher' is relative to 'pupil,' and 'cause' to 'effect.' Relative terms usually go in pairs and are known as **Correlatives.** Adjectives, as well as nouns, may be related in this way. The presence of one quality or characteristic in a thing frequently implies the presence of others. Thus, ignorance and superstition, sympathy and tolerance, are necessary correlatives, because the one involves the other, or is invariably connected with it.

It is, of course, true that no finite thing is completely absolute or independent of other things. The nature of each thing is largely

determined by the nature of the other things with which it stands in relation. A tree, for example, is relative to the seed from which it sprang, the soil in which it grew, the sunshine, rain, etc., which accompanied its growth. All finite things have a beginning and an end, and are also influenced throughout the whole period of their lives by the action of other things. They are, therefore, not completely absolute or independent. It is, however, possible to make a distinction between words which are the names of things that are comparatively independent, and may for ordinary purposes be considered by themselves, and those which have only a meaning when regarded as correlatives.

§ 16. **Extension and Intension of Terms.** — In the foregoing sections of this chapter we have explained the main distinctions which concern the various kinds of terms with which logic deals. It is now necessary to notice two different purposes for which terms are employed. In the first place, terms are used to refer to things, to name and identify them. Thus 'man' refers to the different individual men, John Smith, Thomas Brown, etc., as well as to the various classes of men, Caucasians, Indians, Mongolians, etc. As denoting or naming objects, whether these be individual things or classes of things, terms are said to be employed in Extension. But words are also used to *describe* as well as to name. That is, they represent the qualities or attributes belonging to things for which they stand. They are not bare names without signification; but, as the expression of ideas, they stand for certain qualities or characteristics which things are judged to possess. 'Man,' for example, is not merely a name which may be applied to individual human beings or races of men; but it implies that the objects so named have certain qualities, such as animal life, reason,

and the power of communicating with their fellows. When words are used in this way to define or describe things, rather than merely to name them, they are said to be employed in Intension.

The terms 'Denotation' and 'Connotation' were used by Mill instead of Extension and Intension, respectively, and have been adopted pretty generally since his time. To 'denote' is to point out or specify the objects for which a term stands; and to 'connote' is to take account of the attributes or qualities which a name implies. The words 'depth' and 'range' are also sometimes used as synonymous with Extension, and 'breadth' or 'comprehension' instead of Intension. The terms to be remembered, however, are Extension or Denotation, and Intension or Connotation.

It is essential to accustom ourselves to distinguish these two functions or uses of a term, — to notice, that is, the things or classes of things to which the name applies, and also to reflect upon the signification, or ways of judging about these things, for which the name stands. The Extension of a term, as has been said, indicates the objects to which a name applies, and the Intension the qualities or attributes which it signifies. From the point of view of extension, therefore, 'planet' may be defined by mentioning the names of the various planets, Mercury, Venus, the Earth, Mars, etc. Similarly, a term like 'carnivora' might be given in extension by naming seals, bears, weasels, dogs, wolves, cats, lions, etc. Usually, however, we define from the point of view of intension, that is, by stating the qualities or characteristics for which the term stands. Thus we give the intensive meaning of 'planet,' as a heavenly body which revolves in an elliptical orbit around the sun. 'Carnivora,' defined from the same point of view, are mammalian verte-

brates which feed upon flesh. It is not unusual, however, to supplement an intensive definition by turning to extension and enumerating examples. Thus we might add to the definition of 'carnivora' just given the words, 'as lions, tigers, dogs,' etc.

It is sometimes said that the intension and extension of terms vary inversely. This is simply an attempt to give a mathematical form of statement to the fact that the more a term is defined, or limited, by the addition of attributes, the fewer are the objects to which it applies. 'As the intension of a term is increased, its extension is diminished, and *vice versa,*' is the form in which the relation is often stated. For example, let us begin with some class name like 'animal,' which has a great extension, and add a new attribute, 'rational.' We get 'rational animal' = man. This term now applies to a much smaller number of individuals than 'animal.' The extension of the former term has been diminished, that is, by increasing the intension. If we add to 'man' still another attribute like 'white,' we again lessen the number of individuals to which the term applies. In general, then, it can be seen that the extension of a term is lessened as it is made more definite by the addition of new attributes. And, conversely, by stripping off attributes, by 'decreasing the intension,' the number of individuals to which a term applies is increased. There is, however, no exact ratio between the increase or decrease of intension and the corresponding change in extension. Indeed, the extension of a class may increase greatly without any loss of intension on the part of the term by which the idea is expressed. Thus the meaning or intension of the term 'man' has not lost, but rather gained, during the last hundred years by the increase of population throughout the world.

In general, it is only when some kind of a formal classification is instituted, when terms are taken as arranged in order of subordination, that there is any meaning in speaking of their extension and intension as in inverse relation.

Extension and intension, according to the view just given, represent two different uses or functions of terms. Every term denotes some object or group of objects more or less directly, and at the same time connotes or signifies certain qualities or attributes. Sometimes the one purpose, sometimes the other, is the predominant one. Proper names, for example, are used primarily to denote or mark out things, and do not directly qualify or describe them. In the proposition, 'these animals are all vertebrates,' the predicate term 'vertebrates' is employed less as a name of a number of animals than as a description of their qualities. Nevertheless, in both these cases the terms employed have the double function of naming or denoting objects, and of connoting qualities.

Mill, however, and certain other logicians who follow him, seem to make an absolute distinction between connotative and non-connotative terms. "A non-connotative term is one which signifies a subject only, or an attribute only. A connotative term is one which denotes a subject, and implies an attribute. By a subject is here meant anything which possesses attributes. Thus 'John,' or 'London,' or 'England,' are names which signify a subject only. 'Whiteness,' 'length,' 'virtue,' signify an attribute only. None of these names, therefore, are connotative. But 'white,' 'long,' 'virtuous,' are connotative. The word 'white' denotes all white things, as snow, paper, the foam of the sea, etc., and implies or, in the language of the school-

men, *connotes* the attribute *whiteness*. . . . All concrete general names are connotative. The word 'man,' for example, denotes Peter, James, John, and an indefinite number of other individuals, of whom, taken as a class, it is the name. But it is applied to them because they possess, and to signify that they possess, certain attributes."[1]

There is no real ground, I think, for such an absolute distinction between connotative and non-connotative terms, or, as we may call them, descriptive and non-descriptive terms. Of course, it is true that some terms are more directly descriptive than others; but when we consider the use or function of terms, we find that they are never used *merely* to name things, or *merely* to connote attributes, though in certain cases the former purpose is the primary one, and in other cases the latter object is more prominent. Even when proper names are employed, the qualities or characteristics of the objects named are indirectly implied. The very fact that a proper name is given to an object implies that it possesses a certain definitely marked individuality. Moreover, a proper name when used *intelligently* carries with it some still more definite information regarding the qualities of the thing to which it is applied, as, for example, whether it is a name of a person, an animal, or a place. And, on the other hand, every term has an application to real objects, and so a denotation, though this reference to reality is often indirect and somewhat indeterminate. For, without the assumption of this application, no term could be a part of an intelligible proposition or represent a genuine thought. Every term, then, more or less directly, both denotes objects and connotes attributes.

[1] Mill, *Logic*, Bk. I., Ch. II., § 5.

REFERENCES

J. S. Mill, *Logic*, Bk. I., Ch. II.

F. H. Bradley, *The Principles of Logic*, pp. 155–173.

B. Bosanquet, *Logic*, Vol. I., pp. 46–71.

" " *The Essentials of Logic*, Lecture V.

J. N. Keynes, *Studies and Exercises in Formal Logic*, 4th edition, Part I., Chs. I. and II.

CHAPTER V

DEFINITION AND DIVISION

§ 17. **Fixing the Meaning of Terms.** — We have already referred to the necessity of definitely fixing the meaning of the terms which we employ in reasoning. In ordinary life, words are frequently used in a loose and shifting way, without any clear conception of the qualities or properties which they connote, or of the objects to which they apply. Logic demands, in the first place, that we shall have clear and precise ideas corresponding to our words, and that the signification and scope of the latter shall be carefully determined. But this is a demand to which little attention is paid in the ordinary affairs of life. To define our terms in explicit language, or even to make clear to ourselves the ideas and things for which they stand, is by no means a natural or a universal mode of procedure, but something which requires a distinct, conscious effort.

Bacon, Hobbes, Locke, Hume, and nearly all of the older philosophical writers have warned us against the abuse of words. The whole matter has been expressed very clearly by Locke, from whom I quote the following passage: —

" For he that shall well consider the errors and obscurity, the mistakes and confusion, that are spread in the world by an ill use of words, will find some reason to doubt whether

language, as it has been employed, has contributed more to the improvement or hindrance of knowledge amongst mankind. How many are there, that, when they would think on things, fix their thoughts only on words, especially when they would apply their minds to moral matters; and who, then, can wonder if the result of such contemplations and reasonings about little more than sounds, whilst the ideas they annex to them are very confused and very unsteady, or perhaps none at all; who can wonder, I say, that such thoughts and reasonings end in nothing but obscurity and mistake, without any clear judgment or knowledge?

"This inconvenience in an ill use of words men suffer in their own private meditations; but much more manifest are the discords which follow from it in conversation, discourse, and arguments with others. For language being the great conduit whereby men convey their discoveries, reasonings, and knowledge, from one to another; he that makes an ill use of it, though he does not corrupt the fountains of knowledge, which are in things themselves, yet he does, as much as in him lies, break or stop the pipes whereby it is distributed to the public use and advantage of mankind."[1]

The remedy for the obscurities and confusions of words is to be found in clear and distinct ideas. We must endeavour to go behind the words and realize clearly and distinctly in consciousness the ideas for which they stand. Now the means which logic recommends for the attainment of this end is definition. The first requirement of logical reasoning is that terms shall be accurately defined. There are, however, two ways in which the meaning of a term may be defined or explained. Every term, as we have already seen (§ 16).

[1] *Essay concerning Human Understanding*, Bk. III., Ch. XI.

may be regarded either from the point of view of intension, or from that of extension. To define, in the usual sense, is to explain from the standpoint of intension, to state the attributes or qualities which are connoted by the term. The process of explaining terms with reference to the objects, or classes of objects, for which they stand is known as Division. We may include, then, under the general term definition, (1) *Intensive definition, or definition in the ordinary sense*, and (2) *Extensive definition or division.*

§ 18. **Definition.**—To define a term is to state its connotation, or to enumerate the attributes which it implies. Thus we define a parallelogram as a quadrilateral figure whose opposite sides are parallel. A distinction is often made between *verbal* and *real* definition. When we merely wish to explain the meaning in which we intend to employ some word, we have verbal definition. But when it is the purpose of our assertion to state the real nature or essential characteristics of some object, the proposition employed is said to constitute a real definition. This distinction, though not without importance, cannot, I think, be regarded as ultimate. For we never define a word or term for its own sake merely, but in order to understand the nature of the objects to which it refers. Indeed, a mere word, apart from its uses, or from the things for which it stands, has no interest for us. In defining a term, then, we are always attempting to explicate or explain, more or less directly, the nature of a thing, or our idea about a thing.

Nevertheless, there is an advantage in distinguishing propositions whose *immediate* purpose is to expound the meaning of a word, from those which assert something directly of an object. 'Monarchy consists in the authority

of one man over others,' may be regarded as a verbal definition, because the purpose of the proposition is simply to explain the meaning of the subject term. On the other hand, 'iron is malleable' is a *real* definition (though not a complete one), because it does not primarily refer to the signification of the word 'iron,' but to the real object to which the name is applied.

In this connection, it is interesting to notice that a proposition which amounts to nothing more than a verbal definition, is sometimes put forward as if it were an assertion which contained some real knowledge. The solemn commonplaces in which ignorant persons delight are often of this character. 'A republic is a government by the people,' 'a just man will do what is right,' 'if it rains, the ground will be wet,' may serve as examples. The mistake in such cases consists in supposing that these assertions are anything more than verbal. "Trifling propositions," is the name that Locke gives to this form of statement. 'The property of water is to wet, and fire to burn; good pasture makes fat sheep, and a great cause of the night is the lack of the sun,' are Corin's profound remarks to Touchstone, in summing up his philosophy.

There are two points of view from which the subject of definition may be considered. We might either discuss *the best method of obtaining real definitions of the nature of things*, or might confine our attention to the *requirements which a good definition has to fulfil.* A person's ability to define either a term, or the thing for which the term stands, depends, however, upon the possession of clear and distinct ideas on the subject. The problem, then, as to the best method of finding definitions, resolves itself into an inquiry concerning the means to be used in obtaining and classifying our ideas in general; and the answer to this question, so far as an answer can be given, must be found in the theory

of logic as a whole. In our treatment of the subject we shall, therefore, confine our attention mainly to a consideration of the requirements of a logical definition, and the rules which must be observed in stating it in language.

Before entering upon the subject, however, it is interesting to refer briefly to the method proposed by Socrates for obtaining definitions. Socrates, as we have already seen (§ 5), was the first to emphasize the necessity of defining and fixing the meaning of familiar terms. He found that, though the people of Athens were constantly using terms like 'good,' 'beautiful,' 'justice,' and 'temperance,' none of them, not even those with the greatest reputation for wisdom, were able to give any clear and consistent statement of what these terms implied. Socrates himself did not profess to be wiser than the rest, but he had a genuine spirit of inquiry, and made it the business of his life to try to arrive at clear conceptions, especially with regard to certain fundamental ethical virtues, like justice, and temperance, and wisdom, which he regarded as of the utmost practical importance. It was by means of conversation with others that he sought to gain clear ideas regarding the nature of these virtues. By a series of questions and answers, by comparison of any definition proposed with particular facts which are admitted, he led his interlocutors to expose and refute the inadequacies of their earlier statements.

This method of proceeding by means of question and answer, and thus compelling a speaker to admit particular facts which refute the general thesis which he is maintaining, is called **Dialectic**. This was the means by which Socrates constantly strove to advance to consistent and adequate definitions. Apart from the dialectical and dramatic form which the Socratic argument took, the method employed is essentially

that of induction. For the definition, or conception, is derived from a comparison of particular instances, both positive and negative. By a consideration of individual cases, Socrates sought to obtain a definition which would be a complete and adequate expression of the nature of all the individuals which share in the class name. Aristotle says that it is to Socrates we owe the method of induction and logical definitions. Clear and distinct conceptions, formulated in exact definitions, constituted the scientific goal for Socrates, and the inductive procedure of observing and classifying particular instances was the means which he employed for reaching this goal. It should, however, be added that the Socratic use of induction, as Plato represents it in his Dialogues, is more often popular in character than strictly scientific, judged by our present standards.

The second question has reference to the formulation of a definition in language. Suppose that we already possess a clear conception of the meaning of the terms to be defined, what are the conditions which a logical definition must fulfil? The answer to this question is usually given in logical text-books by means of a set of rules for definition. Before stating these rules, however, it is necessary to explain the meaning of the terms 'genus' 'species,' and 'differentia,' which will be frequently employed throughout the remainder of this chapter. These terms, together with 'property' and 'accident,' constitute what the older logicians called the Predicables, and state all the possible relations which a predicate may express with regard to a subject. It will only be necessary, however, for us to consider briefly the signification of the first three terms.

In logic, any term may be regarded as a **genus** which con-

tains two or more subordinate classes or species. A **species**, on the other hand, is simply a subdivision or subordinate class of some larger whole. Thus 'metal' is a genus with reference to iron, gold, silver, etc., which are its species. 'Rectilinear figure' is the genus to which belong the various species, triangle, quadrilateral, pentagon, etc. The **differentia** of any term is made up of the qualities or characteristics which distinguish it from other terms, from the genus to which it belongs, as well as from the species which are coördinate with it. Thus the logical differentia of a triangle is the property of having three sides; the differentia of man is that which distinguishes him from other animals, whether this be the power of speech and reason, or some other characteristic, either physical or mental.

The use of the terms 'genus' and 'species' in logic is entirely relative. That is, any term may be considered either as a species or a genus, according as it is regarded as forming a part of some more comprehensive class, or as itself including other classes. Thus man, for example, is a species of the genus 'animal'; but the same term also may be regarded as a genus including various species of men, Caucasians, Negroes, Mongolians, etc. In the same way, 'animal' may be considered a species of the still more comprehensive class 'organized being,' and this latter term again as a species of the genus 'material being.' A still higher or more comprehensive term which includes as its species material and spiritual beings alike is 'being.' Since this term includes everything which exists, and can therefore never be included in any more general class, it is sometimes called the highest genus (*summum genus*). On the other hand, we might proceed downwards until we come to a class which does not admit of division

into any subordinate classes. Such a term is called in logic the lowest species (*infima species*).

It is important to notice that the terms 'genus' and 'species' have not the same signification in logic as in the natural sciences. In classifying objects in natural history, we use the terms 'variety,' 'species,' 'genus,' 'family,' and 'order,' to denote varying degrees of relationship between certain groups or classes of objects. These terms, as thus employed, also indicate certain relatively fixed divisions, or permanent ways of grouping the various forms of plant and animal life. But in logic the terms 'genus' and 'species' are employed to indicate the relationship between any higher and lower class whatsoever. Moreover, as we have seen, any term (excepting only the highest genus and the lowest species) may be regarded from different standpoints, as either a genus or a species.

We shall now proceed to state the requirements of a logical definition: —

(1) *A definition should state the essential attributes of the thing to be defined.* This is done by stating the genus to which the object belongs, and also the peculiar marks or qualities by means of which it is distinguished from other members of the same class. Or, as the rule is usually stated: A logical definition should give the next or proximate genus, and the differentia of the species to be defined. Thus we define a triangle as a rectilinear figure (genus) having three sides (differentia); and man as an animal (genus) which has the power of speech and reason (differentia).

(2) *A definition should not contain the name to be defined, nor any word which is directly synonymous with it.* If, for example, we were to define justice as the way of acting justly, or life as the sum of vital processes, we should be guilty of a violation of this rule.

(3) *The definition should be exactly equivalent to the class of objects defined; that is, it must be neither too broad nor too narrow.* In other words, the definition must take account of the whole class, and nothing but the class. 'A sensation is an elementary state of consciousness,' for example, is too broad a definition, since it applies equally to affective and conative elementary processes. On the other hand, the definition of government as 'an institution created by the people for the protection of their lives and liberties,' is too narrow. For it takes no account of absolute forms of government which do not depend upon the will of the people. Each of these cases may be regarded as a failure to give the true differentia of the class to be defined, and hence as violations of the first rule.

(4) *A definition should not be expressed in obscure, figurative, or ambiguous language.* The reasons for this rule are at once evident. Any lack of clearness or definiteness in a definition renders it useless as an explanation. Sometimes the words used in defining may be less familiar than the term to be explained (*ignotum per ignotius*). The definition which was once given of the word 'net' as 'a reticulated texture with large interstices or meshes,' may serve as an example.

(5) *A definition should, whenever possible, be affirmative, rather than negative.* A definition, that is, should state what a term implies, rather than what it does not imply. Sometimes, however, the purpose of a definition may be best attained by a negative statement of what is excluded by the meaning of the term. Thus, for example, we may define a spiritual being as a being which is not material, that is, unlike a material body made up of parts extended in space. This is an exception to the rule. But it should be noted that there are other definitions which, while negative in form, are not

really exceptions to it. Such, for instance, is the definition of a bachelor as an unmarried man. This is a precise statement of what *is* included in the meaning of that term. It is, therefore, the meaning rather than the form of the definition to which we should look in applying this rule. The fault against which it is directed is that of the so-called 'infinite' definition, which merely states what a thing is not, without regard to whether such a negation sensibly increases one's knowledge of the meaning of the term or not. Such a definition is 'infinite' in the sense that to enumerate everything that the term to be defined *is not* would be an infinite process.

(1) A logical definition, as has been said, requires us to mention the proximate genus or next higher class to which the species to be defined belongs, and also the specific or characteristic differences which distinguish it from other species. Now it is clear that there are certain cases in which these conditions cannot be fulfilled. In the first place, no logical definition can be given of the highest genus, because there is no more general class to which it can be referred. And, again, although it is possible to give the differentia of any species such as 'man' or 'metal,' it is not possible to state *individual* characteristics by means of a logical definition. An individual thing may be perceived, and its various properties pointed out. But it is never possible to state in a logical definition wherein the individuality of a particular thing consists. The uniqueness of a particular object cannot be summed up in a general definition, but must be learned through perception. We may perhaps say that the highest genus is above, and the individual thing below, the sphere of logical definition.

There are, moreover, other terms such as 'space,' 'time,' 'life,' 'thought,' which are not readily referred to any higher class, and for which, therefore, logical definitions cannot be given. These terms are sometimes said to denote objects which are *sui generis*, or of their own class.

(2) This use of 'genus' and 'species' in definitions comes to us from the logic of Aristotle. The purpose of definition, as we have seen, is to make our conceptions clear and precise; that is, the definition should state, as exactly and concisely as possible, the essential characteristics of the thing defined. And the most convenient way to do this is often to mention some more inclusive group of objects, the general nature of which is known, and at the same time to add the special characteristics which distinguish the thing in question from the rest of this group. Thus, for example, it is much more convenient to define a dicotyledon as 'a plant with two cotyledons or seed shoots' than it would be to enumerate all the special characters of plants as well as the distinctive character of the germinating seed.

(3) But while this is true in general, it should not be supposed that this is the only way in which good definitions can be reached. The purposes and methods of the particular science or study employing the definition determine both its content and the proper form of its statement. The definition, by giving genus and specific differentia, is especially useful where our chief purpose is one of classification, of ranging the concepts employed in any subject in a fixed order for further reference and use. But it is often true, especially in the natural sciences, that a thing may be better defined by telling how it comes into being than by giving it a place in a fixed scheme of classification. This second mode of definition might be called *genetic definition*. Its use is frequent where we are concerned with processes and the laws of their action, and it often represents an advance in knowledge upon classificatory definition. To define 'heat,' for example, as 'a force in nature recognized in the phenomena of fusion and evaporation, etc.,' tells us less about its real nature than the statement that it is 'a form of energy possessed by bodies *derived from* an irregular motion of their molecules.' To define 'water' as 'a fluid which descends from the clouds in rain,' is less adequate for scientific purposes than the chemical definition of it as 'a fluid

formed by adding one part of oxygen to two parts of hydrogen.' In zoölogy and botany the older definitions of animals and plants by giving their genus and the distinctive or 'diagnostic' marks by which their respective species might be recognized, received a new meaning in the light of the theory of evolution; for these classificatory relationships have been shown to be evidences and results of the degree of affinity in descent from common progenitors, and are revised accordingly. The definition of 'ape,' for example, as a 'variety of the quadrumana having teeth like man, etc.,' is widened to include less obvious characteristics; and this and other similarities to man, which the older definition merely stated, are now explained. In all such cases, the genetic definition tells us more about the real nature of the thing defined, because it relates the thing, through general laws of behaviour, to other things and their characteristics. Again, there are other cases where either mode of definition seems equally adequate in itself, and we can employ them indifferently according to the purpose of the moment. In mathematics, for example, a circle may be defined equally well as 'a plane figure bounded by a line, all points of which are equally distant from a point within called the centre,' or as 'the plane figure generated by revolving a straight line about one of its extremities which remains fixed.' And, finally, we may mention a class of genetic definitions whose value seems merely practical, in that their purpose is only to give a brief statement of how to make a certain thing when it is wanted. Such are the chemical formulæ used in certain manufactures, or the receipts found in cook books.

(4) In addition to the question as to which of these modes of definition is to be preferred in any case, the further problem arises: What are the *essential* characteristics which the definition must state? This also must be determined by the purposes for which it is to be used. The essential characteristics of any subject will vary widely according to the different points of view from which it is examined. The legal definition of 'insanity,' for example, differs from

the medical. Jurisprudence is concerned here not with the study of mental abnormality as such, but with the determination of that degree of it which it is expedient to recognize as constituting irresponsibility for what would usually be considered as a criminal act, or as nullifying contracts, deeds, and wills. And, in general, we may say that the purpose of definitions in law is always to insure that the original intention of the legislator shall be carried out, by stating as clearly as possible the distinguishing marks of the agents, acts, or states to which the law is intended to apply. This purpose, and not that of an exact statement of the nature of the thing defined, determines what shall be considered essential characteristics in its eyes. It is plain that there may often be, therefore, an important difference between a good legal definition and a good definition of the same subject-matter in one of the natural sciences, for example. This example will also serve to illustrate the truth that it is neither necessary nor desirable that all definitions should be equally precise. A definition which, from one point of view, lacks logical completeness may sometimes be sufficiently exact for the purpose on hand. Such is the case, for example, with those definitions which are preliminary in any science or argument, and serve to outline its field and to prepare the way for further discussion. Too great haste in defining is in its way almost as much a fault as failure to define at all; and there is a peculiar fallacy which attempts to bar the way to all fruitful discussion by remarking that 'it is all a question of definition, and if the terms had been first defined, all this argument would be unnecessary.' The remark is perfectly true, but it overlooks the fact that any fully adequate definition is the product of thinking, not its point of departure.

In the general rules of definition, therefore, the terms 'genus' and 'specific differentia' should be taken in a wide sense. It should be remembered that they vary with the purpose of the definition, and that that purpose may be either merely to insure recognition by the statement of convenient marks or signs, as in the 'diagnostic'

definitions of disease for the use of the physician; or it may be the ordered arrangement of the subject-matter of a science, as summing up the knowledge we already have and stating it in convenient form for preservation and further investigation; or, again, it may be the concise statement of the way in which particular processes and objects are explained by the general laws of causation. According to these varying purposes, both 'genus' and 'specific differentia' may be sometimes descriptive, sometimes explicative, sometimes fixed classes, sometimes genetic processes.

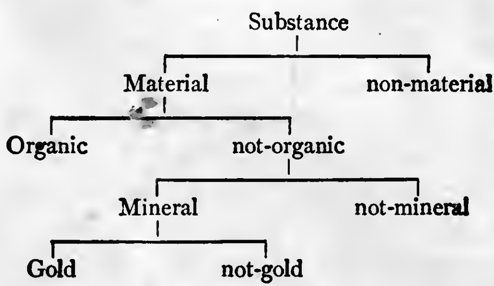
§ 19. **Division.** — We have already spoken of Division as a process of defining a term from the point of view of extension. This is to enumerate the objects or classes of objects which the term denotes. This enumeration must, however, be guided by certain principles which we have now to consider.

It is usual to begin this subject by speaking of Dichotomy, or the division of a term into two parts (*δίχα τέμνειν*, to cut in two). This is a purely formal process, and is based on the so-called law of **Excluded Middle**, which is regarded as one of the fundamental laws of thought. This law may be stated as follows: There is no middle ground between contradictories. Any term, *a*, is either *b* or not-*b*. A triangle is either equilateral or not-equilateral. Of two contradictory predicates, one or the other must belong to every possible subject.

Now it is clear that this is a purely formal principle of division. Some positive knowledge of the particular facts involved is always necessary, in order to enable one to determine what things do stand in this relation of logical opposition. The logical law, in other words, does not help us at all in deciding what may be regarded as not-*a* in any particular case. It is not, therefore, a means of increasing our knowledge, but merely a principle of order and arrangement. This fact, obvi-

ous as it seems, was not understood by the Schoolmen who busied themselves with logic in the latter part of the Middle Ages. They clung firmly to the belief that it was possible to discover the nature of particular facts by purely formal operations of this kind. Accordingly, they spent a great deal of time in classifying and arranging terms as contradictories, contraries, etc. This work was doubtless of much service in fixing the meaning of terms, and in preventing confusion in their employment. But it was a purely verbal investigation, and, of course, could not lead to any discoveries regarding the nature of things.

Moreover, it must be noticed that we do not always get propositions to which any meaning can be attached by uniting subjects and predicates in this way. If the law of dichotomy is not guided by knowledge of the particular facts, it will give absurd propositions like 'virtue is either square or not-square,' 'iron is either pious or not-pious.' Unmeaning propositions of this kind being left out of account, however, we may proceed to divide everything according to this principle. All geometrical figures are either rectilinear or not-rectilinear; all rectilinear figures either triangular or not-triangular; all triangles, equilateral or not-equilateral, etc. This method of division may be represented thus:—



If it were desirable, the terms 'non-material,' 'organic,' and 'not-mineral' might also be further subdivided in the same way.

Now it is not difficult to see that the practical use of this principle will depend upon our ability to find some positive value for the negative not-*a*. That is, to make the law of more than formal value, we must know what concrete term excludes *a*, or is its logical contradictory. And knowledge of this kind comes, as already said, only from experience of the particular facts. The strictly *logical* contradictory of *a* is always not-*a*; of wise, not-wise; of cold, not-cold; etc. Mistakes frequently arise in stating contradictories in a positive form. The difficulty is that terms are chosen which are not true logical contradictories. Thus, if we say that every man is either wise or foolish, our terms are not contradictories, for a middle ground between them is possible. The same would be true of divisions like 'large or small,' 'rich or poor,' 'saint or sinner,' 'idle or diligent.' In general, it is safe to scrutinize all dichotomic divisions very sharply, to see that the alternatives are really contradictories.

The method of dichotomy depends, as we have seen, upon the law of Excluded Middle. But there is also another process called **Division** in logic, which is perhaps better known by its less technical name of Classification. In classification, there is no necessary limit to the number of classes or divisions which may be obtained. In this respect, it, of course, differs fundamentally from the twofold division which we have been examining. Furthermore, a classification is always made according to some principle which is retained throughout the whole process. Any common characteristic of the group of individuals to be divided may be taken as a principle of

classification. If, however, the characteristic chosen is merely an external and accidental one, the classification based upon it will be regarded as *artificial*, and made for some special or temporary purposes. Thus we might divide all flowering plants according to the colour of the flowers, or the persons in any company according to the pattern of their shoes. A classification which proceeds upon such surface distinctions has, of course, no real or scientific value, except as it aids us to discover more fundamental or deep-lying resemblances between the individuals with which it deals, of which we may regard these superficial qualities as signs. Such a preliminary classification corresponds to what we have called the 'diagnostic' definition (§ 18).

A scientific or natural classification, on the other hand, has for its purpose the statement of real likeness or resemblance. It seeks to find and group together the things which are related in some essential point. Consequently, it selects as its principle of division some property which appears to be a real mark of individuality, and to be connected with changes in other properties. Such a real principle of natural classification is rarely found by comparison of merely one property or set of properties in the things to be compared. To classify according to a single property may be a convenient method of giving names to any group of individuals, and of arranging them in such a way as to be useful to the student. It does not, however, give any adequate idea of the properties and true relations of the individuals compared. A really scientific, or natural, classification must be based upon a study and comparison of all the discoverable properties of the different individuals to be classified. It is only in this way that their real resemblance and affinities can be brought to light.

The classification of plants proposed by the famous Swedish botanist, Karl Linnæus (1707–1778), was based upon the comparison of a single feature: the structure of the sexual organs of plants. This method proved of the greatest convenience in indexing plants in a convenient way into genera and species so that they could be named and described. Yet since the classification adopted was based upon a single property or feature of the plant, it was considered (even by Linnæus himself) as merely artificial. Of course it is not so obviously artificial as the examples of what we may perhaps call merely accidental or trivial classification given above. But Linnæus's system did not aim at setting forth the true relations of plants, and it was not based upon any systematic study of all their properties. It is useful merely as a stepping-stone to the real study of plants which is presupposed in natural classification.

Certain rules for division are usually given in connection with the treatment of this subject. It is not, of course, supposed that by their help one can properly divide any subject without special knowledge. The purpose of these rules is rather to warn against the logical errors to which one is most liable in the process of division.

(1) Every division is made on the ground of differences appearing in the fundamental nature which is common to all the members of the whole to be divided.

(2) Every division must be based on a single principle or ground (*fundamentum divisionis*).

(3) The constituent species (or groups into which the whole is divided) must not overlap, but must be mutually exclusive.

(4) The division must be exhaustive, *i.e.* the constituent species must be equal, when added together, to the genus.

The first rule requires no remark. It simply states that

it is only possible to divide any whole on the basis of differences in something which is common to all its parts. The second rule warns against changing the principle of division while the process is being carried out. This law would be violated, if, for example, one were to divide mankind intc Caucasians, Negroes, Mongolians, Europeans, Australians, and Americans. The principle of division which was first adopted in this example was obviously that of the colour of the skin. But this principle was not carried through, and another principle, that of geographical distribution, was substituted for it. In dividing one must be clearly conscious of the principle which one is using, and keep a firm hold of it until the division is completed. The example which we have just given also violates the third rule. For not all of the groups, European, Caucasian, etc., exclude one another. Similarly, it would not be good logic to divide animals into vertebrates, mammals, insects, birds, mollusks, and fishes. The fourth rule simply insists that the division must be complete. The whole must be completely included in its divisions. It would not be a complete division to say that books may be divided into folios, quartos, and duodecimos, or vertebrates into mammals and birds. For in neither of these examples are the divisions enumerated equal to the whole class.

We have discussed Division as though it always proceeded from the whole to its parts, from the genus to its species. But the contrary procedure is quite as frequent, and in the natural sciences is the method more usually followed. In this we start with a more or less miscellaneous assemblage of objects, examine and compare them, and gradually arrange them into groups on the basis of the observed likenesses and differences. These groups may again be

assembled into more inclusive groups in the same way, and the process continued until we have a systematic classification of the collection with which we began. The name of Classification is often reserved for this procedure, Division being applied only to the method already described. As a matter of fact, however, this distinction seems to be merely relative. Even classification in this narrower sense presupposes some vague idea of the whole, which enables us to mark off in a preliminary way the objects to be classified from other objects; otherwise its task would be infinite. And it is perhaps more usual than not that we classify in both ways at the same time. To borrow an illustration from Mr. Joseph, 'if one were asked to divide the genus "novel," he might suggest a division into the novel of adventure, of character, and of plot; but he would at the same time run over in thought the novels he had read, and ask himself if they could be classed satisfactorily under these three heads.' Division, in fact, in any of its forms, presupposes and involves definition. Now definition, as we have already seen, is based on induction, or an examination of the particular things to be defined; and whether we first notice their general likeness one to another, or the special differences that exist between them along with this likeness, is largely a matter of accident, or is determined by the special purpose of the investigation.

REFERENCES

J. S. Mill, *Logic*, Bk. I., Chs. VII. and VIII.
W. Minto, *Logic Inductive and Deductive*, Pt. II., pp. 82–130.
C. Sigwart, *Logic*, Vol. I., §§ 42–44.
J. H. Hyslop, *The Elements of Logic*, Ch. VI.
H. W. B. Joseph, *An Introduction to Logic*, Ch. V.

CHAPTER VI

PROPOSITIONS

§ 20. **The Nature of a Proposition.** — A proposition is the expression in words of an act of judgment. It is composed, as we have already seen, of two terms, a subject and a predicate, connected by a copula. From the point of view of formal logic the predicate is affirmed (or denied) of the subject. When we come to consider the nature of judgment (cf. especially §§ 78, 81), we shall find reasons for questioning whether this analysis of the proposition can be regarded as furnishing a correct account of what actually takes place in judgment. When we judge, we do not begin with words or terms which are not yet judgments, and then pass on to judgment by joining the former together in an external way. The conclusions which we shall have to adopt are, that terms represent ways of judging, that the simplest act of thought is already a judgment, and that thinking develops by advancing from incomplete to more complete and comprehensive judgments. The theory of the syllogism is, however, worked out on the view that the proposition expresses a relation between subject and predicate. This is sufficiently accurate for practical purposes, and is not likely to lead to any serious mistakes so long as we remember that it is the proposition, rather than the actual nature of judgment, with which we are dealing.

The logical proposition, as the expression of an act of

thought, corresponds to the grammatical sentence. Not every sentence, however, is a logical proposition. Sentences which express a wish or an interrogation do not *directly* enter into the process of argument at all, and may therefore be neglected for the present. The same is true of exclamatory sentences. Again, even indicative sentences frequently require to be rewritten in order to reduce them to the form of a logical proposition, which demands two terms and a copula. The sentence, 'the sun shines,' must, therefore, for purposes of logical treatment, be reduced to, 'the sun is a body which shines.' 'On the hillside deep lies the snow,' is expressed as a logical proposition in some such form as this: 'The snow is a covering lying deep on the hillside.' It is very important to change the grammatical sentence to the regular form of a proposition before attempting to treat it logically.

The most general division of propositions is that which classifies them as **Categorical** and **Conditional**. A categorical proposition asserts directly, and without any condition. The predicate is either affirmed or denied unconditionally of the subject. 'A is B,' 'this room is not cold,' 'New York is the largest city in America,' are examples of categorical propositions. Conditional propositions, on the other hand, state the consequences which necessarily follow from a supposition, or hypothesis, and do not directly assert anything about particular matters of fact; as, *e.g.*, 'we shall go to-morrow, if it does not rain.' 'It will either rain or snow to-morrow,' is also a conditional proposition; for neither rain nor snow are asserted directly and absolutely, but in each case the appearance of the one is dependent upon the non-appearance of the other.

The first of these conditional propositions is known as a *Hypothetical*, and the latter as a *Disjunctive* proposition; but for the present we shall deal only with categorical propositions, and with the form of syllogistic argument to which they give rise. After we have completed the account of the categorical syllogism, however, it will be necessary to return to a consideration of conditional propositions, and to the class of arguments in which they are employed.

§ 21. **The Quality and Quantity of Propositions.** — We shall now consider the various kinds of categorical propositions. Such propositions are classified with regard to **Quality** and **Quantity**. From the standpoint of quality, propositions are either **Affirmative** or **Negative**. An *affirmative* proposition is one in which an agreement is affirmed between the subject and predicate, or in which the predicate is asserted of the subject. The proposition, 'snow is white,' for example, indicates such an agreement between the subject and predicate, and is therefore affirmative in quality. A *negative* proposition indicates a lack of agreement or harmony between the subject and predicate. The predicate does not belong to the subject, but all relation or connection between the two is denied. 'The room is not cold,' 'the trees are not yet in full leaf,' are examples of negative propositions.

The **Quantity** of a proposition is determined by the extension of the subject. When the proposition refers to all of the individuals denoted by the subject, it is said to be **Universal** in quantity. When, on the other hand, the proposition affirms that the predicate belongs only to a part of the subject, it is said to be **Particular**. For example, 'all metals are elements' is a universal proposition, because the assertion is made of the subject in its widest or fullest extent; 'some

metals are white' is a particular proposition, because reference is made to only a part of the subject 'metal.'

We divide propositions, then, with regard to quantity, into Universal and Particular propositions. Universal propositions are often indicated by adjectives like 'all,' 'the whole,' 'every,' etc. It frequently happens, however, that no such mark of universality is present. A scientific law is usually stated without any explicit statement of its quantity, though from its very nature it is meant to be universal. Thus we say, 'the planets revolve around the sun,' 'comets are subject to the law of gravitation.' Propositions which have a singular or an individual name as subject are often called Individual propositions, as, *e.g.*, 'the earth is a planet,' 'knowledge is power.' But since it is impossible to limit a singular subject, individual propositions are to be regarded as universal. They belong, that is, to the class of propositions which employ the subject term in its complete extent.

Another class, called Indefinite or Indesignate propositions, has sometimes been proposed. This class is usually said to include propositions in which the form of the words does not give any indication whether the predicate is used of the whole, or only of a part of the subject. 'Men are to be trusted,' 'animals are capable of self-movement,' may serve as examples. This classification may be useful in illustrating the evil of making indefinite or ambiguous statements. Otherwise there is nothing to be learned from it. A really indefinite proposition has no place in an argument, and logic rightfully refuses to deal with it. The first demand of logic is that our statements shall be clear and precise. A proposition is not necessarily indefinite, however, because it has no qualifying words like 'all' or 'some.'

It is the meaning of a proposition as a whole, rather than the form of its subject, which renders it definite or indefinite. Where, on the other hand, it is really impossible to decide whether the proposition is universal or particular, logic forbids us to proceed with the argument until this point has been made clear.

Particular propositions are usually preceded by some word or phrase which shows that the subject is limited in the extent of its application. The logical sign of particular propositions is 'some,' but other qualifying words and phrases, such as 'the greatest part,' 'nearly all,' 'several,' 'a small number,' etc., also indicate particularity. Here again, however, it is the meaning of the proposition, rather than its form, which is to be considered. 'All metals are not white,' for example, is a particular proposition, although introduced by 'all,' since it is clearly equivalent to 'some metals are not white.' 'Every mark of weakness is not a disgrace,' again, is a particular proposition, and signifies that 'not all, or some marks of weakness are not disgraceful.'

The words 'few' and 'a few' require special attention. The latter, as in the proposition, 'a few persons have spoken to me about it,' is equivalent to 'some,' and introduces a particular affirmative proposition. 'Few,' on the other hand, is negative in character. Thus, 'few were saved from the shipwreck' implies that only a few were saved, or that the greater number did not escape, and the proposition is therefore to be considered as a particular negative.

Propositions, then, are classified as affirmative and negative in Quality, universal and particular in Quantity. When these classifications are combined, we get four kinds of propositions, to symbolize which the vowels A, E, I, O are

employed. A and I, the vowels contained in *affirmo*, stand for affirmative propositions; E and O, the vowels in *nego*, for negative propositions. This may be represented as follows: —

Universal	Affirmative:	All S is P.	A
	Negative:	No S is P.	E
Particular	Affirmative:	Some S is P.	I
	Negative:	Some S is not P.	O

We shall henceforth use A, E, I, and O to represent respectively a universal affirmative, a universal negative, a particular affirmative, and a particular negative proposition. In dealing with propositions logically, the first step is to reduce them to one or other of these four types. This can be accomplished readily by noticing the distinctions previously laid down. There are, however, certain grammatical forms and sentences which present some difficulty, and it may therefore be useful to consider them separately.

§ 22. **Difficulties in Classification.** — In the first place, we may notice that in ordinary language the terms of a proposition are frequently inverted, or its parts separated in such a way that it requires attention to determine its true logical order. In the proposition, 'now came still evening on,' for example, the subject 'still evening' stands between two portions of the predicate. As a logical proposition, the sentence would have to be expressed in some such form as the following: 'Still evening is the time which now came on.' Similarly, we should have to write an inverted sentence like, 'deep lies the snow on the mountain,' as 'the snow is something which lies deep on the mountain.'

If a subject is qualified by a relative clause, the verb of the latter must not be confused with the main assertion of the proposition. Take the sentence, 'he is brave who conquers his

passions.' Here it is evident that the relative clause describes or qualifies 'he.' Logically, then, the proposition is of the form A, and is to be written, 'he who conquers his passions is brave.' The reader will notice that all propositions which begin with pronouns like 'he who,' 'whoever,' etc., are universal in quantity, since they mean all who belong to the class in question.

(1) We have reduced grammatical sentences to logical propositions by changing the form in such a way as to have two terms united by 'is' or 'are' as the copula. Such a proposition, however, does not express time, but simply the relation existing between subject and predicate. When the grammatical sentence does involve a reference to time, and especially to past or future time, the reduction to logical form is somewhat awkward. Perhaps the best method is to throw the verb expressing time into the predicate. Thus 'the steamer will sail to-morrow' = 'the steamer is a vessel which will sail to-morrow'; 'we waited for you two hours yesterday' = 'we are persons who waited for you two hours yesterday.'

(2) Exclusive propositions exclude all individuals or classes except those mentioned by the use of some such word as 'except,' 'none but,' 'only.' 'None but the guilty fear the judge'; 'only citizens can hold property'; 'no admittance except on business.' These propositions may all be reduced to the form E by writing 'no' before the contradictory of the subject term. Thus 'none but the guilty fear the judge' = '*no* one who is *not* guilty fears the judge'; 'only citizens can hold property' = '*no* one who is *not* a citizen, etc.'; 'no admittance except on business' = '*no* person who has *not* business is to be admitted.' Or, by taking the predicate as subject, the meaning of the proposition may be expressed affirmatively: 'all who fear the judge are guilty'; 'all who can hold property are citizens.'

§ 23. **Formal Relation of Subject and Predicate.** — We have now to consider how the relation existing between

the terms of a proposition is to be understood. In § 16 it was shown that every term may be interpreted in two ways: either from the point of view of extension, or from that of intension. Extensively, terms are taken to represent objects or classes of objects; while their meaning in intension has reference to the attributes or qualities of things. Now the interpretation of the categorical proposition given by formal logic is based entirely on extension. That is, the subject and predicate are regarded as standing for individual objects or classes of objects. The question to be considered, then, concerns the extensive relation of these groups of objects in the propositions A, E, I, and O.

This mode of interpreting propositions must not be taken as furnishing an adequate theory of the nature of the act of judgment which is expressed in the proposition. It leaves entirely out of account the intensive meaning, or the connection of attributes asserted by the proposition, which in many cases is the most prominent part of its signification. Thus the proposition, 'all metals are elements,' implies that the quality of being an element is united with the other qualities connoted by the term 'metal.' Indeed, this interpretation is perhaps more natural than the one given by formal logic, namely, that the class of metals is included in the class of elements. It must be admitted that the extensive way of reading propositions, as affirming or denying the inclusion of one class of objects in another class, frequently seems artificial. Nevertheless, it is the view upon which the historical account of the syllogism is founded. And the fact that this mode of representing the meaning of propositions leads in practice to correct conclusions proves that it is not wholly false. It represents, as we have seen in discussing terms (§ 16), one side or aspect of the meaning of propositions.

From the point of view of formal logic, then, a logical proposition signifies that a certain relation exists between the class of things denoted by the subject, and that denoted by the predicate. This relation may be one of inclusion or of exclusion. For example, the proposition 'all good men are charitable,' is interpreted to mean that 'good men' are included in the class of 'charitable men.' On the other hand, 'no birds are mammals,' signifies that the two classes, 'birds' and 'mammals,' are mutually exclusive. The meanings of the four logical propositions A, E, I, and O may be represented by means of a series of diagrams, which were first used by the celebrated German mathematician Euler, who lived in the eighteenth century.

FIG. I.

To represent the meaning of a proposition in A, like 'all good men are charitable,' we draw a circle to symbolize the class of charitable beings, and then place inside it a smaller circle to stand for good men. The proposition, that is, signifies that 'good men' are included in the class of 'charitable beings.' The subject belongs to, or falls within, the larger class of objects represented by the predicate.

It must be carefully noted that proposition A does not usually assert anything *of the whole of its predicate*. In the ex-

ample just given, no assertion is made regarding the whole class of 'charitable beings,' but only in so far as they are identical with 'good men.' There may possibly be other charitable beings who are not good men, or not men at all. The meaning of the proposition, then, is that 'all good men are *some* charitable beings.' In other words, the predicate of the ordinary universal affirmative proposition is taken only in a partial, or limited extent: nothing is affirmed of the whole of the circle of charitable beings. We denote this fact by saying that the predicate of proposition A is *undistributed*. The subject, on the other hand, as a universal term, is employed in its fullest extent, or is *distributed*.

In some cases, however, the predicate is not a broader term which includes the subject, but the two are equal in extent. In the proposition, 'all equilateral triangles are equiangular,' for example, this is the case. If we were to represent this proposition graphically, the circle of equilateral triangles would not fall inside that of equiangular triangles, but would coincide with it. The same relation between subject and predicate holds in the case of logical definitions. For example, in the definition, 'monarchy is a form of political government where one man is sovereign,' the subject is coextensive with the whole of the predicate. In examples of this kind, it is of course obvious that the predicate, as well as the subject, is distributed.

As an example of proposition E, we may take the example, 'no birds are mammals.' The meaning of this proposition is represented graphically by means of two circles falling outside each other as in Fig. 2.

The proposition asserts that the class of birds falls completely without the class of mammals, that the two classes

are entirely distinct, and mutually exclusive. With regard to quantity, the subject is of course universal or *distributed*. And, in this case, the predicate is also distributed. For the proposition asserts that the subject 'birds' does not agree with *any part* of 'mammals.' Or, in terms of the diagram, we deny that the circle representing 'birds' corresponds with *any*

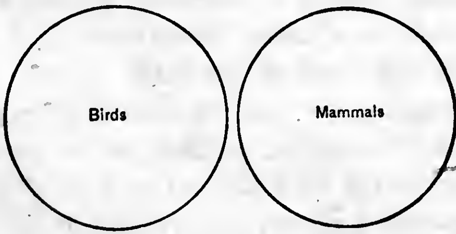


Fig. 2.

portion of the circle 'mammals.' But to exclude the former circle completely from the circle which represents 'mammals,' it is necessary that we know the whole extent of the latter. Otherwise we could not be sure that the subject had not some point in common with it. Proposition E, therefore, distributes, or uses in their widest extent, both subject and predicate.

The meaning of a proposition in I, as, *e.g.*, 'some birds are web-footed,' is shown by means of two circles intersecting or overlapping as in Fig. 3. A part of the class of birds corresponds with a part of web-footed animals. The proposition has reference to the common segment of the two circles, which may be large or small. The two circles correspond in part at least. In proposition I, both subject and predicate are undistributed. The subject is, of course, a particular or limited term. And, as will be clear from what has already been said in the case of proposition A, reference is made only to a limited portion of the predicate. In the example used, the

assertion refers only to those web-footed animals which are also birds. Or we may say that the proposition has reference only to the common segment of the circles representing sub-

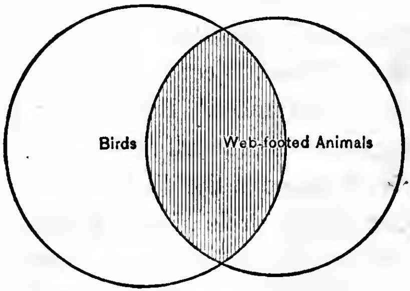


FIG. 3.

ject and predicate. Nothing is asserted of the other portions of the two circles. In other words, both subject and predicate are employed in a limited extent, or are *undistributed*.

'Some metals are not white,' may serve as an example of proposition O.

This proposition may be represented graphically as in Fig. 4. Though this is the same form of diagram as that

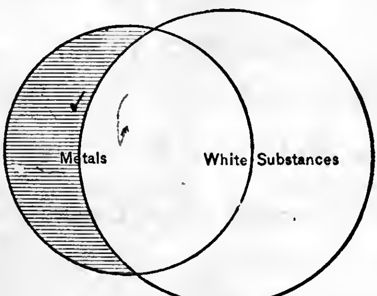


FIG. 4.

employed in the last figure, the proposition refers now to the outlying part of the circle 'metal.' Some metals, it asserts

do not fall within the sphere of white substances. A larger or smaller section of the circle representing the former term, falls *completely without* the circle of white substances.

It is necessary to notice carefully that although the subject of O is undistributed, its predicate is distributed. For, as we have seen, a part of the subject is completely excluded from the class of 'white substances.' But in order to exclude from *every part* of the predicate, the full extent of the predicate must be known. Or, in terms of the diagram, the proposition excludes a portion of the circle of metals (some metals) from each and every part of the circle of white things. The latter term must therefore be used in its full extent, or be distributed.

It is absolutely necessary, in order to comprehend what follows, to understand the distribution of terms in various propositions. It may help the reader to remember this if we summarize our results in the following way: —

Proposition A, subject distributed, predicate undistributed.
Proposition E, subject distributed, predicate distributed.
Proposition I, subject undistributed, predicate undistributed.
Proposition O, subject undistributed, predicate distributed.

REFERENCES TO § 23

J. N. Keynes, *Studies and Exercises in Formal Logic*, Part II., Chs. I and II.

J. S. Mill, *Logic*, Bk. I., Ch. V.

C. Sigwart, *Logic*, § 5.

B. Bosanquet, *The Essentials of Logic*, Lectures V. and VI.

CHAPTER VII

THE INTERPRETATION OF PROPOSITIONS

§ 24. **The So-called Process of Immediate Inference.** — Many logicians speak of two kinds, or processes of reasoning, to which they give the names of **Mediate**, and **Immediate Inference.** Mediate inference, it is said, asserts the agreement or disagreement of a subject and predicate after having compared each with some common element or middle term. The conclusion is thus reached mediately or indirectly. The syllogism is the best example of mediate inference. In the syllogism,

All M is P,
All S is M,
Therefore S is P,

the conclusion is reached through the medium of M, with which both S and P have been compared. It will be noticed that to obtain a conclusion in this way two propositions or premises are necessary.

We sometimes are able, however, to pass directly or immediately from one proposition to another. For example, the proposition that 'no men are infallible,' warrants the statement that 'no infallible beings are men.' Or, if we know that it is true that 'some birds are web-footed,' we perceive at once that the proposition, 'no birds are web-footed,' is false. It is this

process of passing directly from one proposition to another which has been named by many logicians **Immediate inference.**

The question may be raised, however, whether the direct passage from one proposition to another, as in the above examples, should properly be called inference, or whether the change is not merely in the verbal expression. As we have already shown, inference is a process of exhibiting the relation of facts to one another by discovering some common element or connecting principle by means of which they are united (cf. also § 92). Wherever we can discover a connecting thread or common element between two facts or groups of facts, we are able to *infer* with greater or less certainty from the nature of the one what the nature of the other must be. But it is essential to inference that there shall be a real transition from one fact to another — that the conclusion reached shall be different from the starting-point.

The point at issue, therefore, is whether a new fact or truth is reached in the so-called processes of immediate inferences, or whether we have the same fact repeated in the form of a new proposition. When we pass from 'no men are infallible,' to 'no infallible beings are men,' can we be said to infer a new truth? In this case it is evident, I think, that there has been no real development or extension of the original proposition so as to include a new fact. The new proposition is the result of a verbal interpretation of the original one, and restates the same fact in a different way. Inference always completes or enlarges the truth from which it sets out by showing the reasons which support it, or the consequences which follow from it. Now, when we pass directly from one proposition to another, as

in the examples given above, it will be found, I believe, that nothing new has been *added* to the original statement — no new facts have been brought into connection in the process.

Nevertheless, the process does not appear to be merely verbal, but to involve a certain movement of mind, — a fuller and clearer realization of the meaning and bearings of the original proposition. Before deciding the matter, the claims of each of the different types of so-called immediate inference should be examined separately; and the question is one that the student should keep in mind throughout the chapter. Some authors have named these processes 'Eduction,' since they draw out or explicate the meaning of propositions. Whether or not they may properly be called *inference*, they render important service in helping us to understand all that is really implied, both in the way of affirmation and denial, in the propositions we use. Nothing is commoner in argument than disputes as to what certain statements imply — what propositions 'amount to the same thing,' and may therefore properly be substituted for any given statement. Now it is the purpose of the methods of logical interpretation (or immediate inference) which are to be discussed in this chapter, to determine what other statements, positive or negative, are really involved in the case of the different forms of logical proposition. Given a certain proposition as true or false, what other propositions can be immediately derived from it? We may consider under the following five headings the results obtainable by processes of Immediate Inference, or direct Interpretation: Opposition, Obversion, Conversion, Contraposition, Inversion.

§ 25. **The Opposition of Propositions.** — We have seen that

all categorical propositions have to be reduced to one of the four forms, A, E, I, O, in order to be dealt with by logic. Now, between these propositions, all of which have the same subject and predicate, certain relations of exclusion and inclusion exist, to which the general name of Opposition has been given. It is clear that the truth of some of these propositions excludes the truth of others, and also that the relation between certain of the propositions is such that one assertion necessarily involves the truth of another. Logical Opposition, then, is used to denote any relation, either of exclusion or inclusion, that exists between propositions having the same subject and predicate. Thus, if it be true that 'no professional gamblers are honest,' it is impossible that 'all professional gamblers are honest,' or even that some are honest. The proposition E is thus inconsistent with both A and I. Again, if it be true that 'all politicians are dishonest,' it must be true that 'some politicians are dishonest,' as well as false that 'no politicians are dishonest.' That is, when A is true, I is also true, while E is necessarily false. Propositions A and E are called **Contrary** propositions. 'All A is B,' and 'no A is B,' express the greatest possible degree of contrariety or opposition. If one proposition be true, the other is necessarily false. It is to be noticed, however, that we cannot conclude that if one be false, the other is true. For both A and E may be false. Thus, for example, the propositions, 'all men are wise' and 'no men are wise,' are both false. But, on the other hand, propositions A and O, E and I, are pairs of **Contradictory** propositions: if one is false, its contradictory is necessarily true; and if one is true, the other is manifestly false.

The relation of the four logical propositions is clearly shown by arranging them in the following way: —

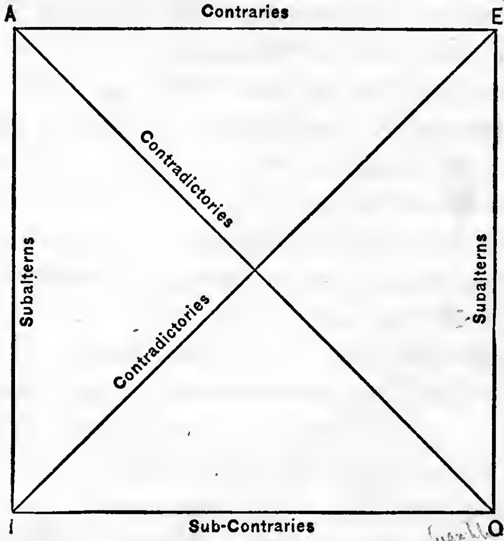


Fig. 5.

A and E are known as *contraries;* I and O as *subcontraries;* A and O, I and E, as *contradictories;* A and I, E and O, as *subalterns.*

The relations of these propositions may now be summed up in the following statements:—

(1) Of contrary propositions, one is false if the other is true, but both may be false.

(2) Of contradictory propositions, one is true and the other necessarily false.

(3) If a universal proposition is true, the particular which stands under it is also true; but if the universal is false, the particular may or may not be true.

(4) If a particular proposition is true, the corresponding universal may or may not be true; but if the particular is false, the universal must be false.

(5) Subcontrary propositions may both be true ; but if one is false, the other is necessarily true.

The knowledge that any one of these propositions is either true or false enables us to determine the truth or falsity of at least some of the others.

For example, if A is true, E is false, O is false, and I is true. If A is false, E is doubtful, O is true, and I doubtful.

If I is true, E is false, A is doubtful, and O doubtful. If I is false, E is true, A is false, and O true.

Similarly, we are also able to determine what follows when we suppose that E and O are either false or true.

It ought to be carefully noted that when we affirm the truth of the particular proposition I, we do not deny the truth of the universal proposition A. The proposition, 'some students are fond of recreation,' for example, does not exclude the truth of 'all students are fond of recreation.' Similarly, the truth of O does not exclude the corresponding proposition in E: the statement, 'some men are not generous,' for example, does not interfere with the truth of the universal proposition, 'no men are generous.' A particular proposition, in other words, asserts something of a limited part of a subject; it neither affirms nor denies anything of the same term taken universally.

The reader will remember that propositions which have the name of some singular or individual thing as subject, have been classified as universal. 'New York is the largest city in America,' 'charity is not the only virtue,' are examples of such propositions. Now it is at once evident that in cases of this kind there are no corresponding particular propositions. What has just been said regarding the relation of universal and particular propositions, applies therefore only to propositions which have a general term or name as subject.

Moreover, we must notice that when A and E propositions have a singular or individual name as subject, the relations between them are somewhat different from those just stated. A and E, we said, are contrary, but not contradictory propositions. By that it was implied that although we can proceed from the truth of the one to the falsity of the other, it is not possible to go in a converse direction, from falsity to truth. We cannot conclude, for example, from the falsity of the proposition that 'all men are selfish' the truth of the corresponding negative proposition, 'no men are selfish.' With contradictory propositions, however, we can go from a denial to an affirmation. Now the point to be observed, with regard to propositions with a singular term as subject, is that although only contraries in form, they have yet the force of contradictories. 'Socrates is wise' (A), and 'Socrates is not wise' (E), are contradictory, as well as contrary, propositions.

§ 26. **The Obversion of Propositions.** — The terms 'Obversion' and 'Æquipollence' were formerly used to denote any process by which the form of a proposition is changed without an alteration in meaning being involved. The name 'Obversion' is, however, now generally employed to describe the change which a proposition undergoes in passing from the affirmative to the negative, or from the negative to the affirmative form while still retaining its original meaning.

Every fact is capable of expression either in the form of an affirmative or of a negative proposition. Whether the affirmative or negative form is chosen in any particular case, is partly a matter of convenience. It is also determined largely by the psychological interest of the moment, *i.e.* by the purpose which we have in view in making the assertion. When, for example, we wish to repel some sug

gestion which may have occurred to us, or to deny something which our companions appear to believe, we naturally choose the negative form of statement. But the meaning of the proposition is the same whether we say, 'all men are fallible,' or, 'no men are infallible.' Similarly, we can say, 'not one of the crew escaped,' or, 'all of the crew perished.'

Obversion, then, is the process of substituting for any affirmative proposition its equivalent in negative form, or of expressing the meaning of a negative proposition as an affirmative. To obtain the obverse of proposition A, we proceed on the principle that two negatives are equal to an affirmative. Instead of 'all animals digest food,' we may write, 'no animals are beings that do not digest food'; for, 'every man has his own troubles,' 'there are no men who have not their own troubles.' Instead of affirming the predicate of the subject, the obverse of A takes the contradictory of the original predicate and denies it universally.

Proposition I may be obverted in the same way, though it yields a particular, instead of a universal negative proposition. Thus the obverse of, 'some of the houses are comfortable,' is 'some of the houses are not not-comfortable,' *i.e.* uncomfortable. We deny the negative predicate in the obverse proposition, instead of affirming the positive.

We obtain the obverse of the propositions E and O by changing the negation contained in them to its equivalent affirmation. This is done by attaching the negative to the predicate, and then affirming it of the subject. For example, to obtain the obverse of, 'no one who was present can forget the scene,' we first write the proposition in logical form, 'no one who was present is a person who can forget the scene.' Now the contradictory of the predicate term, 'a person who

can forget the scene,' is, 'a person who can *not* forget the scene.' Affirming this universally we get, 'all persons who were present are persons who cannot forget the scene.' As an example of how the obverse of O is obtained, we may take the proposition, 'some metals are not white.' Now if we change the quality of the proposition by attaching the negative to the predicate, we obtain, 'some metals are not-white.' That is, instead of denying, we affirm the contradictory of the original predicate. When the predicate is made up of several words, it is important that the logical contradictory of the whole term be taken. For example, in the proposition, 'some men are not fond of work,' the predicate fully expressed is, 'persons who are fond of work.' Now the negative or contradictory term corresponding to this is, 'persons who are *not* fond of work.' The obverse of the original proposition therefore is, 'some men are persons who are not fond of work.'

§ 27. **The Conversion of Propositions.**—To convert a proposition is to transpose its subject and predicate so that each shall occupy the place previously held by the other. Thus the proposition, 'no men are infallible,' is converted by writing it, 'no infallible beings are men.' The original proposition is called the **Convertend**, and the proposition obtained by conversion the **Converse.** By conversion, then, a proposition having P as its subject is derived directly from the original form of the assertion S — P. It is for this reason that conversion is usually ranked as a process of immediate inference. For it makes clear what is involved in the original proposition but is perhaps not clearly recognized; namely, that in the assertion S — P some statement about P as subject in its relation to S is also involved.

Whether this may more properly be regarded as a process of formal interpretation, than as one which involves real inference, is a question which the student may consider for himself.

It is evident that in proceeding to convert propositions it will be necessary to notice whether the predicate of the convertend, or proposition to be converted, is distributed or undistributed, otherwise we should not know what extension to apply to this term when used as the subject of the converse proposition. The rules usually given to limit the process of conversion are as follows: —

(1) No term must be distributed in the converse proposition which was not distributed in the convertend.

(2) The quality of the converse proposition must remain the same as the quality of the convertend.

The reason for the first rule is at once evident from what has been already said. The second rule is not one which is always observed. Of course, the meaning of a proposition must not be altered by changing the quality simply or directly. But, in converting by Contraposition, as we shall see later, it is first necessary to obtain the equivalent of the convertend by obversion, and this necessarily involves a change of quality.

There are two kinds of conversion usually recognized: (*a*) Simple Conversion; (*b*) Conversion by Limitation or *per accidens*.

(*a*) By Simple Conversion is meant the direct transposition of the subject and predicate without any other change in the form of the proposition. Both propositions E and I can be converted in this way. Thus the converse of, 'none of the books on this shelf are novels,' is another proposition in E, 'no novels are books on this shelf.' From 'some dicoty-

ledons are exogens' we obtain by conversion another particular affirmative proposition, 'some exogens are dicotyledons.'

(*b*) Conversion by Limitation or *per accidens* is applied to proposition A. In this process A loses its universality, and yields as a result only proposition I. To illustrate this mode of conversion we may take the proposition, 'brown hematite is an iron ore.' As we already know, the term 'an iron ore,' being the predicate of proposition A, is undistributed. When used as the subject of a new proposition, therefore, it must be limited by the adjective 'some.' We thus obtain the converse proposition, 'some iron ore is brown hematite.' Similarly, the converse of the proposition, 'all sensations are mental processes,' is 'some mental processes are sensations.' When proposition A is converted by limitation, then, it yields proposition I as a result. And it is evident that the proposition has really lost something in the process. For it is impossible by converting again to obtain anything more than a particular proposition. It is, however, sometimes possible to convert proposition A without limiting the predicate. In formal definitions, for example, the subject and the predicate are of equal extent, and may be transposed simply without any limitation of the latter. Thus the converse of, 'an equilateral triangle is a plane figure having three equal sides,' is 'a plane figure having three equal sides is an equilateral triangle.'

Proposition O is the only form of logical proposition that does not admit of Conversion. E and I, as we have seen, may be converted simply, and the converse of A is obtainable by limitation, or even in some cases by simple Conversion. But from an O proposition, 'some S is not P,' no proposition where P is subject and S predicate can be obtained. And

the reason for this may be seen at once. For if the conversion were made, giving the form 'some P is not S,' S would be distributed as the predicate of a negative proposition. But in the convertend ('some S is not P') it was not distributed; accordingly, an attempt to convert O involves a breach of the rule that no term must be distributed in the converse proposition which was not distributed in the convertend.

§ 28. **Contraposition and Inversion.** — In **Contraposition** the contradictory of the predicate of the original proposition is taken as the subject of a new assertion. That is, the Contrapositive of a proposition of the form S — P, has as its subject non-P, the contradictory of P. Contrapositive propositions may be derived from A, E, and O. Proposition I, for reasons that will be evident later, does not yield a contrapositive.

The contrapositive of A, E, and O may be obtained through two steps: by first obverting and then converting. After some practice in deriving the contrapositive in this way the student should learn to obtain it directly, remembering that what is required is a statement as to what is implied in the original proposition regarding non-P, the contradictory of the predicate. Let us first, however, illustrate the longer method.

If we take as an example of A the proposition 'all the planets are bodies that revolve around the sun,' we can obtain the contrapositive by (1) obverting, 'no planets are bodies that do not revolve around the sun,' and (2) converting the E proposition obtained by obversion, 'No bodies that do not revolve around the sun are planets.' This is in the form 'no non-P is S,' and we might therefore write the contrapositive of A directly, by taking the contradictory of the original predicate and *denying it universally* of the subject.

The form here derived, the converse of the obverse, has usually been defined as the contrapositive of a given proposition, and we have so far followed this definition. But some logicians speak of the contrapositive as a proposition which has the same *quality* as the original, and has the more symmetrical form 'non-P — non-S.' This may be obtained by obverting the result obtained in the last paragraph, 'all bodies that do not revolve around the sun are non-planets.' The two forms are not essentially different, but we may follow what appears to be the best usage by speaking of the form 'non-P — S,' as the partial contrapositive, and 'non-P — non-S' as the full contrapositive.

Taking as an example of E the proposition 'none that love angling are wholly given over to the world,' we obtain (1) by Obversion, 'all that love angling are persons not wholly given over to the world,' and (2) by Conversion of this latter proposition, 'some persons not wholly given over to the world are those who love angling.' This is the partial contrapositive, which when obverted gives us the full contrapositive, 'some persons not wholly given over to the world are not those who do not love angling,' a negative proposition like the E from which it is derived, and which has the form 'some not-P is not not-S.' It is especially to be noted that the contrapositive of E is a particular proposition.

To obtain the contrapositive of O, we proceed in the same way, first obverting, then converting the result for the partial contrapositive, and obverting once more for the full contrapositive. For example, 'some things that glitter are not gold'; (1) by obversion, 'some things that glitter are not-gold' (*i.e.* substances other than gold); (2) by conversion, 'some substances other than gold are things that

glitter'; (3) by obversion, 'some substances other than gold are not things that do not glitter.'

Inversion. The original proposition has S as subject and P as predicate; the converse has P as subject and S as predicate; the contrapositive, non-P as subject, and in its full form, non-S as predicate. It is clear that the only remaining term to be used as a subject is non-S. Now, where an assertion is made regarding this — the contradictory of the original subject — the form is known as the Inverse. The question now is: What logical propositions of the form S — P enable us to derive a proposition about what is not-S? By experimenting in applying obversion and conversion we find that only the Universal propositions, A and E, yield the Inverse form, and also that this is always a particular proposition. From 'All S is P,' we may derive, by alternately obverting and converting, 'some not-S is not-P' (which may be called the full Inverse by analogy with the terms employed in regard to contraposition), which by obversion gives 'some not-S is not P,' the partial Inverse.[1] Similarly, beginning with conversion, and then obverting and converting, from 'no S is P' may be derived the partial Inverse,

[1] Keynes (*Formal Logic*, 4th ed., pp. 139–40) calls attention to the apparent error in passing from 'All S is P,' — where P is not distributed — to, 'Some not S is not P,' — where P is distributed. The result seems an error, yet it is impossible to discover any mistake in the processes of conversion and obversion by which it has been obtained. This difficulty may serve to illustrate the impossibility of proceeding logically without assumptions even where the transformations appear to be purely formal. Keynes says: "It is in the assumption of the existence of the contradictory of the original predicate that an explanation of the apparent anomaly may be found. That assumption may be expressed in the form, 'Some things are not P.' The conclusion 'Some not-S is not P' may accordingly be regarded as based on this premise combined with the explicit premise, 'All S is P'; and it will be observed that, in the additional premise, P is distributed."

'some not-S is P,' which yields, by obversion, the full Inverse, 'some not-S is not not-P.'

We have already summarized results with regard to the Opposition of propositions (p. 101). For the sake of convenience the outcome of the other processes may be brought together in the following table, given by Keynes.[1] S′ and P′ are used to denote not-S and not-P.

		A	I	E	O
i	Original proposition	SaP	SiP	SeP	SoP
ii	Obverse	SeP′	SoP′	SaP′	SiP′
iii	Converse	PiS	PiS	PeS	
iv	Obverted Converse	PoS′	PoS′	PaS′	
v	Partial Contrapositive	P′eS		P′iS	P′iS
vi	Full Contrapositive	P′aS′		P′oS′	P′oS′
vii	Partial Inverse	S′oP		S′iP	
viii	Full Inverse	S′iP′		S′oP′	

REFERENCES

B. Bosanquet, *Logic*, Vol. I., pp. 310–319.

W. Minto, *Logic Inductive and Deductive*, Pt. III., pp. 130–166.

J. N. Keynes, *Studies and Exercises in Formal Logic*, 4th ed., Chs III., and IV.

[1] *Op. cit.*, p. 140.

CHAPTER VIII

THE SYLLOGISM

§ 29. **The Nature of Syllogistic Reasoning.** — The syllogism, as we have already seen (§ 10), presents a conclusion together with the reasons by means of which it is supported. A single proposition taken by itself is dogmatic: it merely asserts, without stating the grounds upon which it rests. The syllogism, on the other hand, justifies its conclusion by showing the premises from which it has been derived. It thus appeals to the reason of all men, and compels their assent. To do this, it is of course necessary that the truth of the premises to which appeal is made should be granted. If the premises are disputed or doubtful, the argument is pushed a step further back, and it is first necessary to show the grounds upon which these premises rest. The assumption of syllogistic reasoning — and, indeed, of all reasoning whatsoever — is that it is possible to reach propositions which every one will accept. There are certain facts, we say, well known and established, and these can always be appealed to in support of our conclusions. In syllogistic reasoning, then, we exhibit the interdependence of propositions; *i.e.*, we show how the truth of some new proposition, or some proposition not regarded as beyond question, follows necessarily from other propositions whose truth every one will admit.

The question which arises in connection with the syllogism, therefore, is this: Under what conditions do propositions which are accepted as true contain or imply a new proposition as a conclusion? Or we may put the question in this form: In what ways may the four kinds of logical propositions, A, E, I, O, be combined so as to yield valid conclusions?

We pointed out in a previous chapter that a syllogism has always two premises. It is, however, impossible to obtain a conclusion by combining any two propositions at random, as *e.g.*,—

All A is B,
No X is Y.

It is evident that *any* two propositions will not yield a conclusion by being taken together. In order to serve as premises for a syllogism, propositions must fulfil certain conditions, and stand in certain definite relations to each other. To determine some of the most apparent of these conditions, let us examine the argument:—

All mammals are vertebrates,
The whale is a mammal,
Therefore the whale is a vertebrate.

It will be noticed that the term 'mammal' is common to both premises, and that it does not occur at all in the conclusion. Moreover, it is because the other terms are compared in turn with this common or **Middle Term** and found to agree with it, that they can be united in the conclusion. It is only propositions which have a middle term, therefore, which can be employed as the premises of a syllogism. The syllogism is thus essentially a process of comparison. Each of the terms entering into the conclusion is compared in turn with the same middle term,

and in this way their relation to each other is determined. We reach the conclusion not directly or immediately, but by means of the middle term. The conclusion is therefore said to be *mediated,* and the process itself is sometimes called **mediate reasoning.**

It will be interesting to compare what has just been said regarding the function of the middle term, with what has been previously stated regarding the nature of inference. When we infer one fact from another, it was said, we do so by discovering some identical link or connecting thread which unites both. We may say that to infer is to see that, in virtue of some identical link which our thought has brought to light, the two facts, or groups of facts, are in a certain sense identical. Now the middle term in a syllogism is just the explicit statement of the nature of this identical link. It is true that in the syllogism we seem to be operating with words or terms rather than with the thought-process itself. When we go behind the external connection of the terms, however, we can see that the middle term represents the universal principle, by means of which the conclusion is reached. In the example given above, for instance, we reason that the whale, *being a mammal,* is a vertebrate.

The terms which enter into the conclusion of a syllogism are sometimes called the *Extremes,* as opposed to the middle term. Of the Extremes, *the predicate of the conclusion is known as the* **Major Term**, *and the subject of the conclusion as the* **Minor Term.** The premise which contains the major term is called the **Major Premise,** and stands first when the syllogism is arranged in logical form. The **Minor Premise** on the other hand, is the premise which contains the minor term, and it stands second in the arrangement of the syllogism. The propositions of which the syllogism is composed may occur, however, in any order in actual reasoning; either premise, or

even the conclusion, may stand first. To arrange an argument, therefore, it is necessary to determine which is the major, and which is the minor premise. This can be done most readily by turning to the conclusion, and distinguishing the major and minor terms. For example, take the syllogism: —

> The whale suckles its young,
> No fish suckles its young,
> Therefore the whale is not a fish.

By turning to the conclusion we see that 'fish' (being the broader term and therefore naturally predicate) is the major term. The proposition which contains this term, 'no fish suckles its young,' is, therefore, the major premise, and should stand first. Before proceeding to examine the syllogism further it would be necessary to arrange it as follows: —

> No fish is an animal which suckles its young,
> The whale is an animal which suckles its young,
> Therefore the whale is not a fish.

§ 30. **The Rules of the Syllogism.** — It is customary to give a number of rules or canons to which the syllogism must conform in order to yield valid conclusions. We shall first enumerate the rules, and afterwards remark on their meaning and importance.

(1) In every syllogism there should be three, and only three, terms, and these terms must be used throughout in the same sense.

The terms, as we have already remarked, are known as the major term, the middle term, and the minor term.

(2) Every syllogism contains three, and only three, propositions.

These are called the major premise, minor premise, and conclusion.

(3) The middle term must be distributed in at least one of the premises.

(4) No term must be distributed in the conclusion which was not distributed in one of the premises.

(5) From negative premises nothing can be inferred.

(6) If one premise be negative, the conclusion must be negative; and, conversely, to prove a negative conclusion one of the premises must be negative.

As a consequence of the above rules there result two additional canons which may be set down here.

(7) No conclusion can be drawn from two particular premises.

(8) If one of the premises be particular, the conclusion must be particular.

The reason for the first and second rules will be evident from what has been already said about the structure of the syllogism. We saw that a logical argument is a process of comparison; that two terms are united through comparing them with a common or middle term. If the meaning of the terms does not remain fixed, there are more than three terms, and no comparison is possible. The second rule follows as a corollary from the first.

The third rule, that the middle term must be distributed once, at least, is extremely important, and its necessity will be readily perceived. For, since the middle term is the standard of comparison, it must be used in at least one premise in its universal extent. Otherwise we might compare the major term with one part of it, and the minor term with another part. Such a comparison would of course not warrant us in either

affirming or denying the connection of these terms in the conclusion. For example, the two propositions, —

Sedimentary rocks are stratified substances,
Some metamorphic rocks are stratified substances,

do not distribute the middle term, 'stratified substances,' at all, being both affirmative propositions. It is clear that the

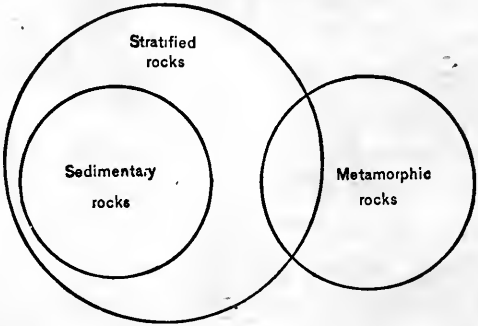


FIG. 6.

term, 'sedimentary rocks,' agrees with one part of the stratified substances, and 'metamorphic rocks' with another part. We are, therefore, not able to infer that 'some metamorphic rocks are sedimentary rocks.' This may be clearly shown by representing the propositions by Euler's method of circles as in Fig. 6. We know from the second proposition that the circle representing 'metamorphic rocks' falls partly within the circle of 'stratified substances.' But it is impossible to determine from the statement whether it corresponds at all with the circle of sedimentary rocks, or falls, as in the figure, entirely without it.

The fourth rule states that no term must be distributed in the conclusion which was not distributed in one of the premises. That is, the conclusion must be proved *by means of* the premises, and no term which was not employed in its universal

signification in the premises can, therefore, be used universally or distributively in the conclusion. This rule may be violated by using either the major or the minor term in a wider sense in the conclusion than in the premise in which it occurs. The resulting fallacies are then known as the **Illicit Process** of the major and minor terms respectively. As an illustration of the illicit process of the major term, we may consider the following argument: —

> All rational beings are responsible for their actions,
> Brutes are not rational beings,
>
> ---
>
> Therefore brutes are not responsible for their actions.

It will be at once seen that the major term, 'beings responsible for their actions,' is distributed in the conclusion, but was not distributed when it appeared as the predicate of an affirmative proposition in the major premise. The fallacious nature of this argument may also be shown by representing the proposition by circles.

The illicit process of the minor term is usually more easily detected. We may take as an example of this fallacy: —

> All good citizens are ready to defend their country,
> All good citizens are persons who vote regularly at elections,
>
> ---
>
> Therefore all who vote regularly at elections are ready to defend their country.

It is clear that the minor term, 'persons who vote regularly at elections,' is undistributed when used as the predicate of the minor premise. In the conclusion, however, it is wrongly taken universally, and it is this unwarranted extension to which the name of *illicit minor* is given. Students are advised to draw circles to illustrate the nature of this fallacy.

The fifth and sixth rules have reference to negative premises. It is not difficult to understand why two negative premises cannot yield any conclusion. For, from the fact that S and P are both excluded from M, we can conclude nothing regarding their relation to each other. Two negative premises afford us no standard by means of which we can determine anything concerning the relation of major and minor terms. Again, where one premise is negative and the other affirmative, it is asserted that, of the major and minor terms, one agrees, and the other does not agree, with the middle term. The necessary inference from these premises, then, is that major and minor terms do not agree with each other. That is, the conclusion must be negative.

It is worth noticing that it is sometimes possible to obtain a conclusion from premises which are both negative in form. For example: —

> No one who is not thoroughly upright is to be trusted,
> This man is not thoroughly upright,
>
> ---
>
> Therefore this man is not to be trusted.

In this example, although the form of both premises is negative, the minor premise supplies a positive basis for argument, and is really affirmative in character. Or we may say that the 'not' in the predicate of the minor premise belongs to the predicate, and not to the copula. The proposition may therefore be said to affirm, rather than to deny.

The seventh and eighth rules, which refer to particular premises can be proved by considering separately all the possible combinations of premises. If this is done, it will be found that these rules are direct corollaries from the third and fourth, which are concerned with the proper distribution of terms. It is impossible

to secure the necessary distribution with two particular premises; for either the distribution of the middle term will not be provided for, or if this has been secured by means of a negative premise, the conclusion will show a case of the illicit major term. By means of the same rules, it may be shown that a particular premise always requires a particular conclusion. The truth of these two subordinate canons also may be readily shown by the use of circles.

§ 31. **The Figures of the Syllogism.** — We have seen what an important part the middle term plays in the syllogism. It constitutes the mediating link between the major and minor terms, and makes possible their union. Now upon the position of the middle term in the premises depends the **Figure** of the syllogism. There are four possible arrangements of the middle term in the two premises, and therefore four figures of the syllogism. If we let P represent the major term, S the minor, and M the middle term, the form of the different figures may be represented as follows: —

FIRST FIGURE

M — P
S — M
∴ S — P

SECOND FIGURE

P — M
S — M
∴ S — P

THIRD FIGURE

M — P
M — S
∴ S — P

FOURTH FIGURE

P — M
M — S
∴ S — P

In the first figure, the middle term is the subject of the major premise, and the predicate of the minor premise.

In the second figure, the middle term is predicate of both major and minor premises.

The third figure has the middle term as the subject of both premises.

In the fourth figure, the middle term occupies just the opposite position in the two premises to that which it holds in the first figure; *i.e.* it is the predicate of the major premise, and the subject of the minor premise.

1 2 3 4.

CHAPTER IX

THE VALID MOODS AND THE REDUCTION OF FIGURES

§ 32. **The Moods of the Syllogism.** — By the **Mood** of a syllogism we mean the combination of propositions A, E, I, and O, which goes to make it up. Thus, when a syllogism is made up of three universal affirmative propositions, we speak of it as the mood AAA; if it is composed of a universal negative, a particular affirmative, and a particular negative proposition, we name it the mood EIO.

Every syllogism, as has been already stated, is made up of some arrangement of the four propositions A, E, I, O, taken three at a time. Now, there are in all sixty-four possible permutations of these four propositions taken three at a time. We might then write out these sixty-four moods, and proceed to determine which of them are valid. But this would be a long and somewhat tedious undertaking. Moreover, if we can determine which are the valid combinations of premises, we can draw the proper conclusions for ourselves. Since, then, there are but two premises in each syllogism, we shall have to deal only with the possible permutations of A, E, I, and O, taken two at a time, or with sixteen combinations in all.

The following, then, are the only possible ways in which the propositions A, E, I, and O can be arranged as premises: —

AA	EA	IA	OA
AE	~~EE~~	IE	~~OE~~
AI	EI	~~II~~	~~OI~~
AO	~~EO~~	~~IO~~	~~OO~~

Some of these premises, however, cannot yield conclusions, since they plainly violate certain rules of the syllogism. The combinations of negative premises EE, EO, OE, and OO can be at once struck out. Again, since no conclusion follows from two particular premises, we can eliminate II, IO, and OI. There remain, then, for further consideration the combinations: —

AA	EA	IA	OA
AE	—	IE	—
AI	EI	—	—
AO	—	—	—

At this point we must recall the fact that every argument must belong to one of the four figures. We must now therefore ask this question: Which of the above combinations of premises will yield valid conclusions in the first, second, third, and fourth figures, respectively? By examining the form of the syllogism in each of these figures, we shall be able to discover what conditions must be fulfilled in each case, and to lay down special canons for each figure. We shall first proceed to state and prove the special canons of the different figures. It will not, however, be necessary for the student to commit these rules to memory, as he can always derive them for himself by a consideration of the form of the argument in the different figures.

§ 33. **The Special Canons of the Four Figures.** —*In the first figure, the minor premise must be affirmative, and the major premise universal.*

The first figure is of the form: —

$$\begin{array}{l} \quad M - P \\ \quad \underline{S - M} \\ \therefore S - P \end{array}$$

To show that the minor premise is affirmative, we employ the indirect method of proof. Let us suppose that the minor premise is not affirmative, but negative. Then since one premise is negative, the conclusion must be negative. But if the conclusion is a negative proposition, its predicate, P, must be distributed. Any term which is distributed in the conclusion must, however, have been distributed when it was used in the premise. P must be distributed, therefore, as the predicate of the major premise. But since negative propositions alone distribute their predicates, the major premise, M — P, must be negative. But by hypothesis the minor premise, S — M, is negative. We have, therefore, two negative premises, which is impossible. Our supposition, that the minor premise is negative, is therefore false; or, in other words, the minor premise must be affirmative.

This having been established, we can very easily prove that the major premise must be universal. For the middle term, M, must be distributed in at least one of the premises. But it is not distributed in the minor premise, for it is there the predicate of an affirmative proposition. It must, therefore, be distributed as the subject of the major premise, that is, the major premise must be universal.

If we turn now to the second figure, we shall find that the following rules may be deduced from a consideration of its form: —

(1) *One premise must be negative, and the conclusion therefore negative.*

(2) *The major premise must be universal.*

The second figure is in the form: —

$$\begin{array}{l} \text{P} — \text{M} \\ \underline{\text{S} — \text{M}} \\ \therefore \text{S} — \text{P} \end{array}$$

The reason for the first rule is at once evident. If one premise is not negative, the middle term, M, is not distributed, and no conclusion is therefore possible. The only means of securing distribution of the middle term in the second figure is by means of a negative premise. And if one premise is negative, it of course follows that the conclusion must be negative.

This having been established, the proof of rule 2 follows almost immediately. For, since the conclusion is negative, its predicate, P, must be distributed. And since P is distributed in the conclusion, it must have been used distributively when it occurred as the subject of the major premise, or, in other words, the major premise must be universal.

The third figure is of the form: —

$$\begin{array}{l} \text{M} — \text{P} \\ \underline{\text{M} — \text{S}} \\ \therefore \text{S} — \text{P} \end{array}$$

From an analysis of this, the two following rules may be obtained: —

(1) *The minor premise must be affirmative.*

(2) *The conclusion must be particular.*

The minor premise is here shown to be affirmative by the method employed in proving the same rule in the first

figure. That is, we suppose the minor premise negative and show that, as a result of this hypothesis, the conclusion is negative, and the major term distributed. It follows, then, that this term must be distributed as the predicate of the major premise. But this could happen only if this premise were negative. The hypothesis that the minor premise is negative thus leads to the absurdity of two negative premises. The conclusion that the opposite is true, that the minor premise is affirmative, is therefore proved indirectly.

Since the minor premise is affirmative, its predicate S is undistributed. This term must therefore be used in an undistributed, *i.e.*, particular sense in the conclusion. And, as this term forms its subject, the conclusion is particular.

In the fourth figure the terms are arranged in the following way: —

$$\begin{array}{l} \text{P — M} \\ \underline{\text{M — S}} \\ \therefore \text{S — P} \end{array}$$

From a consideration of the form of this figure we can obtain the following special canons: —

(1) *If either premise be negative, the major premise must be universal.*

(2) *If the major premise be affirmative, the minor must be universal.*

(3) *If the minor premise be affirmative, the conclusion must be particular.*

The student will be able to prove these canons for himself by applying the rules of the syllogism in the same way as has been done in the proofs already given.

§ 34. **The Determination of the Valid Moods in Each of the Figures.** — We have now to apply these special canons in order to determine what moods are valid in each of the four figures. It has already been shown (p. 122) that the premises which are not excluded by the general rules of the syllogism are: —

AA	EA	IA	OA
AE	—	IE	—
AI	EI	—	—
AO	—	—	—

Now we have proved that in the first figure the major premise must be universal, and the minor affirmative. The only combinations of premises which will stand these tests are, AA, EA, AI, and EI. Drawing the proper conclusion in each case, we have as the four valid moods of the first figure: —

AAA, EAE, AII, EIO.

It will be noticed that the first figure enables us to obtain as conclusion any one of the four logical propositions A, E, I, and O.

The special canons of the second figure state that the major premise must be universal, and one premise negative. Selecting the combinations of premises which fulfil these conditions, we obtain EA, AE, EI, and AO. These give, when the conclusions have been drawn, the following four moods of the second figure: —

EAE, AEE, EIO, AOO.

By means of the second figure, therefore, we are able to establish the truth only of the negative propositions, E and O.

In the third figure the minor premise must be affirmative, and the conclusion particular. Taking all the combinations in which the minor is affirmative, there result, AA, IA, AI, EA, OA, EI. It must be remembered that the third figure yields only particular conclusions, even where both premises are universal. The valid moods in this figure are therefore as follows: —

AAI, IAI, AII, EAO, OAO, EIO.

The canons of the fourth figure, which have to do with the premises, state that where either premise is negative, a universal major is necessary, and that an affirmative major premise must be accompanied by a universal minor. The combinations of propositions which fulfil these conditions are AA, AE, IA, EA, and EI. In drawing conclusions from these premises, however, it is necessary to pay attention to the third canon of this figure, which states that where the minor premise is affirmative, the conclusion must be particular. Accordingly, the valid moods of this figure may now be written: —

AAI, AEE, IAI, EAO, EIO.

Here we are able to obtain a universal negative as a conclusion, but not a universal affirmative. It is interesting to notice that the first figure alone enables us to prove a proposition of the form A.

It may also be pointed out that the combination [I E], although not excluded by the general rules of the syllogism, cannot be used at all as a premise, since it violates the canons of all four figures. There remain in all, then, nineteen valid moods of the syllogism, — four in the first figure,

four in the second, six in the third, and five in the fourth figure.

§ 35. **The Mnemonic Lines.** — It is not necessary to commit to memory the valid moods in each figure. By applying the general rules of the syllogism to the figure in question, the student will be able to determine for himself in every case whether or not an argument is valid. The Latin Schoolmen in the thirteenth century, however, invented a system of curious mnemonic verses for the purpose of rendering it easy to remember the valid moods in each figure. Although it is not necessary for the student to burden his memory with these barbarous names, it is interesting to understand the use of the lines: —

> *Barbara, Celarent, Darii, Ferio*que prioris;
> *Cesare, Camestres, Festino, Baroko,* secundæ;
> Tertia, *Darapti, Disamis, Datisi, Felapton,*
> *Bokardo, Ferison,* habet; Quarta insuper addit
> *Bramantip, Camenes, Dimaris, Fesapo, Fresison.*

The words printed in ordinary type are real Latin words, indicating that the four moods represented by Barbara, Celarent, Darii, and Ferio are the valid moods of the first figure, that the next four are valid in the second figure, that the third figure has six valid moods represented by as many artificial names, and that the fourth figure adds five more. Each word represents a mood, the vowels A, E, I, and O indicating the quality and quantity of the propositions which go to compose them. Thus, Barbara signifies the mood of the first figure which is made up of three universal affirmative propositions AAA; Cesare, a mood of the second figure, composed of the three proposi-

tions EAE. These lines, then, sum up the results reached on pages 126–127 regarding the valid moods in each figure.

But certain consonants in these mnemonic words also indicate how arguments in the second, third, or fourth figures may be changed to the form of the first figure. The first figure was called by Aristotle the **perfect figure**, and the second and third the **imperfect figures**, since he did not regard an argument in these forms as so direct and convincing as one of the first-mentioned type. The fourth figure was not recognized by Aristotle, but is said to have been introduced into logic by Galen, the celebrated teacher of medicine, who lived in the latter half of the second century. If we consider an example of this figure, the reason for refusing it an equal rank with the other three will appear: —

> The whale is a mammal,
> All mammals are vertebrates,
> Therefore some vertebrates are whales.

It is plain that the conclusion of this argument is somewhat strained. That is, it would be more natural to obtain the conclusion 'whales are vertebrates,' than to infer that 'some vertebrates are whales'; for this statement seems to make the species, or less inclusive term, the predicate of the genus, or wider term. It was for this reason, apparently, that Aristotle omitted this figure, as improperly making the real major term a minor, and the real minor a major, and so stating in a less adequate way an argument which could have been better formulated in the first figure.

The process of changing an argument from one of the so-called imperfect figures to that of the first figure is known as **Reduction**. And, as we have said, these curious but

ingenious mnemonic words give rules for carrying out this process. For example, *s* indicates that the proposition represented by the preceding vowel is to be converted simply. Thus an argument in the second figure of the mood Cesare is changed to Celarent in the first figure, by converting the major premise simply. Again, *p* denotes that the preceding vowel is to be converted by limitation, or *per accidens*; *m* is supposed to stand for *mutare*, and indicates that the premises are to be transposed; *k*, which is used in the moods Baroko and Bokardo, shows that an indirect method of proof or reduction is necessary to reduce the arguments to the first figure.

Further, the initial consonants of the moods of the imperfect figures correspond with those of the moods in the first figures, to which they can be reduced. Cesare and Camestres of the second figure, for example, and Camenes of the fourth are reducible to Celarent; and, similarly, Festino, Felapton, Fesapo, and Fresison may all be reduced to Ferio.

The student who understands the structure of the syllogism will be able to arrange an argument in one figure or another, as may be most convenient, without the aid of any mechanical rules. It may be interesting, however, to give a single example for the sake of illustrating the workings of this most ingenious device. Let us take the following argument in the second figure of the mood AEE, or Camestres: —

All members of the class are prepared for the examination, A
No idle persons are prepared for the examination, E

Therefore no idle persons are members of the class. E

Now the *m* in Camestres shows that the major and minor premises are to be transposed; the first *s* indicates that the minor premise is

to be converted, and the second that the same process must be performed on the conclusion.

Converting the minor premise and transposing, we obtain:—

No persons prepared for the examination are idle,
All members of the class are prepared for the examination.

Converting the conclusion,

Therefore no members of the class are idle persons.

This result, as will at once be seen, is an argument in the first figure of the mood EAE, or Celarent.

REFERENCES

Sir W. Hamilton, *Lectures on Logic.* Lectures XX., XXI.
A. Bain, *Logic,* Part First, Deduction, Bk. II., Ch. I.

NOTE.—It would be interesting to work out, in connection with the various forms of inductive reasoning treated in Part II., the organic relation of the syllogistic figures, and their natural applicability to various purposes of argument. This task, however, seemed to lie beyond the proper limits of this book. All of the investigations on this point start from Hegel's treatment in the second part of the *Wissenschaft der Logik* (*Werke,* Bd. 5, pp. 115 ff.). Those interested in this subject may consult W. T. Harris, *The Psychologic Foundations of Education,* Ch. IX.–XI., and the same author's *Logic of Hegel.* See also B. Bosanquet, *Logic,* Vol. II., pp. 44 ff., 88 ff., and *The Essentials of Logic,* Lecture X.; H. W. B. Joseph, *An Introduction to Logic,* Ch. XIV.

CHAPTER X

ABBREVIATED AND IRREGULAR FORMS OF ARGUMENT

§ 36. **Enthymemes.**—The term 'enthymeme' seems to have been used by Aristotle for an argument from signs or from likelihood, without complete proof. From this sense of logical incompleteness, the name has come to be applied in modern times to an argument in which some part is omitted. We have already noticed, in dealing with the syllogism (§ 10), that one premise is often omitted. Indeed, it is but seldom in ordinary reasoning that we arrange our arguments in the strict syllogistic form. We hurry on from one fact to another in our thinking without stopping to make all the steps definite and explicit. We feel it to be a waste of time, and a trial to the patience, to express what is clearly obvious, and so we press on to the conclusion which is, for the time being, the central point of interest.

But the more rapid and abbreviated the reasoning, the more necessary is it to keep a clear head, and to understand what conclusion is aimed at, and what premises are assumed in the argument. To bring to light the hidden assumption upon which an argument is based, is often the best means of refuting it.

Enthymemes are sometimes said to be of the first, second, or third order, according as the major premise, the minor premise, or the conclusion is wanting. As a matter of fact,

an enthymeme of the third order is a rhetorical device used to call special attention to a conclusion which is perfectly obvious, although suppressed. Thus, for example, 'all boasters are cowards, and we have had proofs that A is a boaster.' Here the conclusion is at once obvious, and is even more striking than if it were actually expressed.

It is usually easy to complete an enthymeme. If the conclusion and one premise are given, the three terms of the syllogism are already expressed. For the conclusion contains the major term and the minor term; and one of these again, in combination with the middle term, is found in the given premise. From these data, then, it will not be difficult to construct the suppressed premise. When the premises are given without the conclusion, there is no way of determining, except from the order, which is major and which is minor. It is therefore necessary to assume that they are already arranged in proper logical order, and that the subject of the conclusion, or minor term, is to be found in the second premise, and the predicate of the conclusion, or major term, in the first premise.

§ 37. **Prosyllogisms and Episyllogisms.** — In deductive reasoning it is often necessary to carry on the argument through several syllogisms, using the conclusion first reached as a premise in the following syllogism. For example, we may argue: —

All B is A
All C is B

∴ All C is A.
But all D is C

∴ All D is A.

It is clear that we have here two arguments in the first figure. The first is called the **Prosyllogism**, and the latter the **Episyllogism**. If the argument were carried on further, so as to include three or more syllogisms, the second would form the Prosyllogism with respect to the third, while the third would be the Episyllogism of the second. A concrete example of this kind of reasoning may now be given: —

All timid men are suspicious,
All superstitious men are timid,

Therefore all superstitious men are suspicious,
But some educated men are superstitious,

Therefore some educated men are suspicious.

It will be noticed that in these examples the argument advances from the premises of the Prosyllogism, to the conclusion of the Episyllogism. It proceeds, that is to say, in a forward direction, developing the consequences of the premises which form its starting point. This mode of investigation is therefore called the *Progressive* or *Synthetic*, since it goes steadily forward building up its results as it advances. To state the same thing in different words, we may say that the *Progressive* or *Synthetic* method advances from the conditions to what is conditioned, from causes to effects.

But it is often necessary to proceed in the opposite way. We have often to go back and show the grounds upon which our premises rest, instead of going forward to show what consequences follow from them. And when we do this we proceed *Regressively* or *Analytically*. To take an example which will illustrate both ways of proceeding: —

No man is infallible, for no man is omniscient,
Aristotle was a man,

Therefore Aristotle was not infallible.

In advancing from the premises to the conclusion in this argument our procedure is progressive or synthetic. Instead of reasoning out the consequences of the premises, however, we may go back and show the grounds upon which the major premise rests. It is evident that this premise is itself the conclusion of a syllogism which may be expressed as follows: —

All infallible beings are omniscient,
No man is omniscient,

Therefore no man is infallible.

The regressive method goes backward from conclusions to premises, or from the conditioned to its necessary conditions. In scientific investigation it reasons from effects to causes, while the synthetic method advances from causes to effects.

§ 38. **Sorites, or Chains of Reasoning.** — A Sorites is an abbreviated form of syllogistic reasoning in which a subject and predicate are united by means of several intermediate terms. Such a train of reasoning represents several acts of comparison, and therefore several syllogistic steps. But instead of stopping to draw the conclusion at each stage, the sorites continues the processes of comparison, and only sums up its results at the close. We may define the sorites, therefore, as a series of prosyllogisms and episyllogisms in which all of the conclusions, except the last, are suppressed. It is usually stated in the following form: —

All A is B
All B is C
All C is D
All D is E

∴ All A is E.

It is evident that this train of reasoning fully expressed is equivalent to the following three syllogisms: —

First Syllogism	Second Syllogism	Third Syllogism
All B is C	All C is D	All D is E
All A is B	All A is C (1)	All A is D (2)
∴ All A is C (1).	∴ All A is D (2).	∴ All A is E (3).

There are two rules to be observed in using this form of the sorites: (1) The first premise may be particular, all the others must be universal; (2) The last premise may be negative, all the others must be affirmative. It is evident from an examination of the syllogisms given above that if any premise except the first were particular, the fallacy of undistributed middle would be committed. For, in that case, the middle term in one of the syllogisms would be the subject of a particular proposition, and the predicate of an affirmative proposition. And if any premise but the last were negative, the major term in the syllogism following that in which this occurred would be distributed in the conclusion without having been distributed in the major premise. We may now give some concrete examples of this kind of reasoning: —

Misfortunes sometimes are circumstances tending to improve the character,

Circumstances tending to improve the character are promoters of happiness,

What promotes happiness is good,

∴ Therefore misfortunes are sometimes good.

In some cases the different terms of an argument of this kind are expressed in the form of hypothetical propositions.

Thus, for example, we might argue: If a man is avaricious, he desires more than he possesses; if he desires more than he possesses, he is discontented; if he is discontented, he is unhappy; therefore, if a man is avaricious, he is unhappy. This argument is hypothetical in form only, and may be easily reduced to categorical type as follows: —

> An avaricious man is one who desires more than he possesses,
> A man who desires more than he possesses is discontented,
> A discontented man is unhappy,
>
> ---
>
> Therefore an avaricious man is unhappy.

It will be noticed that the subject of the first premise in this form of argument is taken as the subject of the conclusion, and that the predicate of the conclusion is the predicate of the last premise. This is usually called the Aristotelian sorites. But there is another form which unites in the conclusion the subject of the last premise, and the predicate of the first, and which is known as the Goclenian sorites.[1] This may be thus represented: —

All A is B
All C is A
All D is C
All E is D

∴ All E is B.

Since B is the predicate of the conclusion, the premise in which it appears is always to be regarded as the major. As a result of this, it is to be noticed that the suppressed conclusions in this argument form the major premise of the following syllogism, instead of the minor premise as in

[1] Rudolf Goclenius (1547–1628), Professor at Marburg, first explained this form in his *Isagoge in Organum Aristotlis*, 1598.

the Aristotelian sorites. We may, therefore, expand the reasoning into the three following syllogisms: —

First Syllogism	Second Syllogism	Third Syllogism
All A is B	All C is B	All D is B
All C is A	All D is C	All E is D
∴ All C is B.	∴ All D is B.	∴ All E is B.

A little consideration of the form of these syllogisms will lead the student to see that the rules given for the Aristotelian sorites must be here reversed. In both forms of the sorites there cannot be more than one negative premise, nor more than one particular premise. In the Aristotelian form, no premise except the last can be negative, and no premise except the first particular. In the Goclenian sorites, on the other hand, the single premise which can be negative is the first, and it is the last alone which may be particular.

§ 39. **Irregular Arguments.** — There are a large number of arguments employed in everyday life which are valid and convincing, and yet which cannot be reduced to the syllogistic form. The difficulty with these arguments is that they appear to have four terms, at least in the form in which they are most naturally stated. We may discuss such irregular forms of reasoning under three headings: (1) Arguments which deal with the relations of things in time and space, or with their quantitative determinations; (2) arguments *a fortiori*; (3) arguments which are largely verbal in character, and may be said to depend upon the principle of substitution.

(1) As an example of the first class of argument we may take the following: —

A is greater than B,
B is greater than C,

Therefore A is still greater than C.

It is obvious that, although we have here four terms, the conclusion is valid, and the form of argument perfectly convincing. The truth seems to be that in reasoning about quantities we do not proceed upon the syllogistic principle of the inclusion and exclusion of terms. But knowing the continuous nature of quantity, we take as our principle that, 'what is greater than that which is greater than another is *a fortiori* greater than that other.' It would not, however, make the matter any clearer to write this as our major premise, and bring the real argument under it in this way: —

What is greater than that which is greater than another is still greater than that other,
A is that which is greater than that which is greater than C,

Therefore A is still greater than C.

What we have here given as the major premise is simply a statement of the nature of quantity, not a premise from which the conclusion is derived. We find the same irregularity in arguments referring to the relations of things in space and time: —

A is situated to the east of B,
B is situated to the east of C,

Therefore A is to the east of C.

In spite of the formal deficiency of four terms the argument is valid. It will be observed, too, that it is in virtue of the comparison of the position of A and of C with that of B, that these relative positions have been determined. The principle upon which we proceed may be said to be that, 'what is to the east of B is to the east of that which B is to the east of.' Or perhaps it would be truer to fact to say that we proceed in such

cases upon what we know regarding the nature of space, and the relations of objects in space.

(2) *A fortiori* arguments proceed to establish a conclusion by showing that the facts and reasons which support it are more certain or stronger than those which support another conclusion that is unquestioned, or generally accepted. They are frequent in dealing with questions of time, space, quantity, and degrees of quality, and all three of the examples just given may be regarded as coming under this head. In fact, we may say that in such matters, whenever the relation involved is not one of contemporaneousness in time, coincidence in space, or equality in quantity, or degree of quality, any argument naturally falls into the *a fortiori* form. The reason for putting this form into a class by itself is that it is very often employed outside of these fields. To illustrate the two ways in which it is used, for proof and disproof respectively, let us compare a possible argument addressed by a vivisectionist to a meat-eater with one urged upon an anti-vivisectionist by a vegetarian: —

(1)

You admit that it is right to kill and use animals for food,
This is less needful than to kill and use them to discover the causes and remedies of diseases,

---

How much more, then, should you admit that vivisection is right.

(2)

You do not think that it is right to kill animals for vivisection,
Yet this is more needful than to kill them for food,

How much less, then, should you hold that it is right to kill them for food, or, How much more should you deny, etc.

Such arguments as these seem always to involve a comparison of the grounds on which certain conclusions may be jus-

tified, when such grounds can be ranked in order of logical cogency. In the one case, it is urged that since the reason for the conclusion advocated is stronger than one which it is admitted does establish a certain proposition, the conclusion in question must, therefore, be regarded as even more firmly established; in the other, as the reason for holding the principle attacked is weaker than that which is regarded as insufficient to justify another principle, it is held that the first principle is still more obviously false than that already denied, or that there is more reason to deny it than there is to deny the other. Hence the name *argumentum a fortiori*, 'argument from, or by, the stronger,' ('reason' being understood).

(3) The third class of irregular arguments is largely verbal in character, and may be dealt with very briefly. As an example we may consider: —

Men are willing to risk their lives for gold,
Gold cannot buy happiness,

Therefore men are willing to risk their lives for what cannot buy happiness.

It is doubtful, I think, whether these propositions represent any real inference. The whole process may be regarded as a verbal substitution in the major premise of 'what cannot buy happiness' for the word 'gold.' By a slight change in the form of the proposition, however, the argument may be expressed as a regular syllogism of the third figure: —

Gold is something for which men are willing to risk their lives,
Gold cannot buy happiness,

Therefore something which cannot buy happiness is something for which men are willing to risk their lives.

Another example which also appears to be irregular at first sight is added: —

The men of the Middle Ages were ready to undertake any expedition where glory could be won,

The crusades were expeditions in which glory could be won,

The crusades, therefore, were readily undertaken by the men of the Middle Ages.

This argument seems to be irregular in form only, and by a slight change in form may be expressed in the first figure: —

All expeditions in which glory could be won were readily undertaken by the men of the Middle Ages,

The crusades were expeditions in which glory could be won,

Therefore the crusades were readily undertaken by the men of the Middle Ages.

REFERENCES,

ESPECIALLY FOR § 38

W. S. Jevons, *The Principles of Science*, Introduction.

F. H. Bradley, *The Principles of Logic*, pp. 348–360.

CHAPTER XI

HYPOTHETICAL AND DISJUNCTIVE ARGUMENTS

§ 40. **The Hypothetical Syllogism.** — We have hitherto been dealing with syllogisms composed entirely of categorical propositions, and have not referred to the use which is made of conditional propositions in reasoning. A conditional proposition is sometimes defined as the union of two categorical propositions by means of a conjunction. It is the expression of an act of judgment which does not directly or unambiguously assert something of reality. We have already pointed out (§ 20) that there are two classes of conditional propositions: the *hypothetical* and the *disjunctive*, and corresponding to these we have the **hypothetical** and the **disjunctive syllogism.** The hypothetical syllogism has a hypothetical proposition as a major premise, and a categorical proposition as a minor premise. The disjunctive syllogism in the same way is composed of a disjunctive proposition as major, and a categorical proposition as minor, premise. In addition to these, we shall have to treat of another form of argument called the 'dilemma,' which is made up of hypothetical and disjunctive propositions.

A hypothetical proposition does not assert directly the existence of a fact, but states the connection between a supposition or condition and its consequence. It is usually intro-

duced by some word or conjunctive phrase, like 'if,' 'supposing,' or 'granted that'; as, *e.g.*, 'if he were to be trusted, we might give him the message'; 'suppose that A is B, then C is D.' The part of a hypothetical proposition which expresses the supposition or condition is known as the **Antecedent**; the clause stating the result is called the **Consequent**. Thus, in the proposition, 'he would write if he were well,' the consequent, 'he would write,' is stated first, and the antecedent, 'if he were well,' follows.

The hypothetical syllogism, as has been already remarked, has a hypothetical proposition as its major, and a categorical proposition as its minor, premise: —

> If justice is to prevail, his innocence will be proved,
> And justice will prevail,
>
> ---
>
> Therefore his innocence will be proved.

It will be noticed that in this argument the minor premise *affirms the antecedent*, and that, as a result, the conclusion affirms the consequent. This form is known as the *constructive hypothetical syllogism*, or the *modus ponens*.

In the following example it will be observed that the *consequent is denied*, and the conclusion obtained is therefore negative.

> If he were well, he would write,
> He has not written,
>
> ---
>
> Therefore he is not well.

This is called the *destructive hypothetical syllogism*, or *modus tollens*.

The rule of the hypothetical syllogism may therefore be stated as follows: *Either affirm the antecedent or deny the*

consequent. If we affirm the antecedent, *i.e.*, declare that the condition exists, the consequent necessarily follows. And, on the other hand, if the consequent is declared to be non-existent, we are justified in denying that the condition is operative.

The violation of these rules gives rise to the fallacies of *denying the antecedent*, and of *affirming the consequent.* Thus, for example, we might argue: —

> If he were well, he would write,
> But he is not well,
>
> Therefore he will not write.

Here the antecedent is denied, and the argument plainly false. For we cannot infer that his being well is the only condition under which he would write. We do not know, in other words, that the antecedent stated here is the only, or essential condition of the consequent. We know that if there is fire, there must be heat; but we cannot infer that there is no heat when no fire is present. Of course, if we can be certain that our antecedent expresses the essential condition, or real *sine qua non* of the consequent, we can go from the denial of the former to that of the latter. For example: —

> If a triangle is equilateral, it is also equiangular,
> This triangle is not equilateral,
>
> Therefore it is not equiangular.

Usually, however, when the hypothetical form of expression is employed, we cannot be certain that the antecedent expresses the sole, or essential condition, of the consequent. At the ordinary stages of knowledge we have to content ourselves with reasoning from antecedent conditions, without being able to show that no other condition is possible.

To illustrate the fallacy of affirming the consequent, we may take the following example:—

If perfect justice prevailed, the rich would not be permitted to rob the poor,

But the rich are not permitted to rob the poor,

Therefore perfect justice prevails.

Here the antecedent states only *one* condition under which the consequent may follow. Because the consequent is declared to exist, it is by no means necessary that it should exist *as a consequence* of the operation of this condition. It is also worth noting in this example that the consequent of the major premise is negative. The minor premise *which affirms the consequent* also takes a negative form. To deny the consequent we should have to say, 'the rich are permitted to rob the poor.' Or, to put the matter generally, it is necessary to remember that the affirmation of a negative proposition is expressed by a negative proposition, and that the denial of a negative — the negation of a negation — is, of course, positive in form.

A type of hypothetical argument differing in form from the hypothetical syllogism is that in which premises and conclusion are all hypothetical propositions, as, for example:—

If the tariff is increased, prices will rise,

If prices rise, the majority of the people will be discontented,

If the majority are discontented, the Republican party will be defeated at the next election,

---

Therefore, if the tariff is increased, the Republican party will be defeated at the next election.

This is an hypothetical sorites, corresponding to the Aristotelian form of the categorical sorites, in that its conclusion unites the antecedent of the first premise with the consequent of the last. There are also hypothetical sorites which unite the antecedent of the last premise with the consequent of the first in their conclusion, and thus correspond to the Goclenian sorites. Such sorites are often hypothetical in form only, as has been pointed out in the preceding chapter, and when this is the case they may be reduced to categorical syllogisms of the first figure, as in the example there given (§ 38).

§ 41. **Relation of Categorical and Hypothetical Arguments.** — It is evident that the form of the hypothetical syllogism is very different from that of the categorical. But, although this is the case, it must not be supposed that with the former we have passed to a new and wholly distinct type of reasoning. In hypothetical reasoning, as in categorical, it is the presence of a universal principle which enables us to bring into relation two facts which formerly stood apart. Indeed, in many cases, it is a matter of indifference in which form the argument is stated. Thus, we may argue in hypothetical form: —

> If a man is industrious, he will be successful,
> A is an industrious man,
>
> ---
>
> Therefore A will be successful.

The same argument may, however, be expressed equally well in categorical form: —

> All industrious men will be successful,
> A is an industrious man,
>
> ---
>
> Therefore A will be successful.

It is clear that, in spite of the different forms in which the argument is expressed, the reasoning is essentially the same in both cases. The middle term, or general principle which makes it possible to unite the subject and predicate of the conclusion, in the hypothetical as well as in the categorical syllogism, is 'industrious.' A will be successful, we argue, because he is industrious, and it is a rule that industrious men are successful.

Moreover, if an argument is fallacious in one form, it will also be fallacious when expressed in the other. The defects of an argument cannot be cured simply by a change in its form. When an hypothetical argument, in which the antecedent is denied, is expressed categorically, we have the fallacy of the illicit major term. Thus, to state the example of denying the antecedent given on page 146, we get: —

> The case of his being well is a case of his writing,
> The present is not a case of his being well,

> Therefore the present is not a case of his writing.

Similarly, when an argument in which the consequent is affirmed is changed to the categorical form, the defect in the reasoning appears as the fallacy of undistributed middle: —

> If this tree is an oak, it will have rough bark and acorns, (every)
> This tree has rough bark and acorns,

> Therefore it is an oak.

When this argument is expressed in categorical form, it is at once clear that the middle term is not distributed in either the major or minor premise: —

All oak trees are trees having rough bark and acorns,
This tree is a tree having rough bark and acorns,

Therefore this tree is an oak.

The change from the categorical to the hypothetical form of argument, then, does not imply any essential change in the nature of the reasoning process itself. Nevertheless, it is important to note that hypothetical propositions and hypothetical arguments emphasize one aspect of thinking, which is entirely neglected by the theory of the categorical syllogism. When dealing with the extension of terms (§ 16), we pointed out that every term, as actually used in a proposition, has both an extensive and an intensive function. That is, the terms of a proposition are employed both to name certain objects or groups of objects, and to connote or imply certain attributes or qualities. In the proposition, 'these are oak trees,' the *main* purpose is to identify the trees given in perception with the class of oak trees. When, on the other hand, we say, 'ignorant people are superstitious,' the proposition does not refer directly to any particular individuals, but states the necessary connection between ignorance and superstition. Although the existence of ignorant persons who are also superstitious is *presupposed* in the proposition, its most prominent function is to assert a connection of attributes which is wholly impersonal. We may perhaps say that, in spite of the categorical form, the proposition is essentially hypothetical in character. Its meaning might very well be expressed by the statement, 'if a man is ignorant, he is also superstitious.' What is here emphasized is not the fact that ignorant persons exist, and are included in the class of superstitious persons, but rather the general law of the necessary connection of ignorance and

superstition. The existence of individuals to whom the law applies is, of course, presupposed by the proposition. It is not, however, its main purpose directly to affirm their existence.

We have reached, then, the following position: Every judgment has two sides, or operates in two ways. On the one hand, it asserts the existence of individual things, and sets forth their qualities and relations to other things. But, at the same time, every judgment seeks to go beyond the particular case, and to read off a general law of the connection of attributes or qualities which shall be true universally. In singular and particular propositions, the categorical element — the direct assertion of the existence of particular objects — is most prominent, although even here the hint or suggestion of a general law is not altogether absent. When we reach the universal proposition, however, the reference to particular things is much less direct, and the meaning seems capable of expression in hypothetical form.

Now in the chapters on the categorical syllogism this latter aspect of judgments has been left out of account. Propositions were there interpreted as referring directly to objects, or classes of objects (cf. § 23). The proposition, S is P, for example, was taken to affirm that some definite object, or class of objects, S, falls within the class P. And the fact that it is possible to apply this theory shows that it represents one side of the truth. But the student must sometimes have felt that, in this procedure, the most important signification of the proposition is lost sight of. It seems absurd to say, for example, that in the proposition, 'all material bodies gravitate,' the class of 'material bodies' is included in the wider class of 'things that gravitate.' The main purpose of the judgment is evidently

to affirm the necessary connection of the attributes of materiality and gravitation. The judgment does not refer directly to things, or classes of things at all, but asserts without immediate reference to any particular object, *if* material, *then* gravitating. The propositions of geometry are still more obviously hypothetical in character. 'The three angles of a triangle are equal to two right angles,' for example, cannot, without violence, be made to mean that the subject is included in the class of things which are equal to two right angles. The main purpose of the proposition is obviously to assert the necessary connection of the 'triangularity' and the equality of angles with two right angles, and not to make any direct assertion regarding any actually existing object or group of objects.

We reach, then, the following conclusion: Our thought is at once both categorical and hypothetical. As categorical, it refers directly to objects and their relations. The terms of the proposition are then taken in extension to represent objects or groups of objects, and the copula to assert the inclusion of the subject in the predicate, or, in cases of negative propositions, to deny this relation. As hypothetical, the reference to things is much more indirect. The terms of the proposition are no longer regarded as representing objects or classes, but are interpreted from the point of view of *intension*. The judgment affirms or denies the connection of the qualities or attributes *connoted* by the terms, and not that of the objects which they denote. Sometimes the one aspect of thought, sometimes the other, is the more prominent.

In sense-perception and in simple historical narration, assertions are made directly and categorically regarding things and events. The main interest is in particular

objects, persons, or events, and our judgments refer directly and unambiguously to them. But, as we have already seen, our thought from its very beginning attempts to get beyond the existence of particular things and events, and to discover what qualities of objects are necessarily connected. We pass from perception and observation to explanation, from the narration of events, to the discovery of the law of their connection. And, as a result of this advance, our judgments deal no longer exclusively with particular objects and events, and the fact of their relation, but with the general laws of the connection between attributes and qualities. There is, of course, no fixed point at which we pass from the categorical to the hypothetical aspect of thinking. But, in general, as we pass from judgments of sense-perception and memory, to a statement of theories and laws, the hypothetical element comes more and more clearly into the foreground. We have seen that it is almost impossible to interpret propositions regarding geometrical relations as referring directly to classes of objects. In the same way, it is evident that propositions which state general laws are more truly hypothetical than categorical. When we assert that 'all men are mortal,' the proposition does not intend to state a fact in regard to each and every man, or to refer directly to individuals at all, but to express the essential and necessary relation between humanity and mortality. A proposition which is essentially hypothetical in character may then be expressed in categorical form. It must be remembered that it is not the form, but the purpose or function of a proposition, which determines its character. The hypothetical form, however, does justice to an aspect of thought which is especially prominent in the universal

laws and formulas of scientific knowledge, and which is not adequately represented by the theory of subsumption, or the inclusion of the subject in the predicate.

§ 42. **Disjunctive Arguments.** —A disjunctive proposition, as we have already seen, is of the form, 'A is either B, or C, or D'; a triangle is either right-angled, obtuse-angled, or acute-angled. It is sometimes said to be the union of a categorical and a hypothetical proposition. On the one hand, it asserts categorically regarding A, and without reference to any external condition. But the disjunctive proposition is not simple like the categorical proposition: it states its results as a series of related conditions and consequences. If A is not B, it tells us, it must be either C or D; and if it is C, it follows that it cannot be B or D.

A disjunctive proposition may at first sight appear to be a mere statement of ignorance, and, as such, to be less useful than the simple categorical judgment of perception. And it is true that the disjunctive form may be employed to express lack of knowledge. 'I do not know whether this tree is an oak or an ash'; 'he will come on Monday or some other day.' A true disjunctive proposition, however, is not a mere statement of ignorance regarding the presence or absence of some fact of perception. It is an attempt, on the part of intelligence, to determine the whole series of circumstances or conditions within which any fact of perception may fall, and to state the conditions in such a way that their relations are at once evident. And to do this implies positive knowledge. In the first place, the enumeration of possibilities must be exhaustive, no cases must be overlooked, and no circumstances left out of account. Secondly, the members of the proposition must be taken so as

to be really disjunctive. That is, they must be exclusive of one another. We cannot combine disjunctively any terms we please, as 'perhaps this' *or* 'perhaps that.' But it is only when we understand the systematic connections of things in the field in question, that we are able to express these connections in the form, *either* B *or* C, and thus assert that the presence of one excludes the other.

A disjunctive proposition, then, presupposes systematic knowledge, and is consequently the expression of a comparatively late stage in the evolution of thought. It is true that disjunction may involve doubt or ignorance regarding any particular individual. We may not be able to say whether A is B or C or D. But, before we can formulate the disjunctive proposition, we must be already acquainted with the whole set of possible conditions, and also with the relation in which those conditions stand to one another. Our knowledge, when capable of being formulated in the disjunctive major premise of an argument, is so exhaustive and systematic, that the application to a particular case effected by the minor premise appears almost as a tautology. This will be evident in the disjunctive arguments given below.

There are two forms of the disjunctive syllogism. The first is sometimes called the *modus tollendo ponens*, or the mood which affirms by denying. The minor premise, that is, is negative, and the conclusion affirmative. The form is, —

A is either B or C,
A is not C,

Therefore A is B.

The negative disjunctive argument has an affirmative minor premise. It is known as the *modus ponendo tollens*,

or the form which, by affirming one member of the disjunctive series, denies the others, —

A is B or C or D,
But A is B,

Therefore A is neither C nor D.

It is, of course, a very simple matter to draw the conclusion from the premises in these cases. As we have already indicated, the real intellectual work consists in obtaining the premises, especially in discovering the relations enumerated in the major premise. It is in formulating the major premise, too, that errors are most likely to arise. As already pointed out, it is essential that the disjunctive members shall be exhaustively enumerated, and also that they shall exclude one another. But it is not always easy to discover all the possibilities of a case, or to formulate them in such a way as to render them really exclusive. If we say, 'he is either a knave or a fool,' we omit the possibility of his being both the one and the other to some extent. A great many statements which are expressed in the form of disjunctive propositions are not true logical disjunctives. Thus we might say, 'every student works either from love of learning, or from love of praise, or for the sake of some material reward.' But the disjunction does not answer the logical requirements; for it is possible that two or more of these motives may influence his conduct at the same time. The disjunctive members are neither exclusive nor completely enumerated.

§ 43. **The Dilemma.** — A dilemma is an argument which includes all possible assertions about its subject-matter under the head of alternatives that involve further con-

sequences, so that one set of consequences or the other must be admitted whichever alternative be allowed. In form, 'a dilemma is a compound hypothetical syllogism, partly disjunctive in form.' The major premise is always hypothetical, and the disjunction is usually stated in the minor premise. As the word is used in ordinary life, we are said to be in a dilemma whenever there are but two courses of action open to us, and when both of these have unpleasant consequences. In the same way, the logical dilemma when used controversially shuts an opponent in to a choice between alternatives, either of which leads to a conclusion he desires to avoid.

The first form, which is sometimes called the **Simple Constructive Dilemma**, yields a simple or categorical conclusion: —

> If A is B, C is D; and if E is F, C is D,
> But either A is B, or E is F,
>
> ---
>
> Therefore C is D.

It will be noticed that the minor premise affirms disjunctively the antecedents of the two hypothetical propositions which form the major premise, and that the conclusion follows whichever alternative holds. We may take as a concrete example of this type of argument: —

> If a man acts in accordance with his own judgment, he will be criticised; and if he is guided by the opinions and rules of others, he will be criticised,
>
> But he must either act in accordance with his own judgment, or be guided by the opinions of others,
>
> ---
>
> Therefore, in any case, he will be criticised.

The **Simple Destructive Dilemma** also yields a categorical conclusion. But in this form of the dilemma, the major premise has one antecedent and two consequents, and these consequents are denied in the minor premise. The antecedent is therefore denied in the conclusion. A famous example is the argument of Zeno to show that it is against reason to believe that motion really takes place: —

If a thing moves, it must move either in the place where it is or in the place where it is not,

But it cannot move where it is, nor can it move where it is not,

---

Therefore it cannot move.

It is worth noticing that in this example the minor premise is not disjunctive; that is, it denies the consequents of the major premise *together*, and not disjunctively. All the disjunction here is in the second part of the major premise. The Simple Destructive Dilemma is the only form in which this occurs, and the disjunction may be in the minor premise in this form also.

The hypothetical propositions which make up the major premise of a dilemma do not usually have the same antecedent or consequent, as is the case in the examples just given. When the antecedents and consequents involved are different, the dilemma is said to be complex, and the conclusion has the form of a disjunctive proposition. In the **Complex Constructive Dilemma**, the minor premise affirms disjunctively the antecedents of the major, and the conclusion is consequently affirmative. We may take, as an example, the argument by which the Caliph Omar is said to have justified the burning of the Alexandrian library: —

If these books contain the same doctrines as the Koran, they are unnecessary; and if they are at variance with the Koran, they are wicked and pernicious,

But they must either contain the same doctrines as the Koran or be at variance with it,

---

Therefore these books are either unnecessary or wicked and pernicious.

A fourth form, the **Complex Destructive Dilemma,** obtains a conclusion made up of two negations disjunctively related, by denying disjunctively the consequents of the hypothetical propositions that form the major premise of the argument. We may take the following example: —

If an officer does his duty, he will obey orders; and if he is intelligent, he will understand them,

But this officer either disobeyed his orders, or else he misunderstood them,

---

Therefore, he either did not do his duty, or else he is not intelligent,

By taking more than two hypothetical propositions as major premise, we may obtain a Trilemma, a Tetralemma, or a Polylemma. These forms, however, are used much less frequently than the Dilemma.

The dilemma is essentially a polemical or controversial form of argument. Its object, when so used, as we have stated, is to force an unwelcome conclusion upon an adversary by confining him to a choice between two alternatives, either of which necessarily leads to such a conclusion. We sometimes speak of the horns of the dilemma, and of our adversary as 'gored,' whichever horn he may choose. Di

lemmas, however, like all controversial arguments, are more often fallacious than valid. The minor premise of a dilemmatic argument, as we have already seen, is a disjunctive proposition with two members. But it is very rarely that two possibilities exhaust all the possible cases. The cases enumerated, too, may not exclude each other, or be real alternatives at all. The dilemma is thus subject to all the dangers which we have already noticed in the case of the disjunctive argument. In the minor premise, in addition, it is necessary to see that the canon of the hypothetical syllogism, 'affirm the antecedent or deny the consequent,' is observed. If this rule is not obeyed, the logical form of the argument will not be valid.

A dilemmatic argument may be attacked in three ways, the traditional names for which are continuations of the metaphor of the 'horns.'

(1) One may 'escape between the horns.' This is simply to point out that the alternatives presented in the minor premise are not exhaustive, and that there are one or more other possibilities left unmentioned.

(2) The dilemma may be 'taken by the horns.' That is, one may accept the alternative antecedents proposed as exhaustive, but deny that one or both of the consequents asserted really follow from them. For an example, let us take this argument: —

If we have trusts, prices will be excessive; and if we do not have them, our manufacturing industries will fail to meet foreign competition,

But we must either have trusts or not have them,

Therefore either prices will be excessive or our manufacturing industries will fail to meet foreign competition.

One might reply to this either by denying that there is any inevitable connection between trusts and excessive prices, or by denying that trusts are necessary to enable us to compete with foreign firms.

(3) Sometimes, as a reply to a defective dilemma, a counter-dilemma is proposed, leading to an exactly opposite conclusion. When this is done, the original dilemma is said to be 'rebutted.' Whenever such an opposition is possible, each of the two dilemmas by itself fails to state exhaustively either the possible antecedents, or else the consequents following from the given antecedents. Formal rebuttal, therefore, is rather a rhetorical device for showing up the weakness of an opponent's position, than a logical argument for the direct proof of one's own conclusion.

A classical example of such rebuttal is the famous *Litigiosus.*— Protagoras the sophist is said to have made an agreement to teach Euathlus the art of pleading for a fee, one-half of which was to be paid to him when he was fully instructed, and the other half when he won his first case in court. Euathlus put off beginning his practice, and Protagoras finally brought suit for the other half of his fee. Protagoras offered the following argument in his own behalf:—

Euathlus

If Euathlus loses this case, he must pay me, by the judgment of the court; and if he wins it, he must pay me in accordance with the terms of his contract,

But he must either lose it or win it,

Therefore he must pay me in any case.

Euathlus then offered the following rebuttal:—

If I win the case, I ought not to pay, by the judgment of the court; and if I lose it, I ought not to pay, by the terms of the contract,

But I must either win it or lose it,

Therefore I ought not to pay.

The onesidedness of dilemmas which directly confront each other

in this fashion is evident in this example. For a complete statement of the case, the major premises of both should be combined. There are really two points of view, or standards of reference, involved in each alike — the expected judgment of the court, and the terms of the contract. Protagoras states the consequent of his first antecedent in accordance with the first standard, and the consequent of the second antecedent in accordance with the second standard. Euathlus simply reverses the application of the standards. But both disputants make use of the two standards alternately, when one only can really be applied. Either the literal terms of the contract must be observed, and in that case there can be no judgment of the court at all, since the proper ground of action — *i.e.* Euathlus having won his first suit — is not present. The suit must simply be dismissed. Or else, if a judgment in equity is to be granted, and the contract interpreted in accordance with its spirit and intention, and not with its letter, the appeal is to the judgment of the court on the whole case presented, and this judgment will be either for or against Euathlus. There is, therefore, no real dilemma involved in the circumstances at all, the appearance of it in each argument being due to the presence of two contradictory points of view.

All dilemmas related in this way of direct opposition, using premises of the same terms, will be found to involve a similar neglect of some aspect of the situation; and this is why we have said that a dilemma in rebuttal, while a striking rhetorical device for attacking an opponent's position, does nothing to establish the truth of one's own. Indeed, if the rebutting dilemma be allowed to remain unsupported by any further argument, it may be considered as presumptive proof that neither party to the debate has any right to a positive conclusion in the matter. Another and simpler example may make this clearer: —

If a man is single, he is unhappy because he has no one to take care of him, and if he is married, he is unhappy because he has to take care of a wife. (Major premise of original.)

If a man is married, he is happy because he has a wife to take care of him; and if he is single, he is happy because he has no one to take care of. (Major premise of rebutting dilemma.)

Here, as in the former example, the vague and shifting use of any standard of reference is apparent in both the original and the rebutting dilemma. There is no attempt to define terms, or to bring the differen: standards into relation; the argument moves and has its being in the mere limbo of undefined phrases where it seems possible to prove anything, just because it is possible to prove nothing.

REFERENCES

Especially for § 40

J. S. Mill, *Logic*, Bk. I., Ch. V.

C. Sigwart, *Logic*, Pt. I., Ch. VII.

W. Minto, *Logic Inductive and Deductive*, pp. 129–138, and pp 214–225.

F. H. Bradley, *The Principles of Logic*, Bk. I., Ch. II.

B. Bosanquet, *The Essentials of Logic*, Lecture VI.

(On § 42) H. W. B. Joseph, *An Introduction to Logic*, pp. 330–337.

W. R. Boyce Gibson, *The Problem of Logic*, pp. 271–277, 292–295.

CHAPTER XII

FALLACIES OF DEDUCTIVE REASONING

§ 44. **Classification of Fallacies.** — A Fallacy may be defined as a conclusion or interpretation, resulting from processes of thinking which claim to be valid, but which fail to conform to the requirements of logic. Various other terms, like 'Sophism,' 'Paralogism,' etc., are employed as more or less exact synonyms. We shall hereafter treat of the fallacies or errors to which inductive reasoning is most subject (Ch. XX.). At present, however, it is necessary to consider the fallacies which are likely to attend the employment of the syllogistic form of reasoning. In considering the subject, we shall find that many fallacies belong equally to both kinds of reasoning. This is especially true of errors which arise from the careless use of words.

The first systematic account of fallacies was given in Aristotle's treatise, *On Sophistical Difficulties* (**περὶ σοφιστικῶν ἐλέγχων**). In this work, Aristotle divides fallacies into two classes: those which are due to language (**παρὰ τὴν λέξιν**, or, as they are usually called, fallacies *in dictione*), and those which are not connected with language (**ἔξω τῆς λέξεως**, *extra dictionem*). Under the first head, he enumerates six kinds of fallacies, and under the second, seven. Aristotle's principle of classification is, however, not entirely satisfactory. We must try to find some positive principle or principles of classification which will render us more assistance in understanding the relations between the vari-

ous fallacies than is afforded by Aristotle's division into those which belong to language, and those which do not.

In the strict sense of the word, a fallacy is to be defined as an error in reasoning. In the syllogism, however, propositions or premises form the data or starting-point. If, now, these propositions are not properly understood, the conclusions to which they lead are likely to be false. We may then first divide fallacies into **Errors of Interpretation**, and **Fallacies in Reasoning.** Errors in interpreting propositions might, perhaps, be more properly treated in a work on rhetoric than in a chapter on logical fallacies. But it has been the custom ever since the time of Aristotle to include in the enumeration of logical fallacies a number of errors which are likely to arise in interpreting propositions. Moreover, as we saw in Chapter VII., there are certain processes of interpretation, like Obversion and Conversion, which are sometimes called immediate inference, and which require a knowledge of the logical structure of propositions.

The Fallacies which arise in the process of reasoning, we may again divide into **Formal Fallacies**, or violations of the syllogistic rules, and **Material Fallacies.** The latter class may be further divided into **Fallacies of Equivocation** (including Ambiguous and Shifting Terms, Composition, Division, Accident, and the Dilemmatic Fallacy), and **Fallacies of Presumption** (including Petitio Principii, Irrelevant Conclusion, Non Sequitur, and Complex Questions). The following table will summarize this classification: —

FALLACIES

- *Errors in Interpretation*
  - (1) Illogical Obversion or Conversion
  - (2) Amphiboly
  - (3) Accent
- *Mistakes in Reasoning*
  - *Formal*
    - In Categorical Arguments
      - (1) Four Terms
      - (2) Undistributed Middle
      - (3) Illicit Major
      - (4) Illicit Minor
      - (5) Negative Premises
    - In Hypothetical Arguments
      - (6) Denying the Antecedent
      - (7) Affirming the Consequent
    - In Disjunctive Arguments
      - (8) Imperfect Disjunction
  - *Material*
    - *Equivocation*
      - (1) Ambiguous and Shifting Terms
      - (2) Composition
      - (3) Division
      - (4) Accident
      - (5) Dilemmatic Fallacy
    - *Presumption*
      - (1) Petitio Principii
      - (2) Complex Question
      - (3) Irrelevant Conclusion
      - (4) Non Sequitur

§ 45. **Errors in Interpretation.**—This class of fallacies results from imperfect understanding of the meaning of propositions. They are not, then, strictly speaking, errors of reasoning at all. If, however, the propositions employed as premises in an argument are not correctly understood, the conclusions founded upon them are likely to be erroneous. And even if the proposition, which is wrongly interpreted, is not made the basis of further reasoning, it is in itself the result of an intellectual error against which it is possible to guard. We do not, of course, profess to point out all the possible sources of error in interpreting propositions. The only rule applicable to all cases which can be given is this: Accept no proposition until you understand its exact meaning, and know precisely what it implies. Deliberation

and attention, both with regard to our own statements and those of others, are the only means of escaping errors of this kind.

(1) *Illogical Obversion or Conversion.*—In a previous chapter (Ch. VII.), we have treated of Obversion, Conversion, Contraposition, etc., and shown the rules to be followed in stating the obverse or the converse of a proposition. In Obversion, we interpret or show what is involved in a proposition, by stating its implications in a proposition of the opposite quality. And unless we have clearly grasped the meaning of the original proposition, mistakes are likely to arise in changing from the affirmative to the negative form of statement, or from the negative to the affirmative. Thus, we should fall into an error of this kind if we should take the proposition, 'honesty is always good policy,' to be the equivalent of, or to imply, the statement, 'dishonesty is always bad policy.' Nor can we obtain by obversion the proposition, 'all citizens are allowed to vote,' from, 'no aliens are allowed to vote.'

In Conversion, we take some proposition, A is B, and ask what assertion it implies regarding the predicate. Does 'all brave men are generous' imply also that 'all generous men are brave'? This is, perhaps, the most frequent source of error in the conversion of propositions. I do not mean that in working logical examples we are likely to convert proposition A simply, instead of by limitation. But in the heat of debate, or when using propositions without proper attention, there is a natural tendency to assume that a proposition which makes a universal statement regarding the subject does the same with regard to the predicate. And, although such errors are very obvious when

pointed out, — as, indeed, is the case with nearly all logical fallacies, — they may very easily impose upon us when our minds are not fully awake, that is, when attention is not active and consciously on guard, or when they occur in the midst of a long and complicated argument. Of the other methods of interpretation perhaps contraposition is most likely to be a source of error. We have already (§ 28) given the rules for obtaining the contrapositive of any proposition. Some practice in working examples will enable one to perceive readily what is the logical contrapositive to any proposition, and what forms are fallacious.

(2) *Amphiboly*, or amphibology (*ἀμφιβολία*), consists in misconception arising from the ambiguous grammatical construction of a proposition. A sentence may have two opposite meanings, but one may be more natural and prominent than the other. A deception may be practised by leading a person to accept the meaning more strongly suggested, while the significance intended is the very opposite, as, *e.g.* 'I hope that you the enemy will slay.' In Shakespeare's *Henry VI.*, we have an instance of amphiboly in the prophecy of the spirit, that "the Duke yet lives that Henry shall depose." Many of the famous utterances of the ancient oracles were of this character, as the reported answer to Crœsus when he inquired at Delphi: "If Crœsus should wage war against the Persians, he would destroy a mighty empire." The more ambiguous the oracle, the more readily it could be explained in accordance with the event, which in this case was the destruction of the empire of Crœsus.

(3) The *Fallacy of Accent* is a misconception due to the accent or emphasis being placed upon the wrong words in a sentence. It may, therefore, be regarded as a rhetorical

rather than as a logical fallacy. Jevons's examples of this fallacy may be quoted in part. "A ludicrous instance is liable to occur in reading Chapter XIII. of the First Book of Kings, verse 27, where it is said of the prophet, 'And he spake to his sons, saying, Saddle me the ass. And they saddled *him*.' The italics indicate that the word *him* was supplied by the translators of the authorized version, but it may suggest a very different meaning. The commandment, 'Thou shalt not bear false witness against thy neighbour,' may be made by a slight emphasis of the voice on the last word to imply that we are at liberty to bear false witness against other persons. Mr. De Morgan, who remarks this, also points out that the erroneous quoting of an author, by unfairly separating a word from its context, or italicizing words which were not intended to be italicized, gives rise to cases of this fallacy."[1] Jevons is also authority for the statement that Jeremy Bentham was so much afraid of being led astray by this fallacy that he employed a person to read to him whose voice and manner of reading were particularly monotonous.

But these misinterpretations of single propositions are comparatively trivial instances of this fallacy. In a broader sense, the fallacy appears in connected arguments of any kind in which, while the facts are not actually misstated, certain aspects of them are so disproportionately dwelt upon and emphasized, at the expense of the rest, that a false idea of the subject in its entirety is the result. In this wider form, this fallacy is one that may be described as the particular vice of *special pleading;* and the caution that may be suggested against it is, in the language of the

[1] Jevons, *Lessons in Logic*, p 174.

astronomer, to make allowances for the 'personal equation' both in one's own thinking and in that of others.

§ 46. **Formal Fallacies.** — We shall follow our table, and deal with mistakes of Reasoning under the two headings of Formal Fallacies, and Material Fallacies. Formal fallacies arise from violations of the rules of the syllogism. The breaches of these rules have been already pointed out and illustrated in the discussion of the various forms of syllogistic argument. The analysis of arguments, with a view to the detection of such fallacies, where any exist, is a very important exercise, and affords valuable mental discipline. It seems only necessary here to add a remark regarding the first fallacy on our list, that of Four Terms, or *Quaternio Terminorum*, as it is usually called by logicians.

The first canon of the categorical syllogism states that 'a syllogism must contain three and only three terms.' This rule would of course be violated by such an argument as;

Frenchmen are Europeans,
Englishmen are Anglo-Saxons,

Therefore Englishmen are Europeans.

It is so obvious that this example does not contain a real inference that no one would be likely to be misled by the pretence of argument which it contains. In some cases, however, a term may be used in two senses, although the words by which it is expressed are the same. The following example may be given: —

Every good law should be obeyed,
The law of gravitation is a good law,

Therefore the law of gravitation should be obeyed.

Here we have really four terms. The word 'law,' in the first proposition, means a command given or enactment made by some persons in authority. A 'good law' in this sense then means a just law, or one which has beneficial results. But in the second proposition it signifies a statement of the uniform way in which phenomena behave under certain conditions. A 'good law' from this point of view would imply a correct statement of these uniformities. It is interesting to note that this example may also be regarded as an instance of Equivocation, and classified as a case of an **ambiguous middle term**. It is often possible to classify a fallacy under more than a single head.

There are, however, cases where an argument may seem at first sight to have four terms, but where the defect is only verbal. The matter must, of course, be determined by reference to the meaning of terms and not merely to the verbal form of expression. It is ideas or concepts, and not a form of words, which are really operative in reasoning.

§ 47. **Material Fallacies.** — What are called material fallacies do not result from the violation of any specific logical rules. They are usually said to exist, not in the *form*, but in the *matter* of the argument. Consequently, it is sometimes argued, the detection and description of them do not properly belong to logic at all. We have found, however, that all these fallacies have their source in Equivocation and Presumption. They thus violate two of the fundamental principles of logical argument. For all logical reasoning presupposes that the terms employed shall be clearly defined, and used throughout the argument with a fixed and definite signification. And, secondly, logic requires that the conclusion shall not be

assumed, but derived strictly from the premises. The violation of these principles is, therefore, a proper matter of concern to the logician. We shall treat first of the fallacies of Equivocation.

(*A*) The fallacies of Equivocation have been enumerated as Ambiguous and Shifting Terms, Composition, Division, and Accident. These all result from a lack of clearness and definiteness in the terms employed. We shall deal with them briefly in order.

(1) The phrase, **Ambiguous and Shifting Terms**, describes the first fallacy of this group. A special case of it appears in the *Fallacy of Ambiguous Middle.* It is obvious that the middle term cannot form a proper standard of comparison, if its meaning is uncertain or shifting. A standard of measure must be fixed and definite. One illustration of this case of the fallacy will be sufficient: —

> Partisans are not to be trusted,
> Democrats are partisans,
>
> ---
>
> Therefore Democrats are not to be trusted.

The middle term, ' partisan,' is evidently used in two senses in this argument. In the first premise it signifies persons who are personally, or with undue bias, interested in some cause ; and in the latter it simply denotes the members of a political party.

But either the *Minor* or the *Major Terms* of a syllogism may also be ambiguous as well as the Middle, and be used in a different sense in the conclusion, than they are in their respective premises. One example of ambiguity in the Major term may be given: —

What is not forbidden by law, no one has a right to prevent my doing.

Reprinting the works of foreign authors is not forbidden by law.

Therefore, no one has a right to prevent me from reprinting such works.

Here 'right' in the major premise means 'legal right' and in the conclusion 'moral right'; 'prevent' in the major premise implies restraint by force or penalty, if necessary, but in the conclusion it is used to mean the use of any means of restraint whatever. The use of the word 'right' in various meanings is a frequent source of such fallacies, and the comment of J. S. Mill on it might well be read by the student.[1]

It is often the case, especially where the major or the minor term is concerned, that this fallacy cannot be perpetrated without some verbal change in the terms, which, however, is made plausible by some similarity in the words employed. Aristotle described some of the ways in which such shifts in meaning are frequently disguised under the name of the *Fallacy of Figure of Speech.* Words which have the same roots may sometimes be substituted one for another, though they have taken on different meanings; as, for example, the noun 'presumption,' the verb 'presume,' and the adjective 'presuming.' Or we may get a wrong meaning for a word from its having a similar inflection with other words of different meaning. An example of this is the passage in which J. S. Mill argues that as what is seen is *visible,* and what is heard is *audible,* so what is desired must be *desirable*

[1] *Cf. System of Logic,* Bk. V., Ch. VII., § 1.

— therefore morally good. But desirable means primarily not what is or can be, but what ought to be desired.

Then, again, as Archbishop Whately points out, this fallacy may be committed by using a term at one time in its usual meaning, and at another in its strict etymological sense. Thus, he remarks, it is frequently argued from the strict original meaning of 'represent,' that a representative in the legislature is merely the spokesman of his constituents, and has no right to use his independent judgment in his voting or public utterances. Such reasoning, it is obvious, does not necessarily prove anything; for the original meaning of a term may be widely different from the true nature and proper functions of the things and persons to which it later comes to be applied.

But trivial as such merely verbal argument may seem when exposed, it is often a source of confusion. Thus a lawyer, for example, might pass from a proper insistence on following the original intention and meaning in interpreting the words of a statute, to the mistaken attempt to determine how a new law should be framed by considering what the accepted name of the things to which it is to apply meant when it was first used. And when an argument is long, and is not arranged in syllogistic form, fallacies of this kind are much more difficult of detection than in the simple examples which have been given. It is of the utmost importance, then, to insist on realizing clearly in consciousness the ideas for which each term stands, and not to content ourselves with following the words.

(2) The fallacy of **Composition** arises when we affirm something to be true of a whole, which holds true only of one or more of its parts when taken separately or *distributively*.

Sometimes the error is due to confusion between the distributive and collective signification of 'all,' as in the following example: —

> All the angles of a triangle are less than two right angles.
> *A*, *B*, and *C* are all the angles of this triangle,
> ———————————————————————
> Therefore *A*, *B*, and *C* are less than two right angles.

It is, of course, obvious that 'all the angles of a triangle' in the major premise signifies each and every angle when taken by itself, and that the same words in the minor premise signify all the angles collectively. What is true of all the parts taken separately, is not necessarily true of the whole. We cannot say that because no one member of a jury is very wise or very fair-minded, the jury as a whole are not likely to bring in a just verdict. The members may mutually correct and supplement each other, so that the finding of the jury as a whole will be much fairer and wiser than the judgment of any single individual composing it. Another instance of this fallacy which is often quoted is that by which protective duties are sometimes supported: —

> The manufacturers of woollens are benefited by the duty on woollen goods; the manufacturers of cotton by the duty on cotton; the farmer by the duties on wool and grain; and so on for all the other producing classes; therefore, if all the products of the country were protected by an import duty, all the producing classes would be benefited thereby.

But, because each class would be benefited by an import tax upon some particular product, it does not necessarily follow that the community as a whole would be benefited, if all products were thus protected. For, obviously, the

advantages which any class would obtain might be more than offset by the increased price of the things which they would have to buy. On the other hand, it would be necessary to take into consideration the fact that an increase in the prosperity of one class indirectly brings profit to all the other members of the same society. We cannot regard a whole as simply a sum of parts, but must consider also the way in which the parts act and react upon one another.

(3) The fallacy of **Division** is the converse of Composition. It consists in assuming that what is true of the whole is also true of the parts taken separately. Some term, which is used in the major premise collectively, is employed in a distributive sense in the minor premise and conclusion. The following example will illustrate this: —

All the angles of a triangle are equal to two right angles,
A is an angle of a triangle,

Therefore *A* is equal to two right angles.

To argue that, because some measure benefits the country as a whole, it must therefore benefit every section of the country, would be another instance of this fallacy. Again, we may often find examples of both Division and Composition in the practice so common in debate of 'taking to pieces' the arguments by which any theory or proposed course of action is justified. A person would be guilty of Division if he should argue that, because a complex theory is not completely proved, none of the arguments by which it is supported have any value. It is, however, perhaps more common to fall into the fallacy of Composition in combating the arguments of an opponent. Some measure, for example, is proposed to which a person finds himself in opposition.

It is usually easy to analyze the different arguments which have been advanced in support of the measure, and to show that no single one of these *taken by itself* is sufficient to justify the change. The conclusion may then be drawn with a fine show of logic that *all* the reasons advanced have been insufficient. This, of course, is to neglect the combined effect of the arguments; it is to assume that what is true of 'all,' taken distributively, is also true of 'all,' when taken in conjunction. And often, as in the case of circumstantial evidence, what gives a chain of inference its strength is not the particular arguments or facts taken each for itself, but what is sometimes called the 'consilience' of these particulars; that is, the fact that they form a *connected* body of proof all pointing to one conclusion, so that each part has a significance, taken in its relation to the whole proof, which by itself it would not have.

But an affirmative form of the fallacy just mentioned is also possible in cases where it is attempted to prove the possibility or probability of a conclusion by pointing to even the high probability, taken *separately*, of each one of a number of conditions which must be true *together*, in order that the conclusion may be true. The mere fact of a large number being possible separately may even seem to the careless to make the conclusion more probable, when really, if the conditions must be present together, this becomes less probable the more there are of them. What should be proved in such cases is of course the probability of the conditions as a body; and this probability is always less than that of the least probable among them taken as occurring by itself. Suppose, for example, that we were considering the probability of a report being true which had been handed down in succession

by A, B, C, and D. What we have to consider is not the probability of any one of these persons reporting correctly by himself, but that of the correct transmission of the report through the entire series. Thus, it will be found that if the probability of mistake by any one of these persons is only 1 in 5, the probability of error in the final result will be approximately 3 in 5.

(4) It is often difficult to distinguish the various forms of the fallacy of **Accident** from Composition and Division. We have seen that the last two rest upon a confusion between whole and part; or, as we have already expressed it, on an equivocation between the distributive and collective use of terms. The fallacies of accident are also due to equivocation. But, in this case, the confusion is between essential properties and accidents, between what is true of a thing in its real nature, as expressed by its logical definition, and what is true of it only under some peculiar or accidental circumstance; or, in other words, a proper distinction is not made between the general import of a principle and its application to cases where special modifying conditions are present.

There are two forms of this argument which are usually recognized: (*a*) The *Direct* or *Simple* Fallacy of Accident, which consists in arguing that what is true of a thing generally, is also true of it under some accidental or peculiar circumstance; or that a proposition generally true is true in exactly the same way when special conditions are present. The old logicians expressed this in the formula, *a dicto simpliciter ad dictum secundum quid.* The second form is (*b*) the *Converse* Fallacy of Accident, which consists in arguing that what is true of a thing under some condition or accident, can be asserted of it simply or in its essential nature; or

that a statement which is true when certain conditions are present is true generally. The formula for this is, *a dicto secundum quid ad dictum simpliciter.*

It would be an illustration of the direct fallacy to reason that, because man is a rational being, therefore a drunken man or an angry man will be guided by reason. Similarly, we should commit this fallacy if we were to argue that because beefsteak is wholesome food, it would be good for a person suffering with fever or dyspepsia; or to conclude from the principle that it is right to relieve the suffering of others, that we ought to give money to beggars.

It would be a case of the converse fallacy to argue that because spirituous liquors are of value in certain cases of disease, they must therefore be beneficial to a person who is well. We should also be guilty of the same fallacy, if we should conclude that it is right to deceive others, from the fact that it is sometimes necessary to keep the truth from a person who is sick, or to deceive an enemy in time of war.

The fallacies of Accident, like all the fallacies of Equivocation, are largely the result of a loose and careless use of language. The source of both forms of the fallacy is one and the same. They arise fom the careless use of principles or propositions without due regard to the circumstances which determine whether they are properly to be applied, unmodified to the case before us. By qualifying our terms so as to state the exact circumstances involved, they may easily be detected and avoided.

(5) The Dilemmatic Fallacy arises from the equivocal and shifting point of view present in the premises of a dilemma which is open to rebuttal. It has been fully discussed at the end of Chapter XI.

(*B*) *Fallacies of Presumption.* — The fallacies of this group are the result of presumption or assumption on the part of the person making the argument. It is possible (1) to assume the point to be proved, either in the premises of an argument, or in a question (*Petitio Principii*, and Complex Question); or (2) to assume without warrant that a certain conclusion follows from premises which have been stated (*Non Sequitur*); or (3) that the conclusion obtained is really what is required in order to settle the question at issue (Irrelevant Conclusion).

(*a*) **Petitio Principii**, or ' Begging the Question,' is a form of argument which assumes the conclusion to be proved. This may be done in either of two ways. (*a*) We may postulate the fact which we wish to prove, or its equivalent under another name. Thus, for example, we might argue that an act is morally wrong because it is opposed to sound ethical principles. 'The soul is immortal because it is a simple and indecomposable substance,' may be regarded as another example of this assumption. A ' question-begging epithet ' or cant phrase is often used to bring in such an assumption. Thus, Mill remarks, when Cicero discusses whether certain propensities, if kept within limits, might be regarded as virtuous, he calls them *cupiditates*, which of itself implies that they are vicious. We shall have occasion to mention this fallacious use of epithets more at length when we come to discuss the fallacies of inductive reasoning. But (*b*) the question may be begged by making a general assumption covering the particular point in dispute. Thus, if the advisability of legislation regulating the hours of labour in a mine or factory were under discussion, the question-begging proposition, ' all legislation which interferes

with the right of free contract is bad,' might be propounded as a settlement of the whole question.

A special form of this fallacy results when each of two propositions is used in turn to prove the truth of the other. This is known as 'reasoning in a circle,' or *circulus in probando*. This method of reasoning is often adopted when the premise, which has been employed to prove the first conclusion, is challenged. 'I should not do this act, because it is wrong.' 'But how do you know that the act is wrong?' 'Why, because I know that I should not do it.'

It is always necessary, then, to see that the conclusion has not been assumed in the premises. But, since the conclusion always follows *from* the premises, we may say that in one sense the conclusion is always thus assumed. It is, therefore, easy to charge an opponent unjustly with begging the question. De Morgan, in his work on Fallacies, says: "There is an opponent fallacy to the *Petitio Principii* which, I suspect, is of more frequent occurrence: it is the habit of many to treat an advanced proposition as a begging of the question the moment they see that, if established, it would establish the question." All argument must, of course, start from premises to which both parties assent. But candour and fairness forbid us to charge an opponent with *Petitio* because the results of his premises are unwelcome. It was Charles Lamb who humorously remarked that he would not grant that two and two are four until he knew what use was to be made of the admission.

(2) **The Complex Question** is an interrogative form of *Petitio*. It is not really a simple interrogation, but is founded upon an assumption. It tacitly assumes, that is, both

that certain things are true, and that certain other things are false; and therefore any direct answer to it always involves the admission as true of more than one statement. Any discussion or argument whatever, of course, always proceeds on the basis of certain assumptions; but there should be principles that are accepted as true, at least provisionally, by all the parties engaged in the discussion, and they should be as far as possible made clear and definite before discussion begins. In fact, this precaution of making as clear as possible to oneself what one is taking for granted is the proper remedy against all the fallacies of presumption. Examples of this fallacy may be found in popular pleasantries, such as, 'Have you given up your drinking habits?' 'Do the people in your part of the country still carry revolvers?' Disjunctive questions, too, always contain an assumption of this kind: 'Is this an oak or a chestnut?' 'Does he live in Boston or New York?' The 'leading questions' which lawyers frequently use in examining witnesses, but which are always objected to by the opposing counsel, are usually of this character. Further instances may perhaps be found in the demand for explanation of facts which are either false, or not fully substantiated; as, *e.g.*, 'Why does a fish when dead weigh more than when alive?' 'What is the explanation of mind-reading?'

(3) **The Irrelevant Conclusion**, or *Ignoratio Elenchi*, consists in substituting for the conclusion to be proved some other proposition more or less nearly related to it. This fallacy may be the result of an involuntary confusion on the part of the person employing it, or it may be consciously adopted as a controversial stratagem to deceive an opponent or an audience. When used in this latter way, it is usually

intended to conceal the weakness of a position by diverting attention from the real point at issue. This is, indeed, a favourite device of those who have to support a weak case. A counsel for the defence in a law-suit is said to have handed to the barrister presenting the case a brief marked, 'No case; abuse the plaintiff's attorney.' To answer a charge or accusation by declaring that the person bringing the charge is guilty of as bad, or even worse, things, — what is sometimes called the *tu quoque* form of argument — is also an example of this fallacy.

Apart from such wilful perversions or confusions, many unintentional instances of this fallacy occur. In controversial writing, it is very natural to assume that a proposition which has some points of connection with the conclusion to be established, is 'essentially the same thing,' or 'practically the same, as the thesis maintained.' Thus one might take the fact that a great many people are not regular church-goers, as a proof of the proposition that religion and morality are dying out in the country. Many of the arguments brought against scientific and philosophical theories belong to this class. Mill cites the arguments which have been urged against the Malthusian doctrine of population, and Berkeley's theory of matter. We may quote the passage referring to the former: "Malthus has been supposed to be refuted, if it could be shown that in some countries or ages population has been nearly stationary, as if he had asserted that population always increases in a given ratio, or had not expressly declared that it increases only in so far as it is not restrained by prudence, or kept down by disease. Or, perhaps, a collection of facts is produced to prove that in some one country with a dense population the people are better

off than they are in another country with a thin one, or that the people have become better off and more numerous at the same time; as if the assertion were that a dense population could not possibly be well off."[1] Ignorance of the methods proper to the subject under discussion is a prolific source of such fallacies as this. Mere knowledge of facts without knowing their meaning is not enough, and those whose knowledge is of this description do not see what the real questions at issue are, or what constitutes a real proof in different subject-matters. As Whately puts it, 'This is to learn a good many answers without the questions.' The history of modern attempts to 'square the circle' furnishes good examples of this; and scientists of unquestioned authority in their own field are often led astray in this way when they attempt to deal, without proper preparation, with questions belonging to another science, or to philosophy or religion.

There are several cases or forms of Irrelevant Conclusion to which special names have been given, and which it is important to consider separately. When an argument bears upon the real point at issue, it is called *argumentum ad rem*. But, on the other hand, there are the following special ways of obscuring the issue: *argumentum ad hominem*, *argumentum ad populum*, *argumentum ad ignorantiam*, *argumentum ad verecundiam*, *argumentum ad misericordiam*, the *Fallacy of Objections*, and, by extension, the *argumentum ad baculum*.

The *argumentum ad hominem* is an appeal to the character, principles, or former profession of the person against whom it is directed. It has reference to a person or persons,

[1] *Logic*, Bk. V., Ch. VII., § 3.

not to the real matter under discussion. In order to confuse an opponent, and discredit him with the audience, one may show that his character is bad, or that the views which he is now maintaining are inconsistent with his former professions and practice. Or, on the defensive side, the character of the advocate of the point at issue may be praised. Or the argument may be used with the hope of persuading the opponent himself. We then try to convince him that the position which he maintains is inconsistent with some other view which he has previously professed, or with the principles of some sect or party which he has approved. Or we may appeal to his interests by showing him that the action proposed will affect injuriously some cause in which he is concerned, or will benefit some rival sect or party. In all of these cases the real point at issue is, of course, evaded. The only case in which such an argument seems at all admissible for the logical purpose of establishing truth, and not merely securing conviction, is when the known bad character or untrustworthiness of some person is appealed to in order to impeach the evidence he may give. Here it at least assists us to exclude what is false, and is therefore a relevant argument, though one of merely negative character.

The *argumentum ad populum* is an argument addressed to the feelings, passions, and prejudices of people rather than an unbiassed discussion addressed to the intellect. The use of question-begging epithets frequently accompanies this fallacy. The *argumentum ad misericordiam* seems to be only a special case of this fallacy, when an appeal is made to the pity or sympathy which people may be made to feel for a person accused of crime. Or sometimes it may be attempted to recommend some party or cause by arousing such feelings for

its adherents, or a law, by dwelling on the plight of those whom it would perhaps relieve.

The *argumentum ad ignorantiam* is an attempt to gain support for some position by dwelling upon the impossibility of proving the opposite. Thus we cannot prove affirmatively that spirits do not revisit the earth, or send messages to former friends through 'mediums.' Now it is not unusual to find ignorance on this subject advanced as a positive ground of conviction. The argument seems to be: —

It is not impossible that this is so,
What is not impossible is possible,

Therefore it is possible that this is so.

The fallacy arises when we confuse what is only abstractly possible — *i.e.* what we cannot prove to be impossible — with what is really possible, *i.e.* with what we have *some* positive grounds for believing in, though these grounds are not sufficient to produce conviction.

The *argumentum ad verecundiam* is an appeal to the reverence which most people feel for a great name, or for long-established usages. This method of reasoning attempts to settle a question by referring to the opinion of some acknowledged authority, without any consideration of the arguments which are advanced for or against the position. It is, of course, right to attach much importance to the views of great men, and to the presumptive evidence of value given by ancient and continued use; but we must not suppose that the opinions of the great, or the presumed validity of custom, amount, by themselves and unexamined, to final proof, or forbid us to consider the matter for ourselves, if we are competent to do so.

There is, however, a more common, though much less justi-

iable, form of the argument from authority. A man who is distinguished for his knowledge and attainments in some particular field, is often quoted as an authority upon questions with which he has no special acquaintance. The prestige of a great name is thus irrelevantly invoked when no significance properly attaches to it. Thus, for example, a successful general is sometimes supposed to speak with authority upon problems of statecraft, and the opinions of prominent clergymen are quoted regarding the latest scientific or political theories.

The *Fallacy of Objections* consists, as Whately states it, in "showing that there are objections against some plan, theory, or system, and thence inferring that it should be rejected; when that which ought to have been proved is, that there are more or stronger objections against the receiving than the non-receiving of it." This fallacy, he remarks, is "the stronghold of bigoted anti-innovators." In any matter of dispute, there will be objections to any solution offered; but this, of itself, is no disproof of the conclusion attacked, provided we have some positive grounds for it. "There are objections," Dr. Johnson once said, "against a *plenum*, and objections against a *vacuum;* but one of them must be true."

When all these forms of the fallacy fail, there is still one recourse remaining, which takes the matter beyond the boundaries of logic; though, indeed, the other forms are in their way quite as irrelevant. This is the *argumentum ad baculum*, which we may translate in current phrase as the 'appeal to the big stick.'

(4) The fallacy of **non sequitur**, or *the Fallacy of the Consequent*, occurs when the conclusion does not really follow from the premises by which it is supposed to be supported. The following example may serve as an illustration: —

Pennsylvania contains rich coal and iron mines,
Pennsylvania has no sea-coast,

Therefore the battle of Gettysburg was fought in that state.

This argument, of course, is thoroughly inconsequent, and would deceive no one. But when the conclusion repeats some words or phrases from the premises, we are likely, when not paying close attention, to be imposed upon by the mere form of the argument. We notice the premises, and remark that the person using the argument advances boldly through 'therefore' to his conclusion. And if this conclusion appears to be related to the premises, and sounds reasonable, the argument is likely to be accepted. The following example will illustrate this: —

Every one desires happiness, and virtuous people are happy,
Therefore every one desires to be virtuous.

A rather frequent form of this fallacy occurs when we think, because we have refuted an argument for a theory, that the theory itself is necessarily false, — which would be true only if the refuted argument was the *only* possible one for the theory. Or, again, we may think that because a conclusion is true, a usual argument for it is also true; thus, for example, we might think that because God exists, the general consent of all mankind, which used to be urged as a proof of His existence, is true. These forms of the fallacy may be regarded as simply a breach, within a continued argument, of the rules of the hypothetical syllogism — 'affirm the antecedent, or deny the consequent.' For in the first form, we argue that because a proof is false, the conclusion which would certainly be true if it were true, is

therefore false; and, in the second, we argue that because a conclusion is true, therefore an argument on which it is usually made to depend is also true.

What is known as the **False Cause** (*non causa pro causa; post hoc ergo propter hoc*) is the inductive fallacy corresponding to the *non sequitur*. In this we assume that one thing is the cause of another merely because we have known them to happen together a number of times. The causal relation is assumed without any analysis or examination, on the ground of some chance coincidence. Thus a change in the weather may be attributed to the moon, or the prosperity of the country to its laws requiring Sunday observance. Or in a case where there is really a causal connection we may take the cause for the effect, or the effect for the cause. Whately's example of this is a good one, because it is a popular fallacy often to be met with, especially where the action of natural selection is not realized. It is frequently assumed, because the animals and men native to countries of inclement climate, where the conditions of life are severe, are usually robust, that the hardships they are forced to undergo in youth are the *cause* of this hardiness; whereas, as a matter of fact, their hardiness was the cause of their having survived the hardships. Popular notions of hygiene are sometimes largely dependent on this confusion. (Cf. § 73.)

REFERENCES

J. S. Mill, *Logic*, Bk. V.
A. Sidgwick, *Fallacies* [Int. Scient. Series].
R. Whately, *Elements of Logic*, Bk. III.

PART II.—INDUCTIVE METHODS

CHAPTER XIII

THE PROBLEM OF INDUCTION

§ 48. **The Problem of Induction.**—In Part I. we have studied the general nature of the syllogism, and have learned what conditions must be fulfilled in order to derive valid conclusions from given premises. But the question how the premises themselves are established was not discussed. It is true that the premises of one syllogism are sometimes proved by means of a Prosyllogism, and that it may be possible to find in turn general propositions to support the premises of this latter argument. But somewhere this process of formal proof must have an end. At last we reach propositions concerning which we can say only that their truth is guaranteed by experience. It is from experience that propositions are obtained like, 'man is by nature a social being,' 'water is composed of hydrogen and oxygen,' which serve as the premises of syllogisms. To say that these propositions are learned through experience, does not however mean that they have been obtained without thinking. For to experience is not merely to feel or to have sensations; it is also to put things together, to interpret, to appreciate to some extent what our sensations stand for and signify. When I say, 'yonder tree is an elm,' this proposition is the outcome of my own thinking; it is my interpretation,

on the basis of past experience, of certain sensations of colour and light and shade, together, it may be, with certain muscular sensations from the movements of the eyes. Our thought is constantly bringing new sensations and perceptions into relation with former experiences, and in this way building up and organizing our world of knowledge. To interpret the real world — not only the physical world, but the psychological and the social world as well — is then the business of thought, and this, as we have seen, is to relate the new in some way to what we already understand. Our sense perceptions, just as they come, are without order or system. Think, for example, of the various things you are sensing at the present time. The greater part of these are not consciously attended to or thought about; they are taken for granted or roughly classified on the basis of some past experience. But if one is really thinking, there is some fact or relation that is taken as a problem, and for which one is seeking an interpretation, *i.e.* some way of thinking this fact or relation that will bring it into place and adjust it to what is already known.

Apart from this task of interpreting the real world, thought has no function, and does not exist. Syllogistic reasoning is not a distinct and separate kind of thinking, but is a necessary part of the work of building up our knowledge of the world in systematic form. Without thinking, then, no knowledge, no real experience. But we must remember that thinking is no mere play of ideas in our heads. It exists only in relation to what is objective and real. In a certain sense it always goes back to a *datum*, to perception. Kant's famous saying that 'perceptions without conceptions (*i.e.* thoughts) are blind, while conceptions without perceptions are empty,' is well worth remembering.

The problem of *Induction*, with which we are primarily concerned in this part of the book, is how we are able to derive from experience general propositions or principles. It is on these, as we have seen, that we base our conclusions in syllogistic reasoning. The difficulty is that experience seems to give information regarding individual things and their qualities only. One learns by experience the qualities of this rose, or of this piece of iron; but how is one to discover the general nature of the rose or of iron as such? As a matter of fact, we are constantly deriving general statements from individual experiences; and in doing this we usually bring up, in a more or less systematic way, a number of cases or instances and use them as the basis of the general statement. And this process of *generalization*, or passing to a general conclusion on the ground of certain instances or cases that have been advanced, may be called *Induction* (ἐπαγωγή). This definition is, of course, only preliminary, and does not attempt to distinguish valid and invalid induction. We have to go on to consider more in detail both the conditions necessary to render the process valid, and the meaning of the generalization at which we arrive.

§ 49. **The Enumeration of Instances.** — In the first place, Induction is not the outcome of a complete enumeration of instances; but from an examination of a certain number we *infer* the general mark or principle that is involved in all the instances. Where all the instances have been examined, the result may be summed up at the end in a proposition that is universal in form; but in such a case there has been no Induction, no passage to any truth that is really general. For example, after measuring each individual in a company and finding that A is less than six feet in height, B less than six

feet, and so on for the rest, I might make the assertion, 'No one in this company is more than six feet tall.' This, however, would be nothing more than a summation of results, and not a genuine Induction at all. Nevertheless, some writers regard such procedure, where all the instances are examined, as the only perfect form of induction. Thus Jevons says: "An Induction, . . . is called Perfect, when all of the possible cases or instances to which the conclusion can refer have been examined and enumerated in the premises."[1] On the other hand, where it is impossible to examine all the cases, the inductive process is regarded as Imperfect by the same writer, and the conclusion expressed in the general law as only probable.

Now this view, though mistaken, is interesting because it assumes that it is the business of Induction to count instances. When it is possible to examine all the cases we can have certainty; when this is impossible (as is usually true), the unexamined instances have to be regarded as more or less probable only. No other conclusion is possible so long as we merely enumerate or cite instances without attempting to analyze them. A mere *factual* connection of two events, P and Q, though experienced a thousand times, does not warrant the universal proposition, 'All P is Q.' As a matter of fact, scientific Induction always does get beyond a mere citation of unanalyzed instances. "Induction which proceeds by merely citing instances," says Bacon, "is a childish affair, and being without any certain principle of inference it may be overthrown by a contradictory instance. Moreover, it usually draws the conclusion from too small a number of instances, taking account only of those that are obvious."[2] This is an excellent

[1] *Elementary Lessons in Logic*, pp. 212–213.

[2] *Novum Organum*, Bk. I., Aph. CV. "Inductio enim quae procedit

description of the popular unscientific way of seeking to establish universal connections between events, by citing random instances where the events have happened to be found together. It is generally easy, for example, to cite instances where dreams have come true, or where one member of a dinner party of thirteen has died within a year. This species of Induction is, as Bacon says, "*res puerilis,*" since it simply *asserts* the connection without justifying it or making it intelligible, by bringing to light any principle of coherency. The possibility of contradictory instances is not excluded, and the cases cited lack definiteness and precision, no account being taken of the attendant circumstances and conditions.

It should be clear, on reflection, that scientific Induction aims at establishing a universal law that does not refer primarily to cases or instances at all. And the method which it employs, as will be shown later, is to discover the law by analyzing the instances and reading it out of them, rather than by merely summing them up. When I conclude inductively that 'sentimental people are selfish,' or that 'the maple has a forked fruit-key,' the universal statement is not to be taken as merely summing up instances. Such propositions are rather assertions about universal types or kinds — the nature of sentimental people as such, or of maple trees as such. What has been established, granting that the induction is valid, is a coherence of characters forming a kind or type, so that the conclusions might be expressed in hypothetical form: 'if sentimental, then selfish,' 'if a maple, then a forked fruit-key.'

per enumerationem simplicem, res puerilis est, et precario concludit, et periculo exponitur ab instantia contradictoria, et plerumque secundum pauciora quam par est, et ex his tantummodo quae praesto sunt, pronunciat."

To discover such universal principles of connection through the analysis and comparison of instances is the goal of what may be called Scientific Induction. But we may also speak of Enumerative Induction as a lower and less complete form. In practical life we often depend with confidence on a conclusion which is based on a somewhat careful survey of instances. It is, of course, easier to rest on the authority of the instances, taking the connection as a fact, than to set systematically to work to analyze the instances in a scientific way in order to determine exactly the universal form of the law. It is likewise clear that these unanalyzed or only partially analyzed instances form the starting-point for scientific induction; and that, therefore, Enumeration must often play an important part in the preliminary stages of an investigation. But in certain fields of investigation we have to go on counting instances because there seems to be nothing else to do. We simply find P and Q invariably conjoined as a fact in experience, but are unable to analyze out the conditions and so either *mediate* the connection, or exhibit the precise form of the law. We cannot get a genuinely universal proposition asserting, 'P as such is connected with Q as such,' or, 'if P, then Q.' But the Enumerative conclusion simply affirms that all instances of P (so far as experienced) are connected with Q. Nor is the particular nature of the connection defined in this form of Induction. P and Q, for example, may be connected directly, or in some indirect way, as through a common dependence on some third thing, M. In the next chapter something further will be said of Enumeration, and how it may contribute, when used intelligently, to the ends of scientific Induction. Considered in itself, however, as

dealing merely with instances, we see how far it falls short, both in certainty and exactness, of the ideals of scientific knowledge.

§ 50. **Induction through Analysis.** — Scientific Induction, then, aims at discovering some typical character or law of behaviour. This usually requires the examination of a considerable number of instances. But the general proposition is not, however, obtained by simply counting the instances, or by adding them together. The purpose of taking a number of instances is to facilitate analysis, to aid us in eliminating characters or circumstances which are accidental or irrelevant, and at the same time, through these exclusions, to exhibit and define more clearly the essential character and relations of the subject we are investigating. The process of analysis is thus at the same time a process of synthesis; the process of excluding the irrelevant, a process of defining the essential. But it should be noted that if the instances are to lead to this result they must, so to speak, be selected for this purpose. They are not likely to be instructive, if they are chosen at haphazard. If the instances were all alike, for example, we should not gain anything by adding to their number, or if we could discover nothing in common among them, we should not be likely to select them. It is clear, then, that instances, to be instructive, must be *selected* with reference to the purpose of the investigation, and that the work of selecting instances is an essential part of the work of induction. It is with this end in view that we extend our observations over as wide an area as possible, drawing instances from different parts of the field. In natural history, for example, specimens are taken from different localities, in order to determine by comparison what

features are specific or generic characters, and what mere 'local variations.' What we seek to obtain is not merely a *number* of instances, but instances which show *differences that might be significant for our problem.* What differences or circumstances might be significant, we cannot, of course, know in advance. We can only guess, guided by our past experience, what might make a difference, and hope, by drawing instances from different parts of the field, to include all the significant circumstances. The function which the instances when thus selected fulfil is, of course, to exhibit what is essential by eliminating circumstances which are, for the purposes of the investigation, superfluous and irrelevant.

Experimentation, when it is possible, is another way of performing the same work of analysis and elimination. Hence in fields where experiments can readily be made, Induction does not have to depend upon an assemblage of instances. The experimenter, having control of the conditions, can produce the variations he wishes to observe, changing one thing at a time and noting the result. In this way, he is able to strip the phenomenon of superficial features that are connected with it only accidentally, or in a particular case, and by so doing lay bare its universal properties and modes of acting. But in experimenting, just as in collecting instances, there must be a guiding idea or purpose. In both cases alike, information is gained only by having questions or provisional guesses in mind, and then selecting for observation what is necessary to enable us to decide which guesses are false and which true.

What guides the selection of instances in an inductive inquiry, and also determines the character of the experiments

to be performed, is the tentative conception or hypothesis which the investigator has in mind. We must look, both in collecting instances and in setting up experiments, for facts which are significant, that is, which will help to answer the questions we have in mind. Bacon discusses at length, and classifies under twenty-seven different heads, what he calls Prerogative Instances, which, as especially instructive, should be the first and last objects of our investigation. Some of his headings are: 'Solitary instances,' 'migrating instances' (where the phenomenon is in process of coming into existence or disappearing), 'clandestine instances,' 'deviating instances' (as sports, or pathological cases), 'bordering instances,' and 'crucial instances.' This last name (*instantia crucis*) is drawn from the metaphor of the cross erected where two roads meet to indicate the different directions. When we have alternative conceptions or explanations in mind, either of which appears possible, we look for some crucial instance, or devise some crucial experiment that will point the way by eliminating one of the alternatives.[1] To know what facts would really be crucial in any given case, it is, of course, necessary to have some definite and systematic knowledge of the field in which the phenomenon under investigation falls. Only when this condition is realized, are we able to interpret rightly the bearing of the new instance or experiment on our problem.

The process of Induction, then, might be represented in the form of a Disjunctive Syllogism, where the conclusion is reached by eliminating successively all but one of the Disjunctive members. For example: —

[1] Examples of crucial experiments may be found among the miscellaneous exercises at the end of this volume.

This phenomenon, P, is either A, or B, or C.
These facts prove that it is not A; and these that it is not B.
Therefore P must be C.

This account is fundamentally correct in principle, though the Disjunctive Syllogism represents the process as more formal than it really is. It is not to be supposed that at the beginning of an inductive investigation all the possibilities are definitely and disjunctively formulated. The various possibilities, and their relation to one another, rather come to light as the examination and analysis proceed. And, at the end, the conclusion is never *merely* the result of the process of exclusion. In other words, we do not accept C merely because we cannot think of anything else; but, through the process of excluding A and B, C has become, to some extent at least, positively defined and determined. In dealing with any real problem, we cannot make any significant denial without thereby implicitly affirming and defining something else. These considerations will come up for discussion again, particularly in Chapter XVIII., where an account is given of the more explicit use and nature of hypotheses. In the meantime, however, the disjunctive principle may be regarded as the working basis of inductive procedure, though, especially in the earlier stages of this process, the disjunctive members are not formally enumerated, or set over against one another as exclusive possibilities.

Where now, we may ask, do the conceptions which are thus put forward in more or less definitely disjunctive form, and tested by means of instances and experiments, have their source? They arise in the mind itself, and are expressions of its own theorizing activity. These conceptions, however, are not mere uninstructed guesses, but are for-

mulated in the light of the knowledge already achieved. Induction, as a scientific process, bases itself on the relations and distinctions that are found in ordinary experience, and simply carries these farther and makes them more definite and consistent. Now, in the language of ordinary life, there is already given a preliminary classification and arrangement of the fundamental aspects of experience. In ordinary speech and in everyday practical relations, there is present a certain organization of experience. And it is this which is taken as the starting-point for the scientific interpretations which are to correct and extend the old. The phenomenon that we set out to interpret can only be understood in the light and with the help of what is already assumed as known. It is because we are able to perceive or imagine the likeness of the new to something with which we are already familiar that it is possible to think it in relation to the rest of our experience. If any phenomenon were to appear as absolutely unclassifiable, or totally unlike anything ever experienced before, there would be no means of getting hold of it, so to speak. And just because it might be anything, it would be for us as good as nothing. Even to attend to it would be impossible, for attention involves comparison. But the truth is that new facts and experiences always appear as modifications or variations of existing experience. In other words, although they have the element of unfamiliarity, it is yet always possible to discover in them some point of resemblance or identity with what has gone before. This resemblance or analogy in certain respects with what is already familiar leads us to assume that they may be of the same general type or kind as the latter, and that they will be found to have similar properties or

modes of operation. But this is as yet only an assumption that must be tested before being accepted as true. Further analysis may show that this assumption is based on a mere surface resemblance which does not warrant the interpretation made. Or, as is more usually the case, examination may disclose analogies which only allow the phenomenon to be classified as belonging to this or that general field. But the point to be noted is that through analogy its sphere has been determined. There are now only a definite number of possible interpretations, which take more or less definitely the form of a disjunctive proposition; P falls in the general field M, and is, therefore, A or B or C. Each member is put forward on some positive ground, and is thus a genuine possibility, not a mere unsupported guess. But it is only a possibility — something whose truth is still to be determined — and so its function is to operate as a plan or schema, pointing the way to further examination and testing through new instances and observations.

Our discussion has accordingly shown that Induction is able to pass from instances to a general conclusion only when the instances are selected because of their bearing on conceptions and hypotheses with which we are experimenting. Moreover, in forming these tentative hypotheses, we are guided in the first place by the analogy of the phenomenon under investigation to what is already known. Analogy and Hypotheses are then indispensable in Induction from the beginning, though the account of the more formal and explicit use of these operations is postponed to the later chapters.

CHAPTER XIV

THE ASSUMPTIONS OF INDUCTION — STAGES IN THE INDUCTIVE PROCEDURE

§ 51. **The Assumptions of Induction.** — It is part of the task of Logic to make us conscious of the assumptions of our thinking. We have found, in dealing with syllogisms, that it is often necessary to look for the premise or principle assumed in drawing the conclusion. But, in addition to these special assumptions which are taken as the basis of argument in particular cases, there are more general assumptions made by each science in the very process of defining its own standpoint and working conceptions (cf. § 95). Moreover, still more general assumptions may characterize *groups* of sciences, as, for example, the natural sciences, the historical sciences, etc. Finally, the question may be raised as to what is assumed in *all* thinking — what are the universal assumptions of thought — and what form these assumptions take in Induction. In § 9 we spoke of the Laws of Thought, and under the name of Identity and Contradiction, reference was made to the principles of consistency on which syllogistic logic is based. Now since Induction and Syllogism, as both processes of reasoning, are different rather in form than in fundamental character, their assumptions are not unrelated to each other. Indeed, the assumptions of Inductive thinking are more concrete expressions of the laws of thought than are the formal expressions of

Identity and Contradiction, mentioned in connection with the syllogism.

What we appear to assume in inductive reasoning is that the reality with which thinking is dealing is systematic and coherent. There is no direct method of proving that the world is not composed of a collection of particular things resembling one another more or less in an accidental or external way, but at bottom having nothing to do with one another. The only proof is that it would be impossible either to understand or to deal practically with such a world. For it would be a world in which experience could teach us nothing, since events might happen in any order or in any way, and it would never be possible to infer anything. We assume, therefore, and must assume, that the world is a cosmos, not a chaos. And this means that there are universal relations and connections of events which, if once discovered in their true nature, may always be depended upon. 'What is once true is always true.' A (*e.g.* the properties of iron, or the principles of heredity), once accurately determined and defined, is A, however various may be the instances in which it appears. To say, as is sometimes done, that in Induction it is assumed that what is true of certain instances will be true of all other instances which resemble these, is not entirely accurate. For, as we have seen, genuine induction is not based on instances at all, but on the discovery through analysis of a typical nature or law of action. What our thinking assumes is that identity of law and identity of nature exist in and through the diversity of things, and that it is in virtue of these universal principles of connection that the world is a coherent and intelligible system. Induction is only possible on the assumption that things not only *are* together

but *belong* together. On this assumption it has to work out the special mode of 'belonging' in various fields of phenomena; to bring to light the identity of nature or law that connects things which at first sight appear diverse and unrelated.

(1) The question of how this identity of nature, which connects things, is to be conceived, is a very fundamental one, both in science and philosophy. We have already seen that, to discover a genuine identity, it is necessary to penetrate beyond striking resemblances and superficial sense qualities to some deeper-lying nature. Moreover, the universal nature of a thing cannot be discovered in the form of some essence or substance that remains permanent and unchanging. It must rather be conceived dynamically, as a mode of activity, or rather as a system of activities in which all the parts are involved, and through which they are correlated. And, furthermore, the activity of a thing, which constitutes its nature, carries it, so to speak, beyond its own boundaries. It acts upon other things, and is in turn influenced by them. Its so-called properties are statements of its relation to other things. It cannot, therefore, be conceived as an isolated, unchanging essence, but must be defined through the constancy of behaviour shown in its changing relations to its environments. For example, the universal nature of man is not found in some unchanging substance, either material or spiritual, that inheres in the different human individuals. It consists rather in the system of functions, physical and mental, through which he expresses his relation to the world of persons and things. Nor, in the case of man, are the activities which constitute his nature modes of reacting with unvaried uniformity, but functions of adjustment and organization which develop in the light of the work they are called upon to perform.

(2) The particular forms of relation which are employed by

our thought to connect things are known as Categories. Thus in the last paragraph, we have been insisting that things are to be interpreted by means of dynamical rather than static categories. Similarly we might speak of Cause and Effect, or Energy, or Unity of Plan (Purposiveness), as Categories, since they are different forms or conceptions which we employ in thinking things in relation. Now each group of sciences has its own standpoint and categories, its own special terms in which it describes things and their relations. Thus physics represents the phenomena with which it deals, as mechanically or externally determining one another as causes and effects, while biology explains the actions of living organisms largely in terms of adjustment and purpose. What particular categories are employed by any science depends partly on the nature of the facts, and partly on the purpose which the science has in view.

(3) If the 'law of thought' or 'inductive assumption' be true, all the various parts of the world must ultimately be related through some law, or system of laws. So much seems to be implied in the very conception of a 'universe.' To find some terms in which a *universe* can be thought is the task of philosophy. What, then, is to be the highest or ultimate category of philosophy? To what common conception may all the diverse and seemingly irreconcilable phases of the world be reduced? The two opposing forms of answer given by philosophy to these questions are: (1) the common basis of all things consists in some form of matter or physical energy (Materialism); (2) the unity of the world is to be conceived in terms of an idea, or inner purposiveness, through which all the parts and functions find their explanation (Idealism).

§ 52. **Stages in the Inductive Process.** — Induction we have already seen to be a process of interpreting facts in terms of general conceptions or principles. This description would, however, apply equally well to Deduction; and, as a

matter of fact, these are not different *kinds* of thinking, but different *methods*, which are necessary to supplement each other in the task of making things intelligible. The various sciences have to start with particular facts learned through experience. The knowledge of general laws and principles comes later, and is derived from a study of the particular facts. It is clear, then, that the procedure of all the sciences must be inductive, at least in the beginning. The various sciences are occupied, each in its particular field, in the task of discovering order and relation among phenomena that at first sight appear to be lawless and disconnected. But in carrying out this undertaking our thinking uses every means which will help it toward its desired end. It is often able, after pushing inductive inquiries a little way, to discover some general principle, or to guess what the law of connection must be. When this is possible, it is found profitable to proceed deductively, reasoning out what consequences necessarily follow from the assumption of such a general law. Of course, it is essential to verify results obtained in this deductive way by comparing them with facts as actually experienced. The truth is that it is impossible, in actual thinking, to separate induction and deduction: the two processes constantly go hand in hand and are mutually supplementary.

Again, it must be remembered that the inductive process, considered broadly as the progressive interpretation of experience, is continuous throughout. What is already known is always taken as the starting-point for a new investigation. And although the immediate purpose of any special inquiry may soon be satisfied, the results obtained lead to new questions, which can be answered only by further analysis and investigation. There is then no break — no fundamental sep-

aration — between the facts with which induction starts and the more highly developed theories and generalizations which it is sometimes able to reach. What we call facts are themselves the results of former processes of thinking and interpretation, as well as the starting-point for new analysis and theorizing. There is a constant passage from one stage to the other, theories when approved and generally accepted coming to be regarded as facts, and facts when critically examined disclosing the theoretical basis on which they rest. For example, we say that it is a 'fact' that the earth revolves on its own axis. Yet this, not very long ago, was regarded as an 'incredible hypothesis.' And when we reflect, we see that this 'fact' is really a conception — or a part of a system of conceptions — which enables us to bring together in our thought a number of simpler 'facts.' And these latter, if examined, would in turn prove to be constructed by coördinating and generalizing still simpler data, the truth being that all facts involve ideas.

Whewell has spoken of Induction as "the true colligation of facts by means of an exact and appropriate conception"; and he goes on to point out that the distinction of fact and theory is only relative. "Events and phenomena considered as particulars which may be colligated by Induction, are facts; considered as generalizations already obtained by colligation of other facts, they are theories." [1]

§ 53. **Observation and Explanation.** — The Inductive process being thus continuous, how are its different stages to be distinguished and classified? We may still adopt the customary terms, and speak of Induction as including both **Observation**, or **Description**, and **Explanation**, though it must be remembered that the one process really involves the other.

[1] *Novum Organon Renovatum*, Bk. II., Aph. XXIII.

Sometimes the relation between Observation and Explanation is stated in quite a misleading way. It is said that in undertaking an investigation we must observe and describe the facts as accurately as possible, and only after this is done proceed to theories and explanations. Now, as has been shown, this is to make an artificial separation between collecting and describing the facts, and relating or explaining them. As we have seen, both processes go on simultaneously. The observation of instances presupposes some guiding idea, some provisional hypothesis, perhaps held in the mind as a question to be answered. We discover the relevant facts as we go along with our investigation, just as we discover the appropriate conception or explanation. And just as the facts observed and described involve theories and conceptions, so the explanation to which we proceed is simply a fuller and more accurate description. When the close and necessary relation of these stages of Induction is kept in mind, there is, however, some advantage in maintaining the distinction between Observation of the nature of particular facts and the wider organization of facts and relations effected by what we call Explanation.

It is the business of the former process to employ various methods and devices in order to determine as accurately as possible the nature of the starting-point. It is essential to have a full and accurate survey of the terms of the problem, and to note carefully every clew that may lead to its solution. In the first place, the different *qualities* of things must be accurately observed and distinguished. But accurate observation in science leads almost directly to the determination of *quantitative* relations through measurement. Under this head fall processes of enumeration, the measurement and recording

of space and time relations, the determination of weights, and the measurement of the so-called secondary qualities like heat, sound, and colour. The special technique through which such observations are carried out and rendered precise in the different sciences, must be learned through occupation with the actual phenomena. In each science, questions arise regarding methods of measurement — the determination of the units to be employed, means of measuring indirectly when direct measurement is impossible, the most accurate method of summing up observations and of eliminating errors — as well as problems regarding the most convenient means of representing quantitative relations through mathematical formulæ, graphs, etc. In addition, the use and manipulation of various instruments designed to supplement and render more accurate the observations of the senses have to be learned; the fingers often require to be trained to perform delicate operations; and a special education of the senses and attention is necessary in some fields before results of scientific value can be obtained. This technical knowledge and skill in the employment of the instruments and methods of observation and description within any science is to be attained, as already stated, only by actual practice. We distinguish practically this work of collecting data — which may be extended over months or years — from the construction of the explanatory theory, the former often seeming to demand the power of patient observation and skill in mechanical manipulation rather than logical reasoning.

It is important, however, to remember that scientific observation itself involves intellectual activity. To observe — at least in the sense in which the word is used in scientific procedure — requires something more than the passive reception

of impressions of sense in the order in which they come to us. Without some activity on the part of mind, it would be impossible to obtain even the imperfect and fragmentary knowledge of everyday life. But accurate observation is one of the means which science employs to render this knowledge more complete and satisfactory; and when observation thus becomes an exact and conscious instrument, it involves, to even a greater extent than in ordinary life, intellectual activities like judgment and inference. It is because this is true, because scientific observation demands the constant exercise of thought, in selecting and comparing the various elements in the material with which it deals, that it affords such excellent intellectual discipline. The observational sciences do not merely train the sense-organs; the discipline which they afford is mental as well as physiological, and it is, of course, true that mental training can only be gained through the exercise of mental activity.

(1) It is quite true that it is of the utmost importance to distinguish between a fact, and further inferences from the fact. As will be pointed out in the chapter on Inductive Fallacies, errors very frequently arise from confusing facts and inferences. This does not mean, as we have seen, that facts exist apart from theories. But in any particular case if we would avoid confusion we must distinguish sharply between the data and further constructions to which we proceed. Especially important is it not to confuse facts with fancies, or with judgments motived by subjective feelings. The point which is emphasized in the previous paragraph, however, is that it requires a certain amount of *thinking* in order to get a fact at all. Facts do not pass over ready-made into the mind. Simply to stare at things does not give us knowledge: unless our mind reacts, judges, thinks, we are

not a bit the wiser for staring. To observe well, it is necessary to be more or less definitely conscious of what one is looking for, to direct one's attention toward some particular field or object; and to do this implies selection among the multitude of impressions and objects of which we are conscious. Moreover, scientific observation requires analysis and discrimination. It is not unusual, in text-books on logic, to symbolize the various facts learned through observation by means of letters, *a*, *b*, *c*, etc., and to take it for granted that they are given in our experience as distinct and separate phenomena; but, as we have just seen, judgments of analysis and discrimination are necessary to separate out the so-called 'phenomena' from the mass or tangle of experience in which they were originally given. Again, to determine the nature of a fact through observation, it is essential to note carefully how it differs from other facts with which it is likely to be confused, and also, to some extent, what relations and resemblances it has. But such knowledge presupposes that thought has already been at work in forming judgments of comparison.

(2) A distinction is sometimes made between observation and experiment. In observation, it is said, the mind simply *finds* its results presented to it in nature, while in experiment the answer to a question is obtained by actively controlling and arranging the circumstances at will. There are, no doubt, some grounds for this distinction, though it is not true that the mind is passive in the one case, and active in the other. Even in observation, as we have seen, knowledge always arises through active analysis and comparison of the instances selected as having a bearing on some problem. The difference is rather this: In observing, where experiment is impossible, one must wait for events to occur, and must take them in the form in which they are presented in the natural order of events. But, where experiment is employed, we have control of the conditions, and can produce the phenomena to be investigated in any order, and as often as we choose. In experiment, as Bacon says, we can put

definite questions to nature, and compel her to answer. This is, of course, an immense advantage. In some of the sciences, however — geology and astronomy, for example — it is not possible directly to control the conditions: one must wait and observe the results of nature's experiments. Physics and chemistry are the experimental sciences *par excellence;* and, in general, we may say that a science always makes more rapid progress when it is found possible to call experiment to the aid of observation. It is not possible to conceive how physics and chemistry could have reached their present state of perfection without the assistance of experiment. And the rapid advances made in recent years by biology and psychology have come mainly through the introduction of experimental methods. Indeed, the almost total neglect of experiment by the Greek and mediæval scholars must be regarded as one of the chief reasons why the physical sciences made so little progress during those centuries.

We have seen that the distinction between observation and explanation is not an absolute one. The task which thought has to perform — the task which is undertaken by science — is to reduce the isolated and chaotic experiences of ordinary life to order and system. And it is important to remember that all the various methods employed contribute directly towards this result. It has, however, seemed possible to divide Inductive methods into two main divisions. Observation, it was said, seeks to discover the exact nature of the facts to be dealt with, and to find accurate means of describing and representing their qualitative and quantitative aspects. But, when this has been accomplished, we have not by any means reached an end of the matter. The desire for knowledge is not satisfied with a mere statement of facts, or even with a mathematical representation of them in a formula or a curve. Complete

knowledge demands an *explanation* of the facts as determined by the methods of observation. The scientist is not content to know merely *that* such and such phenomena happen in certain definite ways, but he attempts to discover *why* this is so. 'Why,' we ask, 'should dew be deposited at certain times, or water rise thirty-two feet in a pump?' The demand is that the processes of analysis be pushed farther by thought. What is required is a wider generalization, or the discovery of a more general law of behaviour under which the phenomenon we are studying may fall as a special case. Yet this explanation, when arrived at, is on one side nothing more than a more complete description of the facts, calling attention to forces and happenings that escape ordinary observation. The explanation of the pump, for example, called attention to the weight of the atmosphere, hitherto neglected. But the new inductive step consists in something more than the addition of new facts. What is essential in explanation is rather the new way of colligating or thinking the facts in relation to one another, afforded by the law or conception. The difference between Description and Explanation is obviously one of degree, being simply a question of how far analysis is pushed. In general, we speak of a conception as explanatory rather than descriptive, when it explicitly brings different facts into relation. Of course, Explanation itself has various degrees of completeness and ultimateness. There always exists the ideal of a higher generalization, a more complete colligation of facts than any which science and philosophy have yet been able to achieve.

An excellent illustration of the distinction between descriptive and explanatory conceptions is afforded by a comparison of the work of Kepler with that of Newton. Kepler was filled

with the idea that there must be some relation capable of mathematical expression between the different positions, previously determined by observation, in the orbit of the planet Mars. At length, after trying and discarding numerous other hypotheses, he was able to show that an ellipse could be passed through all these points. The proof was afterwards worked out of the elliptical character of the orbits of the other planets. The conception of an ellipse enabled Kepler to think all the observed positions of the planets, in relation to one another. But the explanation of why the planets moved through elliptical orbits was still lacking. That explanation, as is well known, was given by Newton in his conception of universal gravitation. This was explanatory because it linked together the movements of the planets with the behaviour of all other bodies moving in space, thus enabling the former to be thought as examples or instances of the action of a universal principle.

It is usually said that where we know merely the nature of phenomena, and their connection, without being able to explain these facts, our knowledge is *empirical*. Thus, I may know that an explosion follows the contact of a lighted match with gunpowder, or that a storm follows when there is a circle around the moon, without being able to explain in any way why these facts are connected. On the other hand, if we can connect events by showing the general principle involved, we say that our knowledge is really scientific. It is important to notice, however, that empirical knowledge is simply in a less advanced stage than the scientific knowledge which has succeeded in gaining an insight into the general law; and also that any knowledge might be called empirical, when contrasted with a more complete explanation. Thus Kepler's knowledge, that the orbits of the planets are ellipses, was empirical compared with that

of Newton. Empirical knowledge leaves a problem which intelligence has still to solve. It is, of course, true that a large part of every one's knowledge is empirical in character. We all know many things which we cannot explain. In all the sciences, too, phenomena are met with which seem to defy all attempts at explanation. Indeed, some of the sciences can scarcely be said to have passed the empirical stage. The science of medicine, for example, has hardly yet reached any knowledge of general principles. The physician knows, that is, as a result of actual experiment, that such and such drugs produce such and such effects. But he knows almost nothing of the means by which this result is achieved, and is therefore unable to go beyond the fact itself. In this respect, he is very little better off than the ordinary man, who knows that if he eats certain kinds of food he will be ill, or if he drinks strong liquors in excess he will become intoxicated.

REFERENCES TO CHAPTERS XIII AND XIV

C. Sigwart, *Logic*, Vol. II., Ch. V.

B. Bosanquet, *Logic*, Vol. II., Chs. II.–V.

H. W. B. Joseph, *An Introduction to Logic*, Chs. XVIII. and XIX.

W. Whewell, *Novum Organon Renovatum.*

L. H. Hobhouse, *The Theory of Knowledge*, Chs. XI.–XIV.

W. P. Montague, "On the Nature of Induction," *Journal of Philos. and Psych.*, Vol. III., pp. 281 ff.

CHAPTER XV

ENUMERATION AND STATISTICS

§ 54. **Enumeration or Simple Counting.**— We shall begin the account of the scientific methods with Enumeration. To count the objects which we observe, and to distinguish and number their parts, is one of the first and most essential operations of thought. It is of course true that qualitative distinctions generally precede quantitative. The child learns to distinguish things by some qualitative mark, such as 'black' or 'hot,' before he is able to count them (cf. § 87). We may say, however, that the qualities of things are known, in a general way at least, before scientific procedure begins. The determination of quantity, on the other hand, seems to demand a more conscious effort on the part of the mind. We learn to distinguish the general qualities of things without effort; but, to obtain exact quantitative knowledge, it is necessary to set ourselves deliberately to work. And it is also necessary, as we shall see, to decide what we shall count. We must make up our mind, with some general idea more or less consciously before us, what it is worth while to enumerate. We may, accordingly, take Enumeration, or Simple Counting, which is perhaps the easiest kind of quantitative determination, as our starting-point in dealing with the Inductive Methods.

A considerable step in advance, in the task of reducing the

world of our experience to order and unity, is taken when we begin to count, *i.e.* to group together things of the same kind, and to register their number. Thus Enumeration is, to some extent, also a process of classification. What is counted is always a collective whole, the units of which are either all of the same kind, or else belong to a limited number of different classes. Thus one might determine by Enumeration the number of sheep in a flock, taking each individual as belonging to the same general class, 'sheep'; or the analysis might be pushed farther so as to give as a result the number of white and of black sheep separately. The purpose for which the enumeration is undertaken always determines the length to which the process of analysis and distinction is carried. For example, if the object of a census enumeration were simply to determine the number of inhabitants in a country, it would not be necessary to make any distinctions, but each person would count as one. But where, as is often the case, the aim is not simply to count the sum-total, but also to determine the relative numbers belonging to various classes, analysis has to be pushed further. In such cases, we might count the number belonging to each sex, the native-born, and those of foreign birth, those below, and those above any given age, etc.

In Chapter XIII we have seen that the so-called 'Perfect Induction,' where all instances are examined, is not properly called Induction at all, since there is no inference to anything new. Scientific Induction analyzes, notes special accompanying circumstances, and gets beneath the surface to the real or essential happening in the various cases. But we saw that before the process of analysis is carried out, as well as in cases where the conditions are too complex or difficult to determine, we do proceed to generalize with greater or less confidence on

the basis of the instances observed. If instances of P and Q for example, have always been found in conjunction, and if we are confident that there has been nothing limiting or restricting observation to some special type of instance, we assume that the connection is not a mere 'casual coincidence,' but that in some form it holds universally. In such cases, the number of instances — *provided they can be assumed to be really unrestricted* — does seem to have a bearing on the logical character of the conclusion. The connection P — Q is less likely to be merely 'casual' in proportion to the frequency with which 'free, or unrestricted' cases of it are observed, while at the same time no exceptions to it appear. The 'imperfect' character of the Induction, when based on a number of carefully established instances that show no exception throughout a considerable range, is found rather in the fact that the *nature* of the connection P — Q is left vague and undetermined, than in any lack of certainty regarding the existence of some universal principle of relationship. The invariable conjunction of a number of 'free' instances rules out the assumption of 'chance'; but, in so far as the instances are left unanalyzed, the precise form of the universal mode of connection is not exhibited in and through them.

Where experience shows both positive and negative cases, and where at the same time it is impossible to discover any basis of difference for the two sets of results, we can compare the number of instances in which the connection obtains with that in which it fails. The ratio thus obtained may then be made the basis for calculating the probability of any particular event; or even of determining the likelihood that there is some law operative with regard to the observed phenomena (cf. p. 232).

As a matter of fact, however, Enumeration of instances is an aid to Induction mainly because in actual counting classification and analysis are also being effected. We are never content merely to count, taking each barely as 'one instance.' We also take account of the character of the instances, rejecting those that are not 'fair' or 'typical' and emphasizing others as of special or 'prerogative' importance. Moreover, the assemblage of instances of different types — of connection and lack of connection, of different races, or ages, etc., serves to bring out differences and similarities between groups. In other words, statistics, when collected intelligently and with some problem in view, are really instruments of analysis; and in fields where experimentation is not possible, they may be capable of revealing, not merely the fact that certain groups of things are correlated, but also to some extent the character of that correlation.

The conclusion which we reach, then, is that no process of enumeration has any claim to the title of Perfect Induction. Enumeration is the beginning, rather than the end of the inductive procedure. Nevertheless, it is exceedingly useful as a preliminary step and preparation for scientific explanation. The number of stamens and pistils which a plant contains, or the number of tympanic bones possessed by an animal, is often of the greatest service in classification. And classification, although it is by no means the end of scientific investigation, is in many of the sciences a most essential and important step toward that end. The task of explaining the infinite variety of natural objects would be a hopeless one, if it were not possible to discover similarities of structure, in virtue of which things can be grouped together in classes. To this, enumeration in a very great degree contributes, especially if

the counting is accompanied and directed by methodical thinking, so that the likenesses and characteristics enumerated are not taken at haphazard, but are really important ones, and such as to bring out, by means of the classification, answers to definite questions. Enumeration thus not merely groups together the phenomena to be studied in a compact form, but at the same time begins the process of analysis, revealing resemblances and differences.

§ 55. **Statistics and Statistical Methods.** — Statistical methods depend upon enumeration. They aim at making the process of counting as exact and precise as possible. Rümelin defines statistics as "the results obtained in any field of reality by methods of counting." Modern science has come to understand that its first task must be to become acquainted, as completely as possible, with the nature of the facts presented to it by experience. And, for this purpose, the careful classification and precise enumeration of particulars afforded by statistics is often of the greatest importance. "The extent to which the statistical method prevails, and everything is counted," says Professor Sigwart, "is another instance of the fundamental difference between ancient and modern science."[1] It would, of course, be impossible to enter here into a full description of the methods employed by statistical science. The methodology of every science must be learned by actual practice within the particular field. What we are interested in from a logical point of view is the purpose which statistical investigation seeks to fulfil, and the part which it plays in rendering our knowledge exact and systematic.

We notice, in the first place, that the class of facts to which statistics are applied has two main characteristics: (1) the subject

[1] *Logic* (Eng. trans.), Vol. I., p. 286.

dealt with is always complex, and capable of division into a number of individual parts or units; and, secondly, it is also of such a nature that the underlying law or principle of the phenomena to be investigated cannot be directly discovered. Thus, we employ statistics to determine the death-rate of any country or community, or the ratio between the number of male and of female births. It is clear that it is impossible to make use of experiment when we are dealing with facts of this kind, because the conditions are not under our control. If it were possible, for example, to determine exhaustively the general laws according to which the various meteorological changes are coördinated with their conditions, we should not trouble ourselves to count and register the separate instances of changes in the weather. Nor, if we knew exactly the general conditions under which any given human organism in contact with its environment would cease to exist, should we count the individual cases of death. "In proportion as we are unable to reduce the particular event to rules and laws, the numeration of particular objects becomes the only means of obtaining comprehensive propositions about that which is, for our knowledge, fortuitous; as soon as the laws are found, statistical numeration ceases to be of interest. There was some interest in counting how many eclipses of the moon and sun took place year by year, so long as they occurred unexpectedly and inexplicably; since the rule has been found according to which they occur, and can be calculated for centuries past and to come, that interest has vanished. But we still count how many thunder-storms and hail-storms occur at a given place, or within a given district, how many persons die, and how many bushels of fruit a given area pro-

duces, because we are not in a position to calculate these events from their conditions." [1]

In cases like those mentioned above, where we are as yet unable to determine the general laws which are at work, we call to our aid statistical enumeration. There are three main advantages to be derived from the employment of this method. (1) The use of statistics contributes directly towards a clear and comprehensive grasp of the facts. Instead of the vague impression derived from ordinary observation, statistics enable us to state definitely the proportion of fine and rainy days during the year. Statistical enumeration is thus one of the most important means of rendering observation exact and trustworthy, and of summing up its results in a convenient and readily intelligible form. It is of the utmost importance, when dealing with complex groups of phenomena, to have a clear and comprehensive view of the facts of the case. Thus, when trying to understand the nature of society, it is necessary to determine accurately, by means of statistics, such facts as the number of male and of female births, the death-rate, the proportion of marriages, the age of marriage, etc. This may be regarded as the descriptive use of statistics. (2) In the second place, by giving us the average in the past for large numbers of things or events occurring within certain lengths of time, in areas of space, statistics enable us to form probable judgments as to what will happen in the future in cases where we cannot predict because the causal laws are unknown or are too complex. This second use will be further discussed in § 56. And, finally, (3), statistics often serve to reveal quantitative correspondences or uniformities between two groups of

[1] Sigwart, *Logic* (Eng. trans.), Vol. II., p. 483.

phenomena, and thus suggest that some causal connection exists between them. It is found, for example, that the number of births in any given country tends to vary in relation to the abundance or scarcity of food. Now, this fact at once suggests the existence of laws which will serve to bring these facts into causal relation. In many cases, such correspondences serve only to confirm our expectation of the presence of a causal law, which is based on other grounds. Thus we should naturally expect that there would be a relatively greater number of cases of fever in a town which had an insufficient water supply, or an antiquated system of sewerage, than in a town where these matters were properly provided for; and statistics might bear out our conclusions. In general, however, it may be said that causal laws are suggested, not by corresponding uniformities, but by corresponding variations, as shown by the statistics of different sets of facts. So long as the death-rate, for example, shows a constant ratio to the population, no causal inference is suggested; but if the annual number of deaths increases or decreases considerably, we are led to look for some variation from the normal in some coincident group of phenomena. And if it is found that the variation in the death-rate has been accompanied by unusually favourable or unfavourable conditions of weather, the presence or absence of epidemics, or any similar circumstances, there will be at least a *presumption* that a causal relation exists between these two sets of events. — From a certain likeness or quantitative proportion between the variations of two distinct classes of phenomena, we are led to the hypothesis of their causal connection.

In this use of statistics, they become directly auxiliary to an explanation of the facts they enumerate. But the correlation

and causal connection of the facts come to light only when looked for. Merely to count, without any definite purpose, would never help us to explain. As we saw in the last chapter, induction always proceeds under the guidance of conceptions or general ideas. We do not simply stare, as it were, at the facts we examine, but we look at them to discover their meaning and select such of them as are relevant or significant in the light of some general theory or conception. In other words, we examine the facts to put theories (which may, of course, be very vague as yet) to the test, or to get answers to certain questions which we have in mind. Now this is just as true of enumeration and statistics as it is of the other methods of induction. As has already been remarked of enumerative classification, we must decide what it is worth while to count in the particular field in which we are counting. The questions that we wish answered will determine this. And even when we have our figures, they will be meaningless or even altogether misleading unless we know how to interpret them. It is the neglect of such considerations that leads to the misuse of statistics and the frequent contradiction of the statement that 'figures cannot lie.'

(1) It is true that on a superficial view of the statistical method the figures may seem at times to arrange themselves in definite groups quite apart from any intellectual labour save that of mere counting. Thus it might seem that in taking the average rate of mortality on the basis of the returns of local officials, etc., the figures of themselves disclosed the fact that the rate was higher for infants under two years of age than in later periods of life. But the total average of deaths would never have shown this. It is only because the average for infants has been separately calculated, *in the expectation that there might be a difference*, that the difference

has been found. The tentative question — Is there, as we have reason on the ground of unsystematic observation to believe, a striking difference between the death-rate of infants and that of older persons? — is thus answered in the affirmative.

But the function of guiding ideas and hypotheses becomes even more important when the statistics are to be used directly in the service of explanation. Two examples will serve to make this plain. The first is from Professor Sigwart: "The position of a barometer in a given locality passes from day to day, and from month to month, up and down through all possible variations, in which we can at first find absolutely no rule (though they have a constant mean value). . . . But if we calculate the average for the particular hours of the day over a considerable time, we find a periodical variation between two maxima and minima with respect to the general average. . . . That the period is daily points to the influence of the sun. . . . But unless we had conjectured that the different positions of the sun, and the changes brought about by them, had some influence, we could not have thought of summing up the particular hours of the day apart from each other."[1] In this case, the constant average first obtained told us nothing, except that the conditions, whatever they were, which governed the fluctuations of the barometer, remained constant on the whole. But when an hypothesis was found, and the varying positions divided into groups of such a nature that their comparison could test it, we obtained a partial explanation of them.

Again, suppose that we are gathering statistics of the divorce-rate in various states and countries. The figures, unanalyzed, would tell us little. But suppose we had a definite problem in mind, such as the effect of laws on the frequency of divorce. What would we do with our figures? "First, select states or countries with similar social and economic conditions, but very different laws, and compare their divorce-rate; do the same for states

[1] *Logic*, Eng. trans., Vol. II., pp. 496–497.

with similar laws, but different economic conditions; note whether the divorce-rate varies with the law, or with the other factors, or with neither exclusively. Secondly, examine every instance of a change in the divorce law, and observe whether it was attended by a change in the figures such as might have been produced by the law."[1] Here again there is a division of the phenomena into groups distinguished by some difference in the supposed cause, and then a comparison of these groups. The methods employed, as we shall see presently, are essentially those of Agreement and Difference, and of Concomitant Variations.

In general, then, there are two things to be said about the use of statistics. In the first place, the smaller and more numerous the groups are into which the enumerated phenomena are divided, and the more exactly the rules of division in general are followed in doing this, the more valuable, other things being equal, the statistics will be. In the second place, it is by the comparison of these groups that statistics aid us to discover causal relations. The kind of groups we shall make, and the points in which we shall compare them, are determined by the questions we have to ask, or the tentative conceptions we have to test. In all these respects the use of statistics is governed by the general principles of the inductive method, which consists essentially in the analysis and comparison of phenomena in the light of an hypothesis.

(2) Statistical enumeration is frequently employed to determine the *average* of a large number of instances of a particular kind. This is obtained by dividing the sum of the given numbers by the number of individuals of which account is taken. In this way a *general average* is reached which does not necessarily correspond exactly with the character of any individual of the group. It represents a purely imaginary conception, which omits individual differences and presents in an abbreviated form the general character of a whole class or group. In this way, by the determination of the average, it

[1] Willcox, *The Divorce Problem*, p. 41.

becomes easier to compare complex groups with one another. Thus, when the average height of Frenchmen and Englishmen has been determined, comparison is at once made possible. From the mean or average of a number of individuals, or set of instances, however, we can infer nothing regarding the character of any particular individual, or of any particular instance. What *is* determined by the method of averages is the general nature of the group, as represented by the average or typical individual. But this method does not enable us to infer anything regarding the character of any member of the group, A, or B.

Indeed, the simple arithmetical mean or average by itself may give us quite an erroneous idea of the general character of the individuals or instances which make up the group. For example, if ten divorces were granted in a county, eight at the end of three years of married life, one at the end of six, and one at the end of thirty, it would give quite a misleading notion to say that the average duration of marriage in couples seeking divorce there was six years. In order to correct such defects in the use of the average by itself, especially in applying the statistical method in biology, two other expressions are now used, the *mode* and the *median value*. The *mode* is the condition which occurs most often in the group examined; in the example just cited it would be three years. The *median value* is the condition of the individual at the middle of the series, when it is arranged in order. In this case it approximates to the mean. When the group is symmetrically distributed about the average, these three expressions are approximately the same; but as it becomes less evenly distributed, they differ more or less widely, and now one of them, now the other, may give a better notion of the character of the group than the average by itself would. All three expressions, however, are primarily expressions for the general nature of the group; and the information they give us concerning the nature of any individual member of it is always indirect, imperfect, and uncertain, save as we are informed

where in the group the member occurs. There are also occasions when it is preferable to use the *geometrical mean*.

§ 56. **The Calculation of Chances.** —We still have to consider the second of the three uses of statistics mentioned in the foregoing section. As has been said, statistics not only help us in describing and in explaining complex phenomena, but they are also used to enable us to judge what will be true, on the whole, of a long series of events, in cases where ignorance of the causal laws concerned prevents our making predictions concerning the individual members of the series, when taken separately. This is usually called the calculation of chances, or probabilities. Now there is, of course, no such thing as 'chance,' regarded as a power which controls and governs events. When we speak of something happening 'by chance,' or of some occurrence as 'probable,' we are expressing merely a deficiency in our own knowledge. "There is no doubt in lightning as to the point it shall strike; in the greatest storm there is nothing capricious; not a grain of sand lies upon the beach but infinite knowledge would account for its lying there; and the course of every falling leaf is guided by the same principles of mechanics as rule the motions of the heavenly bodies." [1] To assert that anything happens by chance, then, is simply to confess our ignorance of the causes which are operative.

It is clear that we are in this position regarding many of the ordinary events which belong to the future. Because of my ignorance of the causes at work, I can only say, 'It *may* rain to-morrow.' It is impossible to tell upon which side a penny will fall at any particular throw, or what card may be drawn from a pack. But in cases like these, we have

[1] Jevons, *The Principles of Science*, Vol. I., p. 225.

to accept, for lack of anything better, a numerical statement of the chances for any particular event. Thus we know that, since there are only two sides upon which a penny can fall, the chances of throwing heads in any trial is $\frac{1}{2}$. Similarly, there are four chances out of fifty-two of drawing an ace from a pack of cards. The chance of obtaining an ace by any draw is therefore $\frac{4}{52} = \frac{1}{13}$. These figures express the mathematical chances. Experience of a limited number of instances may, however, sometimes appear to show a lack of harmony between the mathematical and the actual chances. But in proportion as the number of trials is increased, the result is found to approximate more and more nearly to the mathematical expectation. In twenty throws of a penny or a die, we should not be surprised to find that the result differed from the fraction expressing the mathematical chances. But this discrepancy would tend to disappear as the number of cases was increased. Jevons illustrated this by actual trial, using a number of coins at a time. Out of a total of 20,480 throws, he obtained a result of 10,353 heads. On the result of the experiment he remarks: "The coincidence with theory is pretty close, but considering the large number of throws there is some reason to suspect a tendency in favour of heads." [1]

Apart from the simple and somewhat artificial cases where we are concerned with coins and dice, etc., it is impossible to determine with mathematical precision the chances for or against any event, since the possibilities are indefinite as well as the causes. In cases where the whole series of possibilities does not lie before us, we have to base our calculations for the future on what is known regarding the fre-

[1] Jevons, *op. cit.*, Vol. I., p. 230.

quency with which the events under consideration have occurred in the past. Now the results of the last paragraph make it clear that it is of the utmost importance that the statistics, which are taken as the basis, shall be as full and comprehensive as possible. It is evident, for example, that serious errors would be likely to arise, if the death-rate for a single year, or for a single county or town, were taken as typical of the country as a whole. To render statistics trustworthy, they must be extended over a considerable period of time, and over a large extent of country, so as to eliminate the accidents due to a particular time or to a particular locality.

(1) When this has been done, however, and statistics have been obtained that have a right to be regarded as really typical, the chances in any individual instance regarded simply as one member of a large group, and apart from its own special characteristics, can be readily shown. Thus we find that out of one thousand children born, about two hundred and fifty die before the age of six years. The chances, then, at birth, that any child will reach this age, are $\frac{750}{1000}$ or $\frac{3}{4}$. Again, it is found that only about two persons in one thousand live to be ninety years old. So that the probability of any child living to this age would be expressed by the fraction $\frac{2}{1000}$ or $\frac{1}{500}$. Such probabilities are simply averages which briefly describe what has happened in the past. Now what has happened in the past in a large number of cases we naturally expect to happen in the future. This is essentially the principle upon which life-insurance companies proceed. Their business is conducted on the assumption that there will be an approximately constant death-rate, though they cannot foretell what particular individuals are to die in any year. It thus becomes possible to calculate what losses from death may be expected each year. Suppose that it is found that the annual death-rate among men of a certain

age throughout the country is twenty out of every thousand. If each man's life were insured for $1000, the loss to the company from this source would be $20,000. To compensate for this loss, the company would be obliged to demand an annual payment of $20 from each of the one thousand individuals in the class. Of course, the actual computations upon which insurance is based in concrete cases are vastly more complex than this, and many other considerations arise of which account has to be taken. But the general principle involved is, that by taking a sufficiently large number of cases, chance can be almost eliminated. We can have no means of determining whether any healthy individual will or will not die before the end of the year. There would be a very serious risk, amounting practically to gambling, in insuring his life alone, for probabilities are essentially averages. They inform us about the group, and not directly about any particular member of it. But the transaction, as we have seen, is no longer a mere speculation when a large number of individuals are concerned; for the actual loss can be accurately foretold and provided for.

(2) As precise an analysis of the conditions as is possible is as important in estimating probabilities as it is in the other uses of statistics. The smaller the group of which the average is taken, and the more definite the information we have about it, the more accurate our estimate becomes. It is not enough, for example, for the purposes of life-insurance, to know what the average age of death is, all adults being taken as on the same footing. What the insurance companies do is, in the first place, to exclude all who are not in fairly good health, and who may be in danger of hereditary disease, from their membership; and, in the second place, to calculate the average number of years of life remaining to men of different ages. Every individual is thus put into a special class, and the premium calculated accordingly.

(3) A rather common fallacy is to suppose that the known prob-

ability of any particular event of a group or series, gives us some ground for expecting this when the other events of the series have occurred. But it should be remembered that the known probability affords no such ground of inference, except as we know that there is some causal relation between these events; and then we are not reasoning by probabilities. The probability of throwing double six with two dice, for example, is $\frac{1}{36}$. But because in 35 consecutive throws the double six has not appeared, it does not follow that it is any more likely to do so on the 36th throw than it was on the first. The probability is still $\frac{1}{36}$, and so continues. If we take a sufficiently large number of throws, as has already been remarked, we shall find that the double six has, *on the average*, appeared once out of every 36 throws. But we cannot foresee whether the appearances of the double six sufficient to give this average will be evenly distributed through the whole series of throws, or occur in irregular sequences.

(4) A peculiar use of the theory of probability in order to discover causal connections between events is possible on the principle just stated. When we are in doubt, that is, as to whether two events are in any way causally connected, we can by collecting statistics estimate the probability of their appearing together on the assumption that they have no causal relation. Then if they are found to appear together more or less frequently than this estimate, we are justified in assuming that there is some causal relation between them. Suppose, for example, we are studying two characteristics which occasionally appear in a certain species of animal, and wish to determine whether they have any essential connection. We find on examining a large number of cases that one of these characteristics appears once in every sixteen individuals, on the average, and the other once in every twenty. If there is no connection between them, then, on the theory of probability, the chance of their happening *together* is $\frac{1}{320}$. But if we found that they occurred together in 20 cases out of every

100, we should conclude that there must be some cause or causes common to both characteristics, or else that one of them in some way depends on the other.

REFERENCES

C. Sigwart, *Logic*, §§ 101, 102.

J. G. Hibben, *Inductive Logic*, Ch. XV.

L. T. Hobhouse, *The Theory of Knowledge*, Pt. II., Ch. XI.

J. S. Mill, *Logic*, Bk. III., Ch. XVIII.

B. Bosanquet, *Logic*, Vol. I., pp. 128 ff.

CHAPTER XVI

DETERMINATION OF CAUSAL RELATIONS

§ 57. **Causal Connection.** — So far, we have been dealing, primarily, with observational methods, and with the results obtained through the enumeration of particular things. We have been considering how our knowledge of the qualities and quantities of objects may be made as exact and complete as possible, but we have not discussed in detail the methods by which we discover the connection of things. But all Inductive thinking, as has been shown, is based on the assumption that there are universal forms or principles of relation according to which things are connected in a systematic way. We cannot really be said to know at all, until we become aware that certain parts of our experience are united, like the links of a chain, one part involving another. And, as has been already frequently pointed out, the growth of knowledge is constantly bringing to light new connections between facts that were previously taken to be independent of one another. Now, it was also stated in an earlier chapter (§ 51), that the connections and relations of things may be conceived in different ways — that there are various 'categories of experience.' Natural science, however, in describing and explaining the relations of things, does so primarily in terms of Cause and Effect. All phenomena without exception, it is assumed, are causally dependent on other phenomena; everything which happens has its cause, and is in turn followed by its effect. From the standpoint of

practical experience, also, we are constantly obliged to look for causes; for only where the cause is known is there any certain method of producing the effect. The determination of causes, then, is one of the most essential problems of Induction, the category of Cause and Effect being perhaps the most universal and important category by means of which the parts of our experience are thought as related according to universal laws. What rule, or rules, can now be given which will enable one to discover what is the cause or the effect of an event in any particular case?

Before we proceed to the answer of this question, however, it is necessary to explain briefly what is meant in the natural sciences by the relation of cause and effect. In the first place, the natural sciences regard the world as consisting of a phenomenal order of events. In other words, they are concerned with the particular things and changing events that *appear* or *show themselves* in ordinary experience. Both the inner and the outer world appear to be composed of an indefinite manifoldness of particular things, events, occurrences. Now, the natural sciences do not ask whether this aspect of the world is ultimate Reality or merely Appearance. The problem of the scientist is rather to set out from the manifold objects and events as they appear in ordinary experience, and to seek to describe and explain them by showing how they are related in various complex ways through principles of causal dependence. It is assumed that each phenomenon of which the world is composed, is yet, in spite of the independent and separate existence which it seems to have, connected through the principle of causality with something else which determines it, or is in some way necessary to its existence. Every event, that is, has its cause.

The explanation of every phenomenon is to be found in something external to it, but upon which it is dependent. The relation of cause and effect assumes that all phenomena are *externally* determined; or, as the same thing is often expressed, it assumes a *mechanical* relation between the different parts of the world. Moreover, this relation is, as has been said, simply a special form, or category, through which the universal relations of things are expressed. That there are universal modes of connection, and that 'once true always true,' is a law or postulate of all thinking. Causality, being as we have seen one very definite and useful way of thinking that relation, is accordingly of the greatest importance, both for science and practical life.

(1) When the general postulate of all thinking, that things shall hold together systematically so as to be intelligible, is put in more definite form as the law of Cause and Effect between phenomena, we get the notion of the Uniformity of Nature. Of course, strictly speaking, the Uniformity of Nature is involved in the fundamental postulate of thought that things hang together in a rational way. Nevertheless, the conception is usually taken to imply the absolutely invariable sequence of causal events. From the point of view of natural science, Nature is uniform in the sense that all instances of the same phenomenon P, are always determined in the same way by the same cause Q. This, then, is really mechanical uniformity. The relation between P and Q is not only external or mechanical, but absolutely fixed and invariable. The conception of any 'spontaneous variation,' any modification without an externally determining cause, is completely excluded.

(2) In speaking of any phenomenon as having *a* cause, the relation has, of course, been artificially simplified. In reality, there are always a number of 'causes,' or determining conditions necessary to

the occurrence of any event. What we mean by 'the cause,' in any particular case, depends mainly on the character and purpose of the inquiry. In practical life the 'cause' sought for is usually something that can be employed directly as a means to the desired result. And even in scientific inquiries practical motives continue to play a part in deciding what shall be regarded as the 'essential' or 'real' cause of any phenomenon. The cause is that which can be employed to produce the desired effect, and so to afford practical mastery over the situation. This direct reference to practice, however, is not essential to the idea, which is primarily a way of thinking things in relation. Ultimately, then, the 'real' or 'essential' cause is that which shows most clearly the character of the relationship between two phenomena — that which, in a sense, is the sum or synthesis of all the conditions.

The cause, then, from the point of view of science, is that without which the phenomenon would not occur. It is also sometimes defined as 'the invariable and necessary antecedent,' while the effect is spoken of as the 'invariable consequent.' In using these terms, however, it must not be supposed that the cause always and necessarily precedes the effect in time. The relation of cause and effect is not to be regarded as merely temporal.

§ 58. **Mill's Experimental Methods.** — The methods by which causes and effects may be determined were formulated by Mill in his *Logic*. He stated, in general terms, the principles which were already in use in scientific procedure. Mill gives five separate canons, but, as he himself recognizes, there are but two main principles involved. "The simplest and most obvious modes of singling out from among the circumstances which precede or follow a phenomenon, those with which it is really connected by an invariable law, are two in number. One is, by comparing together different instances in which the phenomenon occurs. The other is, by

comparing instances in which the phenomenon does occur with instances in other respects similar in which it does not. These two methods may be respectively denominated the Method of Agreement and the Method of Difference."[1] Of the other three methods mentioned by Mill, one — the Joint Method of Agreement and Difference — is, as the name implies, a direct combination of the first two, while the Method of Residues and the Method of Concomitant Variations are corollaries from the same principles.

The purpose of these comparisons is to *exhibit and define the true cause.* This is accomplished by proceeding directly through negation. That is, the other circumstances which could be supposed to have any influence are successively eliminated. And, as already pointed out (§ 50), it is just with a view to the possibility of elimination, that the instances are selected. Since the cause is that without which the phenomenon would not occur, the rules of elimination follow immediately: (1) That is not the cause of a phenomenon in the absence of which the phenomenon occurs; (2) That is not the cause of a phenomenon in whose presence the phenomenon fails to occur; (3) That is not the cause of a phenomenon which varies when it is constant, or is constant when it varies, or varies in no proportionate manner with it.[2]

The process of eliminating the other things that could conceivably be causes, also defines the sphere and nature of the true cause. The preceding rules, then, might have been stated positively, and it is this positive side of the process that

[1] Mill, *Logic*, Bk. III., Ch. VIII., § 1.

[2] These statements are essentially those given by Joseph (*An Introduction to Logic*, pp. 403–404), who, however, adds a fourth supplementary rule: "Nothing is the cause of one phenomenon which is known to be the cause of a different phenomenon."

has been emphasized by Mill. It is important to bear in mind, however, in studying Mill's *Methods of Experimental Inquiry*, that elimination or negation plays an important part in the process which he describes. We shall now proceed to state and illustrate the canons of the different methods.

§ 59. **The Method of Agreement.** — The principle upon which this method proceeds is stated in the following way by Mill: "*If two or more instances of the phenomenon under investigation have only one circumstance in common, the circumstance in which alone all the instances agree is the cause (or effect) of the given phenomenon.*" The purpose of this rule, it will be remembered, is to help us to determine what particular facts in our experience are connected as causes and effects. If the problem is to find the cause of some phenomenon, the canon may be illustrated in the following way. Let P^1, P^2, P^3, represent different instances of a phenomenon, P, whose cause is to be ascertained. And suppose that we are able to analyze,

> the antecedents of P^1 into *abcd;*
> the antecedents of P^2 into *gfcm;*
> the antecedents of P^3 into *klnc.*

Now it is clear that *c* is the sole circumstance in which the antecedents of all these instances of P agree. And nothing can be the cause of P in the absence of which P still occurs. We should be justified in concluding, therefore, according to this method, that *c* is probably the cause of the phenomenon under investigation, P. We may, then, adopt Jevons's formula for discovering the cause of any given phenomenon by this method: '*The sole invariable antecedent of a phenomenon is probably its cause.*'

If, now, we wished to discover the effect of something which happens, it would be necessary to determine, by observing a number of instances, what common circumstance can be found among the events which follow it.

If Q^1 were followed by *fghk*,
and Q^2 were followed by *lmgc*,
and Q^3 were followed by *grst*,

we should be able to say that Q and *g* were connected as cause and effect. The rule might then be expressed: *The sole invariable consequent of a phenomenon is probably its effect.*

When antecedents and consequents are thus represented schematically by means of letters, it is easy to perceive at once the common circumstance in a number of instances. But the facts and events of the real world are not separated off from each other in this way. The common circumstance in which a number of instances agree has to be separated out by analysis from the variable elements which form part of the different antecedents and consequents. Moreover, an essential part of the work of Induction consists in selecting instances such that all the possibilities—all the things that might be connected with P—are included. It should also enable us to recognize the common element as common, though it may appear in wholly different circumstances. The way in which the work of analysis proceeds will become more evident by considering a number of concrete cases in which this method may be employed.

If a number of cases of typhoid fever were to appear at about the same time in a community, one would naturally wish to explain this phenomenon by tracing it to its cause; and to do this one would try to discover some circumstance

which was the common antecedent of all the cases. Knowing from the records of past experience that the cause is to be sought for among a limited number of circumstances, one would select the various instances with the purpose of testing the different possibilities. The water supply might first be examined. But if it were found that this was derived from entirely different sources in the different cases, we should probably conclude that the explanation must be sought elsewhere. Suppose that as a result of careful analysis it were discovered that all the individuals prostrated with the fever had eaten oysters bought at the same market. If this were the only common circumstance discoverable after careful investigation, we should conclude that probably the oysters were the cause of the fever. The process of analysis could be pushed still further, if one wished, in order to determine more exactly the precise source of the infection; *e.g.* it might be found, as a result of further inquiry, that the water in which the oysters were kept was vitiated by a sewer.

Another example of the method of agreement which is often quoted by logicians may be given. One would naturally suppose that the colours and lines of mother-of-pearl were due to the chemical or physical character of the substance itself. Sir David Brewster, however, happened to take an impression of a piece of mother-of-pearl in beeswax and resin, and was surprised to see the colours reproduced upon its surface. He then took a number of other impressions in balsam, gum-arabic, lead, etc., and found the iridescent colours repeated in every case. In this way he proved that the colours were caused by the form of the substance, and not by its chemical qualities or physical composition. The different substances, wax, balsam, lead, etc., in which the phenome-

non of colour appeared, had nothing in common except the form. This, therefore, according to the method of agreement, was properly regarded as the cause of the phenomenon to be explained.

An example of the application of this method to the discovery of the *effect* of a phenomenon may now be given. Let us suppose that the problem is to determine the effect of some proposed legislation. It is necessary, of course, to refer to other instances where this legislation has been put in force, and our general information about political and social affairs shows more or less definitely what kind of connected circumstances it is worth while noting. Let us suppose that in one case what followed the enactment of the law under consideration was a falling off of revenue, an increase of immigration, large exports, etc., and in a second, the revival of ship-building, decrease of crime, and increase of immigration; and that in other instances where still other conditions prevailed, the number of immigrants still continued to increase. Since this latter circumstance is the only one which follows invariably upon the enactment of the law, we are justified in concluding, after a certain number of observations, that it is necessarily connected with the law as its result.

It is important to note that the conclusions reached by this method are greatly strengthened by increasing the number of observations, and by taking as many instances as possible that are dissimilar in character. By so doing, the real cause is more likely to be included among the antecedents noted, and, at the same time, the probability is lessened that the connection between antecedent and consequent is a merely accidental conjunction. But even when such precautions are taken, the method of Agreement does not afford

any very definite knowledge. By eliminating the other antecedents, we found that *c* is probably connected causally with P. But *c* is left as a mere unanalyzed 'circumstance,' *e.g.* 'the drinking water,' 'the form' of the substances which showed iridescent colours, etc. Just how the connection takes place, whether it be direct or indirect, is not shown. It is clear, then, that further analysis is necessary in the interest of scientific knowledge. The method of Agreement, although perhaps in some cases yielding results sufficiently exact for practical application, merely suggests a problem for further scientific inquiry. Its defect, as we have seen, is that it does not sufficiently get beneath the surface of things so as to make certain and definite their mode of relation.

It may be well to notice under separate headings some of the special difficulties which result from this method's superficial mode of analysis.

(1) *Reciprocity of Phenomena.* So long as we are dealing with events which succeed one another in time, there is no difficulty in perceiving which is cause, and which effect. But we are often called upon to investigate the relation between phenomena that do not appear as successive, but as co-existent. And it is then not at all easy to determine by means of the method of Agreement which is cause and which is effect. Poverty and intemperance, for example, are found conjoined so frequently as to make it probable, apart from other considerations, that some causal relation exists between them. It might be maintained with apparently equal show of reason, that the former is the cause, or the effect, of the latter. Again, is one to say that ignorance is the cause or the effect of moral degradation? There seems to be no means of determining by this method which is antecedent and which consequent. As a matter of fact, it is probably true in such cases that the phenomena act and react upon each other: that each term, in other

words, is at once both cause and effect. In such instances we go beyond the conception of causal dependence in one direction, to that of the Reciprocity of phenomena.

(2) *Complexity of Phenomena.* Again, neither the cause nor the effect need be composed of a single phenomenon, as the method seems to assume. Indeed, as further observation shows, the antecedents and consequents which the method of Agreement takes as 'single circumstances' are usually very complex. The difficulty is that the process of analysis has not been carried far enough to bring out the essential point involved. Everything is lumped together and the exact nature of the connection left vague and uncertain. Thus, for example, the 'ill-health' of a community might be shown by this method to be related causally to the 'sanitary conditions.' Here it is obvious that both antecedent and consequent involve complex relations and conditions, which are left vague and ill-defined.

(3) *Plurality of Causes.* There is still another circumstance that renders uncertain the results of the method of Agreement. In itself, it can only show that *c* is *a* cause of P, not that it is the only or necessary cause. Taking the word 'cause' in its popular sense, we cannot say that a given phenomenon is always produced by the same cause, or that the effects of different causes are always different. Intemperance may result from different causes in different cases, or heat may be generated through combustion, friction, or electricity. The fact here illustrated, that an effect may be produced by any one of several causes, is what is meant by the phrase 'Plurality of Causes.' Once more, this defect is simply the result of a too vague or superficial analysis. When analysis can discover what has really occurred, what the real nature of the effect is, it becomes possible to determine the nature of the only and essential cause.

§ 60. **The Method of Difference.** — According to the method of Agreement, we compare a number of diverse instances, in all of which a given phenomenon occurs, and en-

deavour to discover the one circumstance which is invariably present. The method of Difference, on the other hand, compares an instance in which a phenomenon occurs with another as nearly similar to it as possible, in which it does not occur. Its canon is expressed by Mill as follows: "*If an instance in which the phenomenon under investigation occurs, and an instance in which it does not occur, have every circumstance in common save one, that one occurring only in the former; the circumstance in which alone the two instances differ is the effect, or the cause, or an indispensable part of the cause, of the phenomenon.*" It will perhaps make the matter clearer to say: 'That which is present in a case when a phenomenon occurs, and absent in another case when that phenomenon does not occur, all other circumstances remaining the same in the two cases, is causally connected with that phenomenon.' That is, by means of this method we compare two instances which differ only in the fact that the phenomenon in which we are interested, is present in the one, and absent in the other. If now the two cases are represented in this way,

PHK conjoined with *alg*,
and HK conjoined with *lg*,

we conclude at once that P is causally connected with *a*. Our selection of P, or the element in question, as the supposed cause, is, of course, made in accordance with an hypothesis or general notion of what the possible or likely causal relations in the subject under investigation are, gathered from previous experience. If this notion is as yet too vague to give us any definite guidance, then we are obliged to analyze the phenomena as exactly and minutely as we can, and experimentally vary the circumstances in every conceivable way, until the requirements of the method are, if possible, satisfied.

Almost any instance in which experiment is employed will serve to illustrate this method. If a bell is rung in a jar containing air, the sound will, of course, be heard at any ordinary distance. But after having removed the air by means of an air-pump, let the bell be again struck. It will now be found that the sound is no longer heard. When the two cases are compared, it is at once evident that the only difference in the antecedents is the presence of the air in the one case, and its absence in the other. When the air was present, the sound was heard; when it was absent, the sound was not heard. We conclude, therefore, that the perception of sound is causally connected with the presence of atmospheric air. Again, we can prove that the so-called 'taste' of different objects depends upon smell, by tasting, say, an orange, and after a little time has elapsed, tasting it a second time while holding the nose. It will be found in this latter case that instead of the familiar 'orange taste,' one senses merely 'acid,' or 'sweet.' The only difference in the two trials being that in the former the organ of smell, which was excluded in the latter, was operative, it follows that the so-called 'orange taste' is proved to be due to smell rather than to taste proper.

An essential requirement of the method of Difference is that *only one circumstance shall be varied at a time.* The object of the method is to isolate the various conditions which go to make up a complex phenomenon, in order that we may mark the effect of the presence or absence of each one individually. Now, in observing what goes on in nature, we rarely find changes in which but a single element has varied. If we find that to-day is cooler than yesterday, we may be inclined to refer the change to the thunder-storm of last night. But rain also accompanied the thunder-storm, and the direc-

tion of the wind has changed. So that it is impossible in such cases to apply the method of difference. To employ this method successfully, observation usually must be supplemented by experiment. In performing experiments, we determine what conditions are to be operative, and arrange the apparatus so as to carry out our purpose. Having thus control of the conditions, we are able to vary them at pleasure. In this way, experiment becomes an instrument by means of which analysis can be carried further than is possible for unaided observation. It enables us to separate things which are usually conjoined, and to observe the result of each when taken by itself. In employing experiment, however, the greatest care must always be taken to introduce or remove only one condition at a time, or at least only one new circumstance which can in any way influence the result.

It often happens, too, as Jevons points out, that the experimenter is not aware of all the conditions which are operative when his investigations are made. 'Some substance may be present, or some power may be in action which escapes the most vigilant examination. Not being aware of its existence, we are of course unable to take proper measures to exclude it, and thus determine the share which it may have in the results of our experiments.'[1] For this reason, it is always necessary that experiments should be repeated by different persons, and so far as possible under varying conditions. I quote two examples from the work of Jevons to which reference has just been made.

"One of the most extraordinary instances of an erroneous opinion due to overlooking interfering agents is that concerning the increase of rainfall near the earth's surface. More than a century

[1] Jevons, *Principles of Science*, Vol. II., p. 37.

ago it was observed that rain gauges placed upon church steeples, house-tops, and other elevated places, gave considerably less rain than if they were on the ground, and it has very recently been shown that the variation is most rapid in the close neighbourhood of the ground. All kinds of theories have been started to explain this phenomenon; but I have attempted to show that it is simply due to the interference of wind which deflects more or less rain from all the gauges which are at all exposed to it.

"The great magnetic power of iron renders it a constant source of disturbance in all magnetic experiments. . . . In some cases, magnetic observations have been seriously disturbed by the existence of masses of iron in the neighbourhood. In Faraday's experiments upon feebly magnetic or diamagnetic substances, he took the greatest precautions against the presence of any disturbing substance in the copper wire, wax, paper, and other articles used in suspending the test objects. It was his invariable custom to try the effect of the magnet upon the apparatus in the absence of the object of experiment, and without this preliminary trial no confidence could be placed in the results."[1]

It is sometimes impossible to remove the suspected cause experimentally without materially changing the attendant circumstances; or it may be impossible to remove it at all, as in the case of gravity. But this difficulty may often be overcome by introducing a circumstance which overcomes or neutralizes the effect of the supposed cause without altering the rest of the phenomena. Thus, *e.g.*, the rain gauges placed in elevated positions which were mentioned above, might be protected from the wind by screening. The effect of this disturbing element would thus be neutralized, leaving it possible to observe what results, if any, in the quantity of rainfall followed a change of elevation.

[1] Jevons, *op. cit.*, pp. 40, 41.

CHAPTER XVII

DETERMINATION OF CAUSAL RELATIONS (*continued*)

§ 61. **The Joint Method of Agreement and Difference.** — The method of Difference can be applied only when all concomitant circumstances, except one, remain constant. In order to apply this method, then, it is necessary either to find two instances which differ only in a single circumstance, or to proceed by means of experiments, adding or removing a single circumstance at a time and noting the result. The difficulty is to find instances that differ only in a single circumstance in fields where, from the nature of the case, experiments cannot be used. For example, in trying to reach generalizations regarding the behaviour of human individuals or human societies — in looking for moral, or social, or economic laws — it is, of course, impossible to employ experiment. Nor, when dealing with individuals and societies, can we find two instances which certainly differ from each other in only a single circumstance. In studying phenomena of this kind, then, it is necessary to employ another method as an instrument of analysis. What is done by this new method is to take a number of instances instead of only two. A number of instances where the phenomenon to be investigated occurs are compared together, and likewise a number of instances where it does not occur, and the results of the two comparisons noted.

This is really to combine the principle of the method of Agreement with that of the method of Difference. Mill, accordingly,

has called this the Joint Method of Agreement and Difference, and has given the following statement of its canon:—"*If two or more instances in which the phenomenon occurs have only one circumstance in common, while two or more instances in which it does not occur have nothing in common save the absence of that circumstance, the circumstance in which alone the two sets of instances differ is the effect, or the cause, or an indispensable part of the cause, of the phenomenon.*" By the help of this method, the weakness which has already been noticed in the method of Agreement is overcome. We first compare different instances in which the phenomenon occurs. If these are found to agree in only a single circumstance, we conclude, according to the canon of Agreement, that this circumstance is probably connected causally with the phenomenon in which we are interested. But the proof is not yet complete. To really prove the connection, we must show that wherever the circumstance is absent, there the phenomenon is also absent.

In interpreting this canon, it is important to remember that both positive and negative instances must be selected from the field within which our previous knowledge enables us to say that the cause (or effect) sought for is to be found. The purpose of the instances, as has been frequently pointed out, is to bring to our attention circumstances which might conceivably make a difference. It is, of course, impossible to predict in advance all the things that might make a difference; but the possibilities fall within a more or less definite range. In both the positive and negative set of instances, then, we are concerned only with circumstances that might be relevant. The negative instances to be chosen are therefore, not *any* cases 'where the phenomenon does not appear,'

but where, in addition, circumstances which were previously found in conjunction with the phenomenon, and which might have been supposed to be causally connected with it, are now shown to be sometimes, at least, present when it is absent. To represent the working of the matter schematically, we may analyze the instances where the phenomenon, P, occurs into the following circumstances: —

Instance 1 . . . . *a, b, c, d, e.*
Instance 2 . . . . *f, c, a, g, k.*
Instance 3 . . . . *d, m, b, c, e.*
Instance 4 . . . . *k, n, c, g, a.*

The method of Agreement, in such a case, would lead to the conclusion that *c* is probably connected causally with P. To strengthen and render more definite that conclusion, however, the Joint method introduces the comparison of instances, as much like the former group as possible and known to exhibit at least many of the same circumstances, but where the phenomenon in question does not occur. These instances of the absence of P would then be represented thus: —

Instance 1 . . . . *b, k, n, g, a.*
Instance 2 . . . . *d, e, b, m, f.*
Instance 3 . . . . *k, l, s, g, b.*
Instance 4 . . . . *x, e, n, a, f.*

What is of significance in this latter series is not merely that the instances show nothing common except the absence of P, but that the same 'circumstances' excluded by the former analysis are now seen to exist in the absence of that phenomenon. But what may be present when a phenomenon is absent is not its cause or effect. All these possible circum-

stances, then, *a*, *b*, *d*, etc., are again eliminated by the comparison of negative instances, leaving as before *c* as that which is causally connected with P.

The canon of this method, then, as stated by Mill must be read with these restrictions in mind. The actual working of the method is better described in the following words: *If when two sets of instances — one in which the phenomenon under investigation is present and one in which it is absent — are drawn from the same field of inquiry, it is found that there is one circumstance which is invariably present when the phenomenon occurs and invariably absent when it does not occur, while each of the other circumstances is both sometimes absent when the phenomenon is present, and sometimes present when it is absent, then the first circumstance is causally connected with the phenomenon.*

As an illustration of the method of Agreement and Difference the following instance will serve: —

We may suppose that in a certain part of the country it was noticed that a considerable difference existed in the number of criminal offences committed, in proportion to the number of inhabitants, in the various towns. In several towns the percentage was high, while in others it was relatively small. This being so, a question naturally arose as to the cause of the high percentage. Now there were among the people various opinions concerning the matter. One thought it was due to the small number of police, a second believed it was caused by the inefficiency of the public schools, a third attributed it to the inadequacy of the penalties attached to the violation of law, a fourth was convinced that it was due to lack of activity on the part of the churches, while a fifth insisted that the phenomenon could be accounted

for by the presence of licensed saloons. Not being able to agree about the matter, it was decided to appoint a committee to investigate the circumstances existing in various towns where the same general conditions prevailed, and upon the basis of this comparison to decide the matter. The towns with a high criminal percentage were examined first. The report of conditions there was as follows: —

Town A: Small police force — efficient schools — severe penalties — inactive churches — licensed saloons.
Town B: Small police force — efficient schools — light penalties — active churches — licensed saloons.
Town C: Large police force — inefficient schools — severe penalties — active churches — licensed saloons.
Town D: Large police force — inefficient schools — light penalties — inactive churches — licensed saloons.

This report revealed the fact that in each of these towns having a high criminal percentage there was one circumstance, and only one, invariably present, — the licensed saloon. This rendered it probable that the saloon was the cause of the high percentage of crime. Still, before finally deciding, it was thought well to investigate negative instances as well; that is, towns in which the high percentage of crime did not occur. The report of conditions there was as follows: —

Town E: Large police force — efficient schools — severe penalties — active churches — no licensed saloons.
Town F: Large police force — inefficient schools — light penalties — active churches — no licensed saloons.
Town G: Small police force — efficient schools — light penalties — inactive churches — no licensed saloons.

Town H: Small police force — inefficient schools — severe penalties — active churches — no licensed saloons.

This table showed that in the absence of the phenomenon (high criminal percentage) one and only one of the conditions concerned was invariably absent; namely, the licensed saloon. This confirmed the previous report and established to the satisfaction of all that the saloon was, at least, the main cause of the high criminal percentage in the cities concerned.

Of course, it is obvious that this can be no more than a hypothetical case. In actual life, the conditions of the method would never be so exactly realized. In the first place, in any such investigation, it would probably never be possible to find instances where one condition is invariably present when the phenomenon occurs, and invariably absent when it does not occur, as the illustration supposes. We could, at most, expect that one condition would exhibit a *tendency* to be present when the phenomenon occurs and absent when it does not occur. That is, there might well be instances met with in which a combination of other conditions might render unnecessary the presence of the usually essential one. In the second place, it would not be satisfactory in actual life to deal with such vague terms as 'efficient' schools, or 'active' churches. On the contrary, we should, in a careful investigation, resort to statistics in order to secure greater definiteness and accuracy. The comparative number of the churches, the size of the police force, the number of saloons, would be noted and compared with the percentage of crime in order if possible to determine which of the above-mentioned circumstances is causally connected with the large number of criminals. That is, although we should not be likely to find fulfilled the strict requirements which this method makes, we should strengthen the inference by showing that

definite quantitative relations exist, as indicated by the statistics, between certain of the circumstances in question.

It is usual to speak of this method as that to which recourse must be had when it is impossible to employ experiment. As a matter of fact, this illustration seems to show that the strict requirements of the method can never be realized except where experiment can be employed to isolate and control the conditions. In fields where this is impossible, it is necessary, as we have seen, to employ statistics as an instrument of analysis. Where the method is not supplemented by determining the relation of the various instances experimentally, or by making possible exact comparisons through the use of statistics, it can yield only vague and unsatisfactory results. It is obvious, therefore, that the various methods must continually supplement one another in actual operation if the complex and changing conditions of experience are to be successfully dealt with at all.

§ 62. **The Method of Concomitant Variations.** — The methods of Agreement and Difference are employed, as we have seen, to determine what events are necessarily connected as causes and effects. By examining a considerable number of instances, and by comparing the cases in which the phenomenon of interest to us occurs, with cases in which it does not occur, we seek to rule out all accidental and unessential conjunctions, and thus to determine the true law of causal connection. But the discovery of certain forms of agreement or correspondence in the variations of phenomena, or groups of phenomena, often enables us to detect a causal relation between them (cf. pp. 222–224). The variations or changing states of all phenomena are events in time. Now, when it is observed that certain of these events continue to show correspondences throughout a series of variations, it is inferred that the conjunction is not accidental, but indi-

cates the existence of a causal connection. This correlation of events may be discovered through correspondences in temporal or spatial arrangement of phenomena, in their progression, or in changes of quality or quantity. The discovery of concomitant variations, however, is of importance in science, not merely because it assists us in determining what events are related as causes and effects, but also because the exact form of the causal relation can thereby be rendered more definite and satisfactory. For scientific knowledge the discovery of a 'general correspondence' between certain phenomena is not enough; it is necessary to obtain some *exact* expression of the relation between the two sets of variations. This is found by reducing the variations to terms of quantity through the application of a common unit of measurement. The law or ratio of the variations may then be expressed in numerical terms. Now the scientist tries to include in his statement of causal laws, whenever possible, precise information regarding the quantitative relations of the phenomena concerned. Indeed, we may almost say that science does not exist until the quantitative aspects of phenomena are taken into account — until things are weighed and measured. The physicist does not think his work finished when he has proved that sound is produced by atmospheric vibrations. He carries on his analysis until he can discover the *quantitative* relations between the amplitude and velocity of the vibrations, and the loudness and pitch of the resulting tone. And the psychologist is not satisfied with the general statement that certain sensations are causally connected with certain kinds of stimuli; but he seeks to discover, whenever possible, the exact quantitative relation between sensation and stimulus. In

short, the most important feature, the very essence, one may say, of modern scientific investigation, is the establishment of quantitative relations.

Looking at two things with respect to the order and progression exhibited by their manner of appearance, then, we say that when their variations keep pace with each other, they are in some way causally connected. What it is necessary to establish, in order to justify the inference to causal relationship, is that there is some definitely expressible relationship between the changes shown by the two series. 'Nothing is the cause of a phenomenon that varies when the latter is constant, or is constant when it varies; or between whose changes and that of the phenomenon there is not some correspondence.' It is not necessary, however, that the variations shown by the two series should always be in the same direction. One series, for example, may increase as the other increases, or the two series of changes may be in inverse ratio. The essential requirement is that there shall be some definite relationship clearly made out between the two series of events.

The following is Mill's statement of the canon: "*Whatever phenomenon varies in any manner whenever another phenomenon varies in some particular manner, is either a cause or an effect of that phenomenon, or is connected with it through some fact of causation.*" The illustrations of this law given by Jevons are so pertinent that we cannot do better than adopt them: —

"The illustrations of this law are infinitely numerous. Thus Mr. Joule, of Manchester, conclusively proved that friction is a cause of heat by expending exact quantities of force by rubbing one substance against another, and showed that the heat produced was

exactly greater or less in proportion as the force was greater or less. We can apply the method to many cases which had previously been treated by the simple method of difference; thus instead of striking a bell in a complete vacuum, we can strike it with a very little air in the receiver of the air-pump, and we then hear a very faint sound which increases or decreases every time we increase or diminish the density of the air. This experiment conclusively satisfies any person that air is the cause of the transmission of sound.

"It is this method which often enables us to detect the material connection which exists between two bodies. For a long time it had been doubtful whether the red flames seen in total eclipses of the sun belonged to the sun or moon; but during the last eclipse of the sun, it was noticed that the flames *moved with the sun*, and were gradually covered and uncovered by the moon at successive instants of the eclipse. No one could doubt thenceforth that they belonged to the sun.

"Whenever, again, phenomena go through *Periodic Changes*, alternately increasing and decreasing, we should seek for other phenomena which go through changes in exactly the same periods, and these will probably be a connection of cause and effect. It is thus that the tides are proved to be due to the attraction of the moon and sun, because the periods of high and low, spring and neap tides, succeed each other in intervals corresponding to the *apparent* revolutions of those bodies round the earth. The fact that the moon revolves upon its own axis in *exactly* the same period that it revolves round the earth, so that for unknown ages past the same side of the moon has always been turned toward the earth, is a most perfect case of concomitant variations, conclusively proving that the earth's attraction governs the motions of the moon on its own axis.

"The most extraordinary case of variations, however, consists in the connection which has of late years been shown to exist between the Aurora Borealis, magnetic storms, and the spots on the sun. It has only in the last thirty or forty years become known that the

magnetic compass is subject at intervals to very slight, but curious, movements; and that, at the same time, there are usually natural currents of electricity produced in telegraph wires, so as to interfere with the transmission of messages. These disturbances are known as magnetic storms, and are often observed to occur when a fine display of the Northern or Southern Lights is taking place in some part of the earth. Observations during many years have shown that these storms come to their worst at the end of every eleven years. . . . Close observations of the sun during thirty or forty years have shown that the size and number of the dark spots, which are gigantic storms going on upon the sun's surface, increase and decrease exactly at the same periods of time as the magnetic storms upon the earth's surface. No one can doubt, then, that these strange phenomena are connected together, though the mode of the connection is quite unknown. . . . This is a most remarkable and extensive case of concomitant variations."[1]

(1) In employing this method it is, of course, hazardous to infer the existence of a universal law of correlation without examining in some detail the nature of the concomitant variations. In general the more definitely the relationship can be shown in a considerable number of cases, the more ground there is for the conclusion that the conjunction is not accidental. Moreover, it is also necessary that observations should be extended over a considerable range in order to determine whether the supposed law of correlation has any limits, and if so how they are to be defined. For example, in Weber's law we have an exact expression for the correlation of the quantity of the stimulus in the case of the various sense organs and the intensity of the resulting sensation. But in every case this exact correlation of stimulus and sensation has an upper and lower limit, beyond which it either changes its character or ceases altogether.

(2) The close and almost inseparable connection of the different methods in actual use, which was emphasized in the preceding sec-

[1] Jevons, *Lessons in Logic*, pp. 249–251.

tion, is also here clearly evident. In many fields it is only through experiment that the fact of correspondences between phenomena can be brought to light, and the character and law of their correlations exactly determined. But to introduce experiment for these purposes is, of course, to supplement the method of Concomitant Variations by the method of Difference. Similarly, in performing experiments where it is impossible to withdraw a certain element, and thus by comparison to note what its cause or effect is, as the strict canon of Difference requires, we may be able to isolate the element practically by causing it to vary while other circumstances are kept constant. It is then possible to note the variations in the corresponding series and thus to determine what is causally correlated with the element in question. For example, if the problem were to determine the effect of moisture on growing plants it would, of course, be impossible to eliminate moisture entirely without killing the plant and putting an end to the experiment. But by varying the amount of moisture, and noting concomitant changes in the plant, both methods of analysis are combined.

§ 63. **The Method of Residues.** — We have said that modern science employs measurement whenever possible, in order to determine exactly the quantitative relations of phenomena. Groups of facts whose connections are at first not perceived, or at best but vaguely apprehended, are brought into close relations with one another by the establishment of definite quantitative relations. The knowledge that electricity possesses energy, for example, is very vague and incomplete when compared with the definite equations which the physicist can furnish between the electrical current generated under certain definite conditions, and the amount of work which it is capable of performing. But the discovery of quantitative relations not only renders our knowledge more perfect and complete, it also enables us in

some cases to detect laws of connection which would not otherwise be observed. We have already seen how the perception of corresponding changes in the quantities of phenomena has led to the discovery of causal laws by means of the method of Concomitant Variations. The method of Residues, which we now have to discuss, is also largely dependent on quantitative determination.

In general, this method calls attention to any remainder or residue which is left over after other portions of a complex phenomenon have been explained. There are two results of this method which may be discussed separately.

(*a*) The application of this method to a complex phenomenon which is the result of several causes, often enables us to determine what part each of these causes plays in the determination of the whole fact under consideration. Mill's fifth canon seems to apply to this case. It is as follows: *Subduct from any phenomenon such part as is known by previous inductions to be the effect of certain antecedents, and the residue of the phenomenon is the effect of the remaining antecedents.* Thus, if it is known that the complex phenomenon BAC is the result of *bac*, and if it is further known that *a* is the cause of A, and *b* of B, it follows, of course, by subtraction that the residue still unexplained, C, is caused by *c*, the remaining antecedent.

Of course the application of this method in concrete cases does not usually resolve itself into such a simple process of subtraction. It requires work — 'previous inductions,' as Mill says — to determine what are the whole number of antecedents in any case, as well as to isolate the various antecedents so as to determine exactly what part of the effect is to be ascribed to each one. This may be illustrated by an example: after my student's lamp has been lighted two

hours, I find the thermometer has risen from 65° to 70° Fahr. The phenomenon to be explained then is the additional 5° of heat. There is no fire, and it seems that the increase in temperature must be due to the lamp, and the heat given off from my body during this period. Suppose that the lamp is burned for the same length of time while the room is unoccupied, all other conditions remaining the same, and that the thermometer shows an increase of 4° in the temperature. By subtraction we could conclude that the heat given off by the body on the former occasion was the cause of the additional degree of temperature.

To carry the process of analysis a step further. Let us suppose that a half pint of oil, which is composed of hydrogen and carbon, has been consumed. We could determine, by measuring the heat produced by the oxidation of the exact amount of carbon contained in one-half a pint of oil, what quantity of heat is due to the combustion of the carbon contained in the oil, and, by subtraction, what must be ascribed to the burning of the hydrogen.[1]

(*b*) The second case in which this method may be applied is where there is an unexplained remainder or residue left over after the result of all the known causes has been calculated. Mill does not distinguish between such instances and the method of simple subtraction discussed above. Since, however, the cause must explain the whole of the effect, the method of residues enjoins us to continue the search for explanation. *When any part of a complex phenomenon is still unexplained by the causes which have been assigned, a further cause for this remainder must be sought.* If, for example, it were found by actual measurement that the heat produced by the lamp, and by the body of the occupant, were

[1] This is, of course, not strictly correct, for it leaves out of account the heat generated by the *chemical combination* of the carbon and hydrogen. It may, therefore, serve to illustrate a case where the method of Residues breaks down.

not sufficient to account for the change in temperature of the room, it would be necessary to seek for some further cause to account for this unexpected remainder.

This method can scarcely be said to be more than a demand for complete and precise explanation. The attempt, however, to account for unexplained residues has led to many extremely important discoveries in science. Residual phenomena are often so obscure, and appear so uninteresting and unimportant to the ordinary mind, that they are passed over without explanation. It usually requires the eye of a scientific genius to see the importance of things which appear trivial and unessential. With Darwin, facts which might appear to an ordinary observer mere unimportant exceptions, were made the object of special attention, and often served as starting-points for his investigations. Francis Darwin, speaking of his father, says: "There was one quality of mind which seemed to be of special and extreme advantage in leading him to make discoveries. It was the power of never letting exceptions pass unnoticed. . . . A point apparently slight and unconnected with his present work is passed over by many a man almost unconsciously, with some half-considered explanation, which is really no explanation. It was just these things that he seized upon to make a start." [1]

Among the many important discoveries which have resulted from the investigation of some obscure and seemingly unimportant fact, we may mention that of ozone. It had been observed for a long time that the passage of electric sparks through the air is accompanied by a peculiar odour. This odour was also found near electrical machines, and was known as the 'electrical smell.' No

[1] *Life and Letters of Charles Darwin*, Vol. I., p. 125.

one seemed to have attached any importance to it or to have attempted to explain it in any way, until Friedrich Schönbein, a professor of chemistry at Basel, turned his attention to the subject. The result of his investigations was the discovery of ozone, the peculiar modification of oxygen, which was the cause of the odour.

Another very striking example of the application of this method is afforded by the history of the discovery of the planet Neptune. In 1781 a new planet was discovered moving outside all the other planets by Sir William Herschel. This was the planet Uranus. When its orbit came to be calculated, it was found that it did not move as it might be expected to do according to the theory of gravitation. That is, the attraction of the sun and the known planets did not account for the path it took: it moved outwards into space farther than it ought to have done. It was evident that either some mistake must have been made in the observation of the astronomers, or some unknown body must be dragging it out of its course. No traces of any such planet could be perceived, and the problem remained unsolved. In 1843, a student of St. John's College, Cambridge, named Adams, undertook to work out the movements of Uranus, to discover, if possible, the position of the body which was pulling it out of what would otherwise be its proper path, the attractions exercised by the sun and the planets in their different positions, and to show what effect they would have in determining the orbit of Uranus. Whenever the planet was deflected outwards, it was necessary to show where the body was situated which was thus influencing it. In 1845 he was able to send a paper to the astronomer royal at Greenwich, informing him in what quarter of the heavens the new planet should be observed. When the discovery was afterwards made, it was proved that his calculations were almost exactly correct. A failure on the part of the astronomer royal to coöperate by looking through his telescope for the planet gave the prior right of discovery to a Frenchman named Leverrier. The latter worked out his calculations in the same way as Adams.

and obtained almost exactly the same results. He sent these results to Professor Galle of the Berlin University on the 23d September, 1846, asking him to look in the part of the heavens which he indicated. That same evening, by following out the directions, the planet was discovered in almost the exact spot predicted.[1]

The history of this discovery illustrates as well several methods and processes which we have not yet discussed, such as the formation and verification of hypotheses. It is also interesting as showing how reason is able, under certain conditions, to anticipate perception. The relations and forces of the heavenly bodies had been so perfectly formulated in the law of gravitation that these two investigators, working in their studies, were able to predict not only the presence, but the exact position of a planet which up to that time had never been observed. It is where mathematical methods can be used that such anticipation is most often possible. Hence this use of the method of Residues has frequently led to important results in astronomy.

REFERENCES TO CHAPTERS XV. AND XVI.

Mill, *Logic*, Bk. III., Chs. VIII. and IX.
Joseph, *An Introduction to Logic*, Ch. XX.
Sigwart, *Logic*, Vol. II., § 95.
Hobhouse, *The Theory of Knowledge*, Chs. XIII.–XV.

[1] Cf. Clerke, *A Popular History of Astronomy during the Nineteenth Century*, pp. 96 ff.; Buckley, *A Short History of Natural Science*, pp. 302 ff.

CHAPTER XVIII

ANALOGY

§ 64. **Explanation by Analogy.**—An 'Analogy' may be defined in general terms as an agreement, resemblance, or proportion between the relations of things to one another, or between the things themselves. Thus it might be said that there is an analogy between the relations of a ruler to his people and those of the captain of a vessel to members of his crew. Or an analogy might be said to exist simply between a ruler and a captain, or between a state and a ship. In logic, analogy is used more specifically as a form of reasoning in which, from the resemblances of two or more things in certain respects, their likeness in other respects is inferred.

The tendency to note resemblances and to assume that things alike in certain respects are alike in all, is present from the first in all stages of thinking. We have seen (§ 50) that this principle guides inductive inquiry by furnishing suggestions as to what may be expected when new facts and conditions are met with. We seek to assimilate what is new to that with which we are already familiar. But in noting, in our earlier discussion, the operation of this principle, no detailed description of its principles was given, or any adequate account of the part it plays in organizing experience. In this chapter emphasis is laid more particularly on the function that Analogy performs at a somewhat advanced stage of inductive inquiry, in leading on to the higher generalizations of science. At a lower level the connections and relations suggested by Analogy are of a factual and descriptive

character. For example, Analogy might suggest in a particular case that the severe frost is the cause of the bursting of water pipes, without affording any clear understanding of the universal law through which these things are connected. In more advanced stages of knowledge, however, Analogy is used consciously and critically as a means of deriving general laws and principles of explanation. In proceeding to the discussion of this more explicit use of Analogy, we may then be said to be passing from Description to Explanation. But, as has already been pointed out (§§ 52, 53), no hard and fast line can be drawn between the determination of the nature and connection of facts, and their explanation. The task which our thought is called upon to perform is to transform obscurely known and isolated facts into an orderly and consistent system of knowledge, and this process is continuous throughout. But, keeping this in mind, one may still say it is necessary, in the first place, for the facts to be thoroughly analyzed and carefully examined; and, secondly, for them to be grouped together according to some general principle or principles which shall make clear and intelligible the relations in which they stand to one another.

To explain is just to show that some fact or group of facts is related in an orderly way to some other fact or group with which we are acquainted. So far as the methods we have discussed enable us to establish connections between events, they may fairly claim to be methods of explanation. Nevertheless, although the difference between these methods, and those of explanation in terms of wider generalizations, is one of degree rather than of essential nature, it is important to keep it in mind. The canons which were stated in the last

two chapters — what Mill named the experimental methods — are rules for determining causal connections between phenomena. The problem in those chapters was to determine what particular phenomena of our experience are essentially and necessarily connected as antecedents and consequents. This constitutes a more or less distinct step in the work of systematization which is carried on by thought. The method of Difference, for instance, enables us to say that hot water will break thick glasses when poured into them, but will not damage thin ones. 'So much for the *fact*,' we say, 'but the *explanation* is still wanting.' We must try to make the fact intelligible by going outside of it, and showing that this behaviour on the part of the glasses is simply a case or illustration of what we already know of the properties of bodies when heated. Again, the method of Concomitant Variations, as we have seen from Jevons's example, has led us to believe in some causal connection between electrical storms, sun-spots, and the Aurora Borealis. In this instance, knowledge has not been able to advance beyond the fact to its explanation. No satisfactory theory has yet been established to account for the undoubted fact that these phenomena are in some way causally connected.

The principle of Analogy is resemblance. The phenomenon to be explained is connected with some more familiar occurrence through a perceived or imagined likeness between the two cases. All our first rude classifications and explanations are based on this principle. In the early stages of the history of the race, everything was explained on the analogy of human actions (cf. § 89). All natural events, that is, were supposed to be produced by superhuman agents, who were, however, endowed with essentially the same qualities as man.

In the thunder, the men of a primitive age heard the voice of a god. An eclipse of the sun or moon was interpreted as a divine sign or warning. When the sea became tempestuous and lashed its shores, they believed that the sea-god was angry. In every case, they interpreted these mysterious happenings of nature by referring them to causes similar in character to those which they best understood as effective forces — the motives and volitions of themselves and their fellows.

The principle of analogy is employed in the same way in modern times. It is true that we no longer think that natural events are directly caused by the action of some spiritual agent more or less like ourselves. But, when we endeavour to show that the phenomena which we are interested to explain are similar in important respects to some group of facts with whose mode of operation we are familiar, we proceed by analogy. On the basis of this similarity, we argue that the phenomena with which we are dealing probably have the same properties, or operate in the same way, or are governed by the same laws, as the better-known facts which they resemble. The formula of analogy may be stated in this way: Two things resemble each other in one or more respects, they are therefore of the same general type or character; it follows that a certain proposition which is true of the one is probably true of the other. The following example of analogy has been frequently used as an illustration: —

"We may observe a very great similitude between this earth which we inhabit, and the other planets, Saturn, Jupiter, Mars, Venus, and Mercury. They all revolve round the sun, as the earth does, although at different distances and in different periods. They borrow all their light from the sun, as the earth does. Several of them are known to revolve around their axes like the earth, and by

that means must have a like succession of day and night. Some of them have moons that serve to give them light in the absence of the sun, as our moon does to us. They are all in their motions subject to the same law of gravitation as the earth is. From all this similitude, it is not unreasonable to think that those planets may, like our earth, be the habitation of various orders of living creatures."[1]

The word 'analogy' at the present time is somewhat loosely used for any mark of similarity or resemblance which enables us to reason from one thing to another. As already noted, the term is also applied either to a likeness between two things, or a likeness between certain relations of things. In the latter case, there is of course a proportion expressed, as when it is said that the relation of a clergyman to his parishioners is analogous to that of a physician to his patients. The purpose of such comparisons is to afford a basis for inferring that the rights or duties that exist in the one case obtain also in the other. In such cases, however, we have always to ask if there are not *differences*, as well as likenesses, in the two sets of relations. This employment of analogy is more strictly that which was noted and defined by Aristotle. "The original word ἀναλογία, as employed by Aristotle, corresponds to the word Proportion in Arithmetic; it signifies an equality of ratios, ἰσότης λόγων: two compared with four is analogous to four compared with eight. There is something of the same meaning in the technical use of the word in physiology, where it is used to signify similarity of function as distinguished from similarity of structure, which is called homology; thus the tail of a whale is analogous to the tail of a fish, inasmuch as it is similarly used for motion, but is homologous with the hind legs of a quadruped. A man's arms are homologous with a horse's fore legs, but they are not analogous, inasmuch as they are not used for progression."[2]

[1] Reid, *Intellectual Powers of Man*, Essay I., Ch. III.

[2] Minto, *Logic, Inductive and Deductive*, p. 367.

Apart from these technical uses, what is known as analogical reasoning may, perhaps, be best defined as an argument from similar instances. In analogy, we do not stop to work out a law of connection between phenomena by comparing a number of cases, or by using any of the ordinary inductive canons. But finding a striking resemblance between some circumstance — relation, quality, arrangement, function, etc. — in the phenomena to be explained, and some phenomena with which we are already acquainted, we use the latter as a basis for conclusions about the former. Analogy is thus an argument from examples or instances, its value depending upon the real identity in some important aspect of the cases compared. When, however, our thought is able to extend to a new case, or set of cases, some general law or principle with whose operation it is already acquainted in other instances, we have passed beyond analogy to a higher form of explanation. In the former case, we argue from the resemblance of instances; in the latter, the thread which binds the new instance with the old is the identity of a general principle.

§ 65. **Analogy as Suggestive of Explanatory Hypotheses.** — We have shown above that analogical reasoning depends on the resemblance which exists between individual cases or instances, and that it does not itself succeed in formulating any general law or principle. The next section will show in more detail in what respects the principle of analogy falls short, and why, taken by itself, it can only be regarded as incomplete explanation. Here we have to notice the important part which it plays in suggesting laws and principles. Although analogy 'sticks in the particular instances,' it leads the mind on to general laws and explanatory theories. It is

thus of the greatest importance as a necessary stage on the way to complete explanation.

When we are able to discover some general resemblance between a group of phenomena which we are interested to explain, and another group whose principle of operation we already understand, our thought strives to extend the known principle and to bring the new facts under it. The unknown or unexplained facts are thus brought under a known law. It is of course true that the application of the law to a new set of facts broadens our conception of its scope, and often requires us to state it in a more adequate way. Thus the analogy which Newton perceived between the heavenly bodies falling through space and the falling of the apple towards the ground, led to the formulation in exact mathematical terms of the universal law of gravitation. Our knowledge of the various functions of plants — digestion, reproduction, etc. — has been obtained by ascribing to the various organs of the plant, purposes analogous to those which are fulfilled by the parts of animal bodies. And, in turn, the study of plant physiology has thrown light upon animal physiology, and enlarged and modified many of its theories. Again, the explanation of many geological changes, — the wearing away of rocks, the formation of deltas or of great ravines, of vegetable mould, etc., — is facilitated by a discovery of their analogy with familiar events which happen constantly before our eyes.

An extremely interesting instance of the part which analogy plays in suggesting possible explanations, is found in the account of the discovery of the principle of Natural Selection given by Darwin in his Autobiography. In 1837 Darwin opened a note-book for the purpose of recording all facts in any way connected with the variation of species in nature and under domestication. He first

investigated the variations of plants and animals which are produced under domestication, by printed inquiries, by conversation with skilful breeders, and by extensive reading. "I soon found," he says, "that selection was the keystone of man's success in making useful races of plants and animals." When useful or pleasing varieties of plants or animals occur, the gardener or breeder preserves them, and their peculiar qualities are transmitted to their offspring. And, in a number of generations, these qualities become more pronounced through accumulation. The differences between varieties of the same species of domesticated animals — varieties which are as different, for example, as the mastiff and Skye terrier — are due to the selective agency of man. But is there anything analogous takes place on an indefinitely larger scale in nature? If so, what is it which plays the part of the gardener or breeder, and preserves certain varieties?

When Darwin had reached this point in his investigations, and, had come to appreciate what selection could do, he happened to read Malthus's book, *On Population*. The purpose of this book was to dispel the optimistic ideas of some of the writers of the eighteenth century who looked for the speedy realization of social well-being and happiness. Such an ideal is impossible of fulfilment, said Malthus, because of the inevitable tendency of population to increase faster than the supply of food. Human beings increase in a geometrical ratio; the means of subsistence, at best, only by an arithmetical ratio. The population will thus constantly tend to exceed the limit of the food supply, and will be kept in check only by starvation. A constant struggle for food is the lot, then, to which each individual is doomed in virtue of this law. Darwin's observations of the rate at which plants and animals tend to reproduce their kind, led him at once to extend Malthus's principle to the whole of nature. The fecundity of natural beings leads to a struggle for existence, not merely among men, but throughout the whole organic world. And if there is a struggle, we have natural

selection or the survival of the fittest. Darwin saw "that natural selection was the inevitable result of the rapid increase of all organic beings." It is not difficult to see that this discovery was the result of Darwin's wonderful power of perceiving analogies between different classes of facts. His genius led him to recognize first the resemblance of the variations of species in nature to the more familiar variations which go on among domesticated plants and animals. And, secondly, he perceived that the competition for the means of subsistence, which the pressure of population imposes upon the members of the human race, is simply one phase of 'the struggle for existence,' which is going on everywhere throughout the organic world.

§ 66. **The Incompleteness of Analogical Reasoning.** — The most striking feature of analogical arguments is found in the fact that they yield only probable conclusions. And the reason for this is not far to seek. For, as has been already shown, analogy is a method of reasoning from one particular case to another on the basis of some imagined or perceived similarity between the two cases. Complete logical demonstration, or certainty, however, is attained only when the new fact or group of facts is really and essentially united by means of some general principle with what is already known. There is no genuine inference from 'particular to particular,' as Mill supposed. Inference, as has been well said, always 'proceeds through a universal.' It is the universal implied in the common name, or vaguely present in the mind of the reasoner, which really carries the inference in cases where conclusions appear to be drawn from a particular case. When one reasons that food or drink which has made A ill will produce the same result in B, it is the universal nature of human beings on which the inference is based. In the

case of Analogy, the inference lacks certainty because the universal nature is not analyzed or defined. Instead, it is vaguely assumed in the form of external likeness or resemblance.

But, although Analogy yields only probable conclusions, it must not be forgotten that 'probability' is not a fixed quantity. An argument from analogy may have any degree of value, from zero almost up to the limit of complete logical certainty. To *fully* explain or demonstrate any fact, we are obliged, I think, to go beyond analogy, and to verify its conclusions by bringing them into relation to a general principle. It is evident, nevertheless, that the value of an analogical argument will depend upon the nature of the resemblance which is taken as the basis of inference. In general, it is true that the greater the resemblance between the two cases, the more certainly can we reason from one to the other. This is not to say, however, that the value of the conclusion is in direct proportion to the number of points of resemblance which can be discovered. For example, we might reason: These two men are of the same height, of the same age, live in the same house, come from the same town; the one man stands well in his classes, therefore the other probably does so also. If the *number* of points of resemblance were the essential thing, the argument ought to possess some weight, but it is clear that it has none. The difficulty is that none of the resemblances mentioned are fundamental, or in any way essential to the real nature of the things compared. If we knew that the two men were similar in character, this one characteristic would be worth more, as a basis for the conclusion, than all the circumstances which we have mentioned combined.

It is true, then, as Mr. Bosanquet remarks, that in analogical reasoning we must *weigh* the points of resemblance rather than count them.[1] Other things being equal, the more points of resemblance we can make out the better; but if these are to contribute at all to the certainty of the conclusion, they must represent some deep-lying characteristic of the things compared. In general, it must be said that it is only experience which can inform us what resemblances are fundamental, and what merely external. Systematic knowledge in any field enables us to separate the essential from the accidental. And, what is perhaps a corollary from this, it must not be forgotten that the value of an inference from analogy depends largely upon the amount of intellectual insight possessed by the mind which makes it. The ordinary mind, at least in its undisciplined and untutored condition, regards all things as of equal importance. It is therefore led away by the strongest stimulus—by striking external and accidental resemblances—as is well shown by the readiness with which such minds are carried away by the fallacies of figurative or analogical language. On the other hand, a scientific genius whose mind is well stored with facts, and who is gifted in addition with imagination, is able to penetrate beneath the surface and to apprehend the real or fundamental resemblance. His imagination enables him to see beyond the chaos of the particular facts, and to detect the underlying principle by means of which these facts can be connected and systematized.

Analogy thus becomes deepened until it passes from the stage of a mere argument from particular to particular, to the perception of a general law which includes the individual instance. But no such direct insight can claim the title of

[1] *Logic*, Vol. II., p. 99.

knowledge, until it is tried and tested by the facts. The guesses of scientific men unfortunately often prove mistaken. It is always necessary that fancy shall be confronted with facts. Even Darwin's magnificent analogical inference was nothing more than an hypothesis, as he himself well understood, until its power of explaining the facts of organic life was demonstrated. We have now to explain in the next chapter the methods by which such guesses are tested.

REFERENCES

J. S. Mill, *Logic*, Bk. III., Ch. XX
A. Bain, *Logic*, Part Second, Induction, pp. 140–148.
J. G. Hibben, *Inductive Logic*, Ch. XIV.
B. Bosanquet, *Logic*, Vol. II., Ch. III.
" " *The Essentials of Logic*, pp. 155–58.
W. Minto, *Logic Inductive and Deductive*, pp. 367–373.

CHAPTER XIX

THE USE OF HYPOTHESES

§ 67. **Reasoning from an Hypothesis.** — An hypothesis, taken in its most general sense, is a guess or supposition as to the existence of some fact or law which will serve to explain a fact or connection of facts already known to exist. It is thus an expression of the tendency of the mind to leave nothing standing in isolation, but to 'explain' the various parts of experience by bringing them into relation with one another. 'Theory' is another word that is often used as equivalent to hypothesis. Strictly speaking, however, it is better usage to employ the term 'hypothesis' for the unverified, or only partially verified guess, and to reserve 'theory' for the hypothesis that has been more completely demonstrated. This distinction, however, is not usually maintained, and even in scientific writings the terms 'theory' and 'hypothesis' are used interchangeably. Nevertheless, it is necessary to distinguish in some way the 'mere hypothesis,' or supposition, which is often as likely to be false as true, from the hypothesis which has been established by proof.

It is important to remember that it is not only in solving scientific problems that we employ hypotheses. In our ordinary experience, we are constantly trying to imagine the most likely explanation of facts which we perceive through the senses. If, for example, one should find on returning to one's room that a pane of glass had been broken, one would straightway set about finding some explanation of this occurrence. One might perhaps first imagine that a stone or

something of the kind had been thrown against it. Acting on this supposition, one would look for the stone in the room. If it were found there, the hypothesis would be confirmed; if no traces of it could be discovered, and if, moreover, on examination the glass proved to be shattered in a way that would probably not result from the projection of a stone against it, our first hypothesis would have to be abandoned. We should then make another guess — perhaps that the outside blind had been violently closed by the wind — and again examine the facts to see if they gave any support to this supposition. We are constantly making hypotheses of this character to explain phenomena which we meet with in everyday experience. If we find a stream swollen, we conclude that it must have rained in some part of the country drained by the stream. If a man has typhoid fever, we are pretty sure to guess that he has been drinking impure water. We no sooner perceive something unusual or striking than we begin to guess out, as it were, its explanation. The formation of hypotheses, then, is simply the mind's response to the demand for explanation.

The examples given above illustrate what may be called the popular, as opposed to the scientific use of hypotheses. In these cases the hypothesis assumes the existence of a particular thing or event as that through which the phenomenon in question is to be explained. The 'law' at which the induction arrives is that of a causal connection of phenomena taken in a descriptive or factual way. Analysis is not carried on to reach a genuinely explanatory hypothesis, as it would be in a strictly scientific investigation. Such an explanatory hypothesis would not point to any particular phenomenon as a 'cause,' but would state as a law certain permanent forms of relation in which things and events

stand, and under which the phenomenon in question is assumed to fall. Think of the difference in character between the hypothesis that the window was broken by the slamming of the blind, and, for example, Newton's law of Gravitation, or the vast generalization of facts included in Darwin's law of Natural Selection.

Nevertheless, it cannot be maintained that the distinction is in any sense absolute between the hypothesis of a fact and the hypothesis of a general law of relation. What is an hypothesis at one stage becomes, when verified, for further investigation a fact or starting point. Between the popular and the scientific use of hypotheses there are important differences of degree, as has been pointed out. In discussing the use of hypotheses in this chapter, we shall have in mind primarily the reflective and critical procedure through which certain conceptions are defined and tested as instruments for the colligation of facts. We shall thus be studying, in its highest and most explicit form, the function that guides Induction from its earliest beginnings.

It is worth noticing that it is only unusual or striking events, or those in which they have some practical concern, which attract the attention of the majority of mankind, and lead them to form explanatory hypotheses. What is familiar, or of no practical importance, does not usually awaken curiosity. Indeed, in a great many cases, such phenomena are not observed at all. But the great scientist is distinguished, one may say, by his intellectual curiosity. He tries to understand phenomena which the ordinary mind neglects and simply takes for granted. He has questions in his mind with regard to familiar things which he wishes to have answered, guesses which he is desirous of having proved or dis-

proved. Unless the mind has some question to answer, or theory to test, it is impossible to see any significance in an experiment. In other words, every experiment must have a purpose, and the purpose is to get some information that will help us to answer a question which we bring with us to the investigation.

In the actual process of acquiring knowledge, then, observation and theorizing go hand in hand. Unless we go to nature with something in our mind, we are not likely to learn much. As a rule, we see only what we look for. Francis Darwin says of his father: "He often said that no one could be a good observer unless he were an active theorizer. This brings me back to what I said about his instinct for arresting exceptions: It were as though he were charged with theorizing power ready to flow into any channel on the slightest disturbance, so that no fact, however small, could avoid releasing a stream of theory, and thus the fact became magnified into importance. In this way it naturally happened that many untenable theories occurred to him, but fortunately his richness of imagination was equalled by his power of judging and condemning the thoughts which occurred to him. He was just to his theories and did not condemn them unheard; and so it happened that he was willing to test what would seem to most people not at all worth testing. These rather wild trials he called 'fool's experiments,' and enjoyed exceedingly. As an example, I may mention, that finding the cotyledons of Biophytum to be highly sensitive to vibrations of the table, he fancied that they might perceive the vibrations of sound, and therefore made me play my bassoon close to a plant." [1]

[1] *Life and Letters of Charles Darwin*, Vol. I., p. 126.

A good example of how essential theories are for an observer, and how blind he may be to what he is not looking for, is found in the work from which we have just quoted. In the brief autobiography contained in the first volume, Darwin tells of a geological trip through Wales which he took while a student at Cambridge, in company with Sedgwick, the professor of geology. It must be remembered that this was before Agassiz had come forward with his theory of a glacial period in the world's history. Darwin writes: "We spent many hours in Cwm Idwal, examining all the rocks with supreme care, as Sedgwick was anxious to find fossils in them; but neither of us saw a trace of the wonderful glacial phenomena all around us; we did not notice the plainly scored rocks, the perched boulders, the lateral and terminal moraines. Yet these phenomena are so conspicuous that, as I declared in a paper published many years afterward in the *Philosophical Magazine*, a house burnt down by fire did not tell its story more plainly than did this valley. If it had been filled by a glacier, the phenomena would have been less distinct than they are now." [1]

§ 68. **Formation of Hypotheses.** — We are now ready to consider a little more closely the formation of hypotheses or theories. In the first place, it is to be noticed that hypotheses are not received from without through sense-perception, but are made by the mind. They are the creations of the imagination. A good theorizer, like a poet, is in a certain sense born, not made. The man to whom 'nothing ever occurs,' whose intellectual processes are never lit up with a spark of imagination, is unlikely to make any important discoveries. It has been by a flash of scientific genius, by im-

[1] *Life and Letters of Charles Darwin*, Vol. I., p. 49.

aginative insight which we may almost call inspiration, that great scientific theories have been discovered. Not even a scientific genius, however, can afford to neglect the facts. But, guided by accurate observation, the scientific imagination tries to invent some law or principle which will serve to connect and explain facts. Tyndall has an essay on "The Scientific Use of the Imagination," from which we may quote a short passage. "With accurate experiment and observation to work upon, imagination becomes the architect of physical theory. Newton's passage from a falling apple to a falling moon was an act of the prepared imagination. . . . Out of the facts of chemistry the constructive imagination of Dalton formed the atomic theory. Davy was richly endowed with the imaginative faculty, while with Faraday its exercise was incessant, preceding, accompanying, and guiding all his experiments. His strength and fertility as a discoverer are to be referred in great part to the stimulus of the imagination. Scientific men fight shy of the word because of its ultra-scientific connotations; but the fact is, that without the exercise of this power, our knowledge of nature would be a mere tabulation of coexistences and sequences."[1]

In speaking of hypotheses as 'guesses,' or 'creations of the imagination,' their dependence upon facts must not be forgotten. It is only when the phenomena to be explained have been carefully observed that our guesses at their explanation are likely to be of value. It is well known that a considerable amount of knowledge is usually required to ask an intelligent question. And in the same way, the mind must be well stored with facts, in order to render our hypothetical explanations worthy of consideration. Indeed, observation of facts and the formation of theories go hand in hand, and natu-

[1] *Fragments of Science*, p. 104.

rally assist each other. We have already spoken of the lack of theory which makes us blind to facts that seem to lie directly before us. But we have perhaps not yet emphasized sufficiently the dependence of theories upon the facts of observation. The process of explanation may be described as a fitting together of the facts given by observation, with the explanatory theories which the mind originates. The theory with which we start enables us to ask questions, and leads us to scrutinize the phenomena which are to be explained; while the latter react upon the theory, and cause it to undergo constant modification. Neither the 'theory' nor the facts are to be regarded as fixed and unchanging; both are constantly changing in relation to each other as the investigation proceeds. The account of Darwin's discovery of the principle of 'the survival of the fittest' is a good illustration of an hypothesis constructed by a constant dependence upon the facts during every step of its progress.

We have already referred to the way in which analogy leads the mind on to general principles of explanation (§ 60). Analogy is a method of inferring that what is true of one object is probably true of others which resemble it. But the ordinary mind sees resemblances only when they are very obvious and striking. The man of scientific insight, on the other hand, like the poet, penetrates more deeply into the nature of things, and is able to discover analogies and resemblances to which the ordinary man is blind. Who but a genius like Newton would have thought of connecting the fall of an apple with the fall of the heavenly bodies through space? The history of science shows that great discoveries are made by means of imaginative insight, but it also teaches that mere imagination without dependence upon known facts is frequently a source of much mischief. Mere theories without facts are not only empty, but often stand in the way of true knowledge. The fruitful exercise of the

imagination, if we may judge from the way in which great discoveries have been made, always takes place in closest connection with what observation and experiment reveal regarding the nature of phenomena. If the imagination is to have power to discover any truth, it must constantly 'touch earth,' and be guided in its course by the nature of facts which are already known.

In framing hypotheses, then, the imagination is constantly prompted by analogies with processes which are more or less familiar. The hypothesis, accordingly, is not created by the imagination 'out of nothing.' It is rather an extension or development of a known law, than an absolute creation.

§ 69. **The Proof of an Hypothesis.** — We have discussed the way in which hypotheses are formed, but as yet have said nothing regarding the means of determining their truth or falsity. But to form hypotheses is usually easy, to verify them is often exceedingly difficult. The scientific worker constantly finds that theories which he has formed cannot be verified, and must therefore be discarded. It is not only essential that a scientific investigator shall possess a mind fertile in ideas; he must also love truth more than any theory, no matter how interesting or attractive it may appear. In behalf of truth, every theory must be subjected to the most thorough and searching tests possible; if it is not borne out by facts, it must be at once discarded. What now is the general method of procedure in testing an hypothesis? How do we proceed to compare our theories with the facts? Two steps or stages may be distinguished in this process: (1) We assume that the hypothesis is true, and proceed to show what are the necessary results which follow from it. In doing this we proceed deductively; that is, assuming the

truth of the hypothesis, we reason out what consequences must follow from it in accordance with laws whose mode of action we already know. (2) The conclusions thus reached are compared with the actual facts, as given to us directly in perception, or as determined by experiment. If they are found to agree with these, the hypothesis is regarded as true; if they do not agree, it becomes necessary to discard the hypothesis, or to modify it in some way suggested by the results so far obtained by the investigation.

This procedure may become clearer by considering some concrete examples. We may first take an illustration of what has been called the popular use of an hypothesis. If we were to come on the campus some morning and find that several branches had been broken from one of the trees, we should naturally try to explain this circumstance by making some hypothesis. Perhaps the first thing which would occur to us would be that there had been a violent wind storm. The hypothesis having been made, the next step would be to look around to see if it could be verified. 'If there has been a cyclone,' we might argue, 'there should be other signs of its presence; we should find broken twigs and blown leaves lying about, and all the trees should present a storm-tossed appearance.' If observation showed that these things were actually present, we would consider our hypothesis so far confirmed. But if not, our first guess would be disproved, and it would be necessary to look about for another explanation. In this case, the second hypothesis, being based on a better analysis of the facts, would be more likely to prove correct than the first. But the process might have to be continued through several steps.

An excellent illustration of the way in which a scientific

hypothesis may be rendered more certain and at the same time more comprehensive and definite is found in the history of the experiments by which it was proved that the atmosphere has weight. Galileo noticed that water will rise in a pump only about 33 feet. He could not find out, however, why it was that the water stopped at this point. After his death, his friend and pupil Torricelli took up the problem, and asked himself: Why does the water rise at all? It then occurred to him that air must weigh something, and that it might be this weight on the surface of the water which forced the water up the pump when there was no air pressing it down. Now, if this were so, he reasoned, the weight of the air ought to lift mercury, which is fourteen times heavier than water, to one-fourteenth of the height. So he took some mercury, and filling a tube about 34 inches long, turned it upside down into a basin of mercury which was open and therefore under the pressure of the atmosphere. The mercury began to settle in the tube, and finally rested at a height of 30 inches. Torricelli had thus invented the barometer, an instrument which would measure the weight of the atmosphere. It was afterwards suggested by the famous French writer, Pascal, that at the top of a high mountain, where there is less air pressing downwards, the column of mercury should fall considerably if the atmosphere were really what caused the water and the mercury to rise. When this experiment was made by carrying the barometer to the top of a mountain called the Puy de Dôme, the mercury fell nearly three inches. Still further confirmation of Torricelli's theory was afforded by the discoveries of Otto Guericke of Magdeburg. In 1650 Guericke invented the air-pump. The first use which he made of his new invention was to show that the atmosphere is pressing

down upon us heavily and equally in all directions. He fitted closely together two metal hemispheres and exhausted the air between them by means of his pump. It was found that the pressure of the atmosphere was so great that it took a great force to separate the hemispheres.[1]

To establish a scientific theory, then, there is necessary not only a ready imagination, but also patience and perseverance in the careful deduction of the consequences of the theory, and in the comparison of the results thus obtained with the actual facts. Scientific work also demands the utmost candour and openness of mind on the part of those who engage in it. One must be willing to abandon any theory as soon as it is found to disagree with the facts. And this is by no means an easy thing to do. When one has a theory which suffices for nearly all the facts, there is always a temptation to cling to it, and to neglect or explain away any troublesome or contradictory facts. There is no doubt that the scientific explanations which have become accepted and established were not the ideas which first happened to occur to the men with whose names they are associated. When Newton first attempted to work out the verification of the gravitation hypothesis, he used the most accurate measurements he could obtain regarding the size of the earth. But in calculating on this basis the pull of the earth on the moon, and the consequent deflection of the moon from the straight line, his results came out wrong. That is, the moon moved more slowly than it ought to move according to his theory. The difference was not great, but Newton could not overlook this lack of agreement with the observed facts. He put the whole matter aside; and it was only when he heard, sixteen

[1] Cf. Buckley, *Short History of Natural Science*, pp. 114–121.

years later, that Picart had discovered from new and more accurate measurements that the earth was larger than had been supposed, that he repeated his calculations, and found his hypothesis verified.

(1) In stating the general theory of Induction in the opening Chapter (§ 50), emphasis was laid on the part played by hypotheses or guiding conceptions from the very beginning of an investigation. Frequent references to this point have been made in the discussion of the various methods. We learned that even to define a problem or ask an intelligent question is to *presume* something, or to have some kind of an hypothesis regarding the kind of answer to be given. The question how hypotheses are tested, is then really identical with the question how inductions in general are established. Now, in explaining and illustrating the procedure of Induction and its use of the various methods, attention was more than once directed to the part played by Elimination. The inductive method of proof, it was said, might be represented by a Disjunctive Syllogism where all the possibilities but one were eliminated by exhibiting their incompatibility with the facts. But in these earlier references it was also indicated that certain qualifications of this view are necessary. It must be borne in mind that Elimination is simply a means to an end, and that it therefore only partially describes the inductive process. The fact must be emphasized that the real purpose of Induction, as of all thought, is to discover positive connections and laws, and to define these as accurately as possible.

When we observe facts and perform experiments in order to test the first hypothesis suggested by a problem, we obtain evidence which not merely serves to eliminate that hypothesis, but which also points more or less definitely in a positive direction. It is not generally true, then, that we approach a problem with several definite hypotheses in mind, and proceed to try them one after another

as we might try various keys at random in a lock. But, in thinking, as in all genuine experimentation, failures are instructive. The new hypothesis is forged in and by the process of investigation itself, just as in the progress of the arts finer and more accurate instruments are constantly made possible through the use of those already in existence. The Ptolemaic theory of astronomy, for example, made possible the observations and measurements which finally overthrew it and gave rise to the conception of Copernicus. The new hypothesis, then, may generally be better represented as a modification or closer definition of its predecessor than as something quite new and independent. The formal representation of the Induction by means of the Disjunctive Syllogism, accordingly, fails to bring out clearly the fact of the *development* of knowledge as the work of investigation proceeds. And, as a consequence, the disjunctive member not eliminated is represented as if it were simply of coördinate importance with the others, and as if the fact that it was not eliminated were a mere accident. Or, put in other words, it fails to make clear the fact that (apart from the unmeaning 'infinite judgment,' *e.g.* 'no good resolution is an octagon') all negation or elimination has positive significance, and that the inductive analysis, as it proceeds, furnishes positive grounds of support for one hypothesis in and through the exclusion of the others. An hypothesis must always be proved by showing its positive conformity with facts: negative results and considerations taken alone never furnish complete inductive proof.

In dealing with certain problems, however, or at certain stages of inquiry, we are often compelled to depend in large part on negative evidence. The fact that other hypotheses are excluded, or are less satisfactory, is very often given as a reason in support of a particular theory. But in such cases there always exist, in addition, positive reasons in support of the theory, though they are not regarded as sufficiently strong to prove it completely. Moreover, at a particular point in an investigation, we are sometimes able

definitely to limit the number of possibilities. We do this in mathematics, for example, when we say that one number or dimension is equal to, greater than, or less than, another. And the same is sometimes possible in other fields where we know definitely the exact relations of things. If we are able to say that the phenomenon we are trying to determine is either *a*, *b*, or *c*, we can, of course, prove that it must be *b* by eliminating *a* and *c*. Outside of mathematics, however, the proof would scarcely ever depend wholly on the principle of Exhaustion; but in eliminating the other possibilities some positive grounds for the existence of *b* would almost certainly appear.

(2) The method of proving an hypothesis has been described (page 285 f.) in the following way: 'If the hypothesis agrees with the facts it is to be regarded as established; if it is not in conformity with them, it is to be discarded as false. Now, when stated thus baldly, the professed method of proof seems to involve the fallacy of affirming the consequent (cf. p. 146). 'If a man swallows prussic acid he will die; he is dead, and therefore must have swallowed the acid.' This is obviously fallacious reasoning. We cannot infer that, because certain facts are known to exist which would exist if a certain hypothesis were true, the hypothesis is therefore true. When we speak of an hypothesis as proved by its ability to explain all the facts, it is evident that some further qualifications are necessary. From a practical point of view, an hypothesis is certain somewhat in proportion to the number and the variety of the facts that it is able to explain, assuming, of course, that there are no important relevant facts which it fails to explain. In speaking of Natural Selection, Darwin says: "This hypothesis may be tested . . . by trying whether it explains several large and independent classes of facts; such as the geological succession of organic beings, their distribution in past and present times, and their mutual affinities and homologies. If the principle of natural selection does explain these and other large bodies of facts

it ought to be received." This quotation brings out the fact that the certainty of an hypothesis is not inferred from a single fact or group of facts, and is even not derived from its agreement with a mere sum of facts. It is rather guaranteed by what has been well called the 'Consilience of Results.' An hypothesis is accepted as established when a number of large and independent bodies of fact all point toward it as the one conception exactly fitted to bring them all into intelligible relations.

From the standpoint of logic, it is essential to prove, not only that the hypothesis will explain the facts, but that it is the only hypothesis which will explain them. To get this result, the other possibilities must obviously be eliminated by a more complete and exact survey of facts, and all the positive circumstances brought to light which tend to confirm the hypothesis in question. This is the function of the 'large and independent bodies of fact' which Darwin mentions in the passage just quoted. What is achieved in this way is the exact fitting together of facts and hypothesis through a process of progressive adjustments. In the process the hypothesis is frequently used as a basis for the prediction of new facts, which, when they are found, serve in their turn to confirm the truth of the hypothesis. A most interesting illustration of this procedure is afforded by Darwin's prediction of the existence of a species of Madagascar moth with a tongue eleven inches in length. The basis of the prediction was his theory of the fertilization of flowers by insects, and the adaptation that is consequently found between the structure of the parts and certain species of insects. Shortly after the appearance of his book *On Fertilization of Orchids by Insects*, a correspondent wrote to him objecting to the theory elaborated in that work: "What have you to say in regard to an orchid which flourishes here in Madagascar possessing a long nectary, as slender as a knitting-needle, and eleven inches in length? On your hypothesis there must be a moth with a tongue eleven inches long, or this nectary would never have been elaborated." Darwin

replied:[1] "The existence of an orchid with a slender nectary eleven inches in length, and with nectar secreted at its tip, is a conclusive demonstration of the existence of a moth with a tongue eleven inches in length, even though no such moth is known." Not long afterwards Darwin's prediction was verified by the discovery of a huge sphinx-moth with a tongue of the length predicted.

§ 70. **Requirements of a Good Hypothesis.** — Various conditions or requisites of a good hypothesis are laid down by writers on logic. The three laws which are most frequently stated are as follows: (1) That the hypothesis shall be conceivable and not absurd. (2) That it shall be of such a character that deductions can be made from it. (3) That it shall not contradict any of the known laws of nature.

It does not seem to me that the first law is of much value. It is largely individual taste or education which leads us to pronounce certain theories 'absurd' or 'inconceivable.' Thus, for a long time, it seemed inconceivable that the earth should be round, and should revolve on its own axis; and less than a generation ago the theory of evolution, as propounded by Darwin, seemed to many persons utterly 'absurd.' Nor can the third law always be applied as a test of an hypothesis, for many great discoveries seemed, at the time when

[1] I have taken this story from W. H. Gibson's *Blossom Hosts and Insect Guests* (pp. 28–29), but have been unable to verify it from Darwin's published letters. In the second edition of the *Fertilization of Orchids* (Ch. VI.), however, Darwin refers to this orchid (*Angræcum sesquipedale*), and from the length of its nectary predicts the existence of a moth with a proboscis of corresponding length. In the same passage he goes on to say: "This belief of mine has been ridiculed by some entomologists, but we now know from Franz Müller that there is a sphinx-moth in South Brazil which has a proboscis of nearly sufficient length, for when dried, it was between ten and eleven inches long. When not protruded, it is coiled up into a spiral of at least twenty windings" (p. 163).

they were announced, to contradict known laws of nature. The difficulty is that no one is able to affirm, unconditionally, that a law of nature forbids us to make this or that hypothesis. Of course, we feel that a theory is very probably false which is at variance with the law of gravity, or with that of the conservation of energy, or any of the laws which we regard as established beyond a reasonable doubt. But, although the chances are always very greatly against any theory which runs counter to what are regarded as well-established laws, there is yet always a *possibility* that it may be true. There is no law of nature so certain as to be infallible. Even those laws which appear to be beyond the possibility of doubt, may require to be modified or supplemented. We may find that, practically, it is not wise to trouble ourselves with theories which undertake to overthrow the law of gravitation, or to disprove other fundamental laws of the physical world. But theoretically, at least, there is always a chance — in cases such as we have been supposing the chance is almost infinitely small — that the new theory may be right, and the old one wrong. The practical objection to admitting the claims of this canon is the difficulty in applying it fairly. The phrase, 'contrary to the laws of nature,' like 'inconceivable,' and 'absurd,' is likely to be used to condemn any theory with which one disagrees. In this way, it is evident that the very point is begged which is really at issue.

Of these three canons, therefore, the second appears to state the only condition which is essential to an hypothesis. An hypothesis, if it is to be of any value, must be capable of being proved or refuted. But, unless its consequences can be shown by way of deduction, it is impossible to know

whether it agrees, or does not agree, with the facts which it is supposed to explain. An hypothesis from which nothing can be deduced, then, is of no value whatever. It always remains at the stage of mere possibility, and without any real connection with fact. It is a mere guess which has no significance whatever, for it is entirely incapable either of proof or of disproof. The ability of an hypothesis to lead to the prediction of facts not previously known to exist has sometimes been emphasized as a test of its value. But this circumstance, although making the hypothesis more impressive is not in itself a proof of its validity. Indeed, true predictions have frequently been made on the basis of hypotheses which were afterwards found incorrect. The essential requirement, however, is that something shall be deducible from the hypothesis, that it shall lead somewhere, and thus afford a programme for further investigation.

(1) In general, it is possible to deduce the consequences of a theory only when the principle employed is analogous, in mode of operation, to something with which we are familiar. Thus, for example, it is because the ether is conceived as resembling other material bodies in important respects that it can be used as a principle of explanation. It is assumed to be elastic and capable of receiving and transmitting vibrations, and as spread out like other material bodies in space. In virtue of these similarities to other material substances, it is possible to deduce the consequences which such a substance as ether would imply, and to compare them with the actual facts. But if one should make the assumption that certain phenomena are due to some agency totally unlike anything of which we have any experience, a disembodied spirit, or ghost, for example, it would be impossible either to prove or to disprove the assertion. For, knowing nothing whatever of the way in which disembodied

spirits act, one could not say whether the phenomena to be explained, table-rapping, planchette-writing, etc., were or were not consistent with a spirit's nature and habits.

Another example of a barren hypothesis from which no conclusions can be drawn, is afforded by the 'catastrophe' or 'convulsion' theory in geology, which was first combated by Lyell, in his *Principles of Geology*, published in 1830. "People had so long held the belief that our earth had only existed a few thousand years, that when geologists began to find a great number of strange plants and animals buried in the earth's crust, immense thicknesses of rock laid down by water, and whole mountain masses which must have been poured out by volcanoes, they could not believe that this had been done gradually, and only in parts of the world at a time, as the Nile and the Ganges are now carrying down earth to the sea, and Vesuvius, Etna, and Hecla are pouring out lava a few feet thick every year. They still imagined that in past ages there must have been mighty convulsions from time to time, vast floods swallowing up plants and animals several times since the world was made, violent earthquakes and outbursts from volcanoes shaking the whole of Europe, forcing up mountains, and breaking open valleys. It seemed to them that in those times when the face of the earth was carved out into mountains and valleys, tablelands and deserts, and when the rocks were broken, tilted up, and bent, things must have been very different from what they are now. And so they made imaginary pictures of how nature had worked, instead of reasoning from what they could see happening around them." [1]

The convulsions, or catastrophes, which were thus assumed to take place were regarded as the result of strange incalculable forces whose mode of operation could never be exactly determined. Instead of these mysterious agencies, Lyell assumed that causes similar to those with which we are now acquainted had been acting uniformly for long ages. The nature of the causes at work

[1] Buckley, *Short History of Natural Science*, pp. 441–442.

being known, it became possible to calculate the nature of the effects, and thus to reduce the facts of geology to order and system. As we have already shown, hypotheses which are to prove really serviceable are formed by extending some known principle through analogy to a new class of facts. The assumption of mysterious agencies and principles whose mode of operation is unlike anything which is known to us, does not aid in the extension of knowledge.

REFERENCES

Mill, *Logic*, Bk. III., Chs. XI.–XIV.
W. S. Jevons, *The Principles of Science*, Ch. XXIII.
C. Sigwart, *Logic*, § 83.
B. Bosanquet, *Logic*, Vol. II., pp. 155–167.
L. T. Hobhouse, *The Theory of Knowledge*, Chs. XVII.–XIX.
H. W. B. Joseph, *Logic*, Ch. XXIII.

CHAPTER XX

FALLACIES OF INDUCTION

§ 71. **The Source of Fallacy.** — It is necessary at the close of our discussion of the inductive methods, to say something regarding the errors to which we are most subject in this kind of thinking. We have seen that knowledge is the result of the mind's own activity, and that it grows in completeness through a persistent effort to keep distinct things which are different, and to connect phenomena which belong together. Truth, in other words, is gained by intellectual activity. And, on the other hand, we fall into error, and are led away by false arguments as a result of mental indolence. Thinking is hard work, and there is always a tendency to avoid it. As a matter of fact, we all think much less frequently than we suppose. Usually, we are content to follow familiar associations, and to repeat current phrases, without doing any real intellectual work. The difficulty is that we can get along comfortably without thinking for the most part — more comfortably, perhaps, than when we do think. Then, again, the mind is less directly under control of the will than the body. One may force himself to sit down at his desk and open a book; but it is more difficult to compel oneself to think.

The only way in which we can be saved from becoming 'intellectual dead-beats,' is by the formation of good mental

habits. It requires eternal vigilance and unceasing strenuousness to prevent our degeneration into mere associative machines. What the logical doctrine of fallacies can do is to put us on our guard against this tendency. It enumerates and calls attention to some of the commonest and most dangerous results of slovenly thinking, in the hope that the student may learn to avoid these errors. Some of the fallacies of which we shall treat in this chapter, apply equally to deductive or syllogistic reasoning, and have been already treated in Chapter XI. We shall, however, enumerate them here again for the sake of completeness. It is convenient to discuss the various fallacies under the following heads:—

(1) Fallacies due to the careless use of Language.
(2) Errors of Observation.
(3) Mistakes in Reasoning.
(4) Fallacies due to Individual Prepossessions.

After what has been said in the preceding chapters regarding the relation of 'facts' and 'theories,' it will not be supposed that the distinction between 'errors of Observation' and 'mistakes in Reasoning' is fixed and absolute. Errors in observation result frequently, as we have seen, from inadequate or confused conceptions. There is, however, a relative difference between the two functions of knowledge, which serves as a convenient principle of classification.

§ 72. **Fallacies due to the Careless Use of Language.**—The careless and unreflective use of words is a very frequent source of error. Words are the signs or symbols of ideas; but the natural sluggishness of the mind leads often to a substitution of the word for the idea. It is much easier to deal with counters than with realities. Since we must use words to express our thoughts, it is almost impossible to prevent them

from becoming our masters. Bacon, who gives the name of 'Idols of the Market-Place' (*Idola fori*) to the fallacies which arise through the use of words, puts the matter in the following striking sentence: "Men imagine that their reason governs words whilst, in fact, words react upon the understanding; and this has rendered philosophy and the sciences sophistical and inactive."[1] The dangers connected with the use of words have also been well represented by Locke, from whom I quote the following passage:—

"Men having been accustomed from their cradles to learn words which are easily got and retained, before they knew or had framed the complex ideas to which they were annexed, or which were to be found in the things they were thought to stand for, they usually continue to do so all their lives; and, without taking the pains necessary to settle in their minds determined ideas, they use their words for such unsteady and confused notions as they have, contenting themselves with the same words other people use, as if their very sound necessarily carried with it constantly the same meaning. . . . This inconsistency in men's words when they come to reason concerning either their tenets or interest, manifestly fills their discourse with abundance of empty, unintelligible noise and jargon, especially in moral matters, where the words, for the most part, standing for arbitrary and numerous collections of ideas not regularly and permanently united in nature, their bare sounds are often only thought on, or at least very obscure and uncertain notions annexed to them. Men take the words they find in use amongst their neighbours; and, that they may not seem ignorant what they stand for, use them confidently, without much troubling their heads about a certain fixed meaning; whereby, besides the ease of it, they obtain this advantage: That, as in such discourses they seldom are in the right, so they are as seldom to be convinced that they are in the

[1] Bacon, *Novum Organum*, Aph. LIX.

wrong; it being all one to go about to draw those men out of their mistakes who have no settled notions, as to dispossess a vagrant of his habitation who has no settled abode."[1]

(1) In treating of the misuse of words, we mention, in the first place, errors arising from the use of a word or phrase in more than one sense. This has already been described as the fallacy of Equivocation. In some cases, the equivocation may be mere wilful quibbling on the part of the person propounding the argument, as in the following example of Jevons:—

> All criminal actions ought to be punished by law,
> Prosecutions for theft are criminal actions,
> Therefore prosecutions for theft ought to be punished by law.

Examples of this kind do not mislead any one; but in some instances the change of meaning in words may not be perceived, even by the person who employs the argument. For example, one might reason:—

> It is right to do good to others,
> To assist A in obtaining office is to do him good,
> Therefore it is right to assist him in this way.

Here the phrase which is used equivocally is, 'to do good,' as will at once be perceived.

(2) Another frequent source of error in the use of words is found in what has been excellently named the Question-begging Epithet. As is well known, there is much in a name. The name may beg the question directly in the terms which it applies, or it may arouse misleading associations. Epithets, like 'class-legislation,' 'compromise measure,' 'a

[1] *Essay Concerning Human Understanding*, Bk. III., Ch. X.

dangerous and immoral doctrine,' are terms freely used to describe the measures or views of opponents. And, as it is always easier to adopt a current phrase, than to examine the facts and draw our own conclusions, it is not surprising that the name settles the whole matter in the minds of so many people. Of course, the epithet employed may beg the question in favour of the subject it is used to describe, as well as against it. Politicians well understand the importance of adopting an impressive and sonorous election cry to represent the plank of their party. Thus, party cries like 'honest money,' 'prohibition and prosperity,' 'the people's cause,' etc., are essentially question-begging epithets. Even words like 'liberty,' 'justice,' and 'patriotism,' are frequently used in such a way as to bring them under the class of fallacies which we have here described. Under this heading, also, may be grouped 'cant' words and phrases. When we accuse a person of using cant, we always imply that he is more or less consciously insincere, that he is professing opinions and sentiments which he does not really possess. Any insincere expression which is made primarily for the sake of effect may be rightly termed cant. It is not even necessary that the speaker should be fully conscious of his insincerity. A man may easily deceive himself, and, as he repeats familiar words and phrases, imagine himself to be overflowing with patriotism, or with sympathy for others, or with religious feelings.

(3) Figurative language is another frequent source of error. Of the various figures of speech, perhaps metaphors are the most misleading. The imagery aroused by metaphorical language is usually so strong as to make us forget the difference between the real subject under consideration and the matter which has been used to illustrate it. Thus, in

discussing problems of mind, it is very common to employ metaphors drawn from the physical sciences. For example, we read in works on psychology and ethics of 'the struggle of ideas,' of 'the balancing and equilibration of motives,' of 'action in the *direction* of the strongest motive,' etc. Another illustration, which has been often quoted, is Carlyle's argument against representative government founded on the analogy between the ruler of a state and the captain of a ship. The captain, he says, could never bring the ship to port if it were necessary for him to call the crew together, and get a vote every time he wished to change the course. The real difference between the relation of a captain to his crew, and the executive officers in a state to the citizens, is lost sight of by the metaphor. Metaphors should be used only to illustrate and suggest, and never to prove. Metaphorical reasoning is simply a case of analogy, the imperfections and dangers of which have been already pointed out. It is, however, one of the errors which it is most difficult to avoid. A hidden metaphor lurks unsuspected in many of the words in common use. We may thus appreciate the force of Heine's humorous petition: "May Heaven deliver us from the Evil One, and from metaphors." [1] It is, of course, not necessary or desirable to abstain entirely from the use of metaphors. What is essential is to prevent them from 'reacting upon the understanding.' A person who is able to employ many metaphors drawn from various fields is perhaps less likely to be misled by them, than the unimaginative man — the man of one figure and one phrase — whose mind sticks in mechanical grooves.

§ 73. **Errors of Observation.** — Sometimes insufficient observation is the result of a previously conceived theory;

[1] Quoted by Minto, *Logic*, p. 373.

sometimes it may be due to inattention, to the difficulties of the case, or to lack of the proper instruments and aids to observation. We have already had occasion to refer to the influence of a theory on observation (cf. § 67). As a rule, we see only those instances which are favourable to the theory or belief which we already possess. It requires a special effort of attention to take account of negative instances, and to discover the falsity involved in some long-standing belief. Indeed, it perhaps requires quite as much mental alertness to overthrow an old theory, as to establish a new one. It is obvious that the fallacy here is due, as is generally the case, to insufficient observation and analysis. The conclusion is based on an uncritical use of the method of Agreement, without any attempt to compare the positive cases with instances where the phenomenon is absent. This comparison is made by the method of Difference. This tendency of the mind to seize upon affirmative instances, and to neglect the evidence afforded by negative cases, is well set forth by Bacon in the following passage: —

"The human understanding, when any proposition has been once laid down (either from general admission and belief, or from the pleasure it affords), forces everything else to add fresh support and confirmation; and although most cogent and abundant instances may exist to the contrary, yet either does not observe or despises them, or gets rid of and rejects them by some distinction, with violent and injurious prejudice, rather than sacrifice the authority of its first conclusions. It was well answered by him who was shown in a temple the votive tablets suspended by such as had escaped the peril of shipwreck, and was pressed as to whether he would then recognize the power of the gods; 'But where are the portraits of those who have perished in spite of their vows?' All superstition is much the same, whether it be that of astrology,

dreams, omens, retributive judgment, or the like, in all of which the deluded observers observe events which are fulfilled, but neglect and pass over their failure, though it be much more common. But this evil insinuates itself still more craftily in philosophy and the sciences, in which a settled maxim vitiates and governs every other circumstance, though the latter be much more worthy of confidence. Besides, even in the absence of that eagerness and want of thought (which we have mentioned), it is the peculiar and perpetual error of the human understanding to be more moved and excited by affirmatives than negatives, whereas it ought duly and regularly to be impartial; nay, in establishing any true axiom the negative instance is the most powerful." [1]

The nature of this fallacy has been so well illustrated by the quotation which has just been given, that we may pass on at once to speak of other cases of insufficient observation. Our discussion of the processes of reasoning have made it clear how necessary it is to observe carefully and attentively. The majority of the false theories which have appeared in science and in philosophy, as well as those of common life, have arisen from lack of observation. The doctrine of innate ideas, and the theory that combustion was a process of giving off phlogiston — a substance supposed to be contained in certain bodies — may be given as examples. With regard to phlogiston, Mill says: "The hypothesis accorded tolerably well with superficial appearances: the ascent of flame naturally suggests the escape of a substance; and the visible residuum of ashes, in bulk and weight, generally falls extremely short of the combustible material. The error was non-observation of an important portion of the actual residue; namely, the gaseous products of combustion. When these were at last noticed and brought into account, it appeared to be a universal

[1] *Novum Organum*, Bk. I., Aph. XLVI.

law that all substances gain instead of losing weight by combustion; and after the usual attempt to accommodate the old theory to the new fact by means of an arbitrary hypothesis (that phlogiston had the quality of positive levity instead of gravity), chemists were conducted to the true explanation, namely, that instead of a substance separated, there was, on the contrary, a substance absorbed."[1] This illustration also exemplifies the consequences both of neglecting Residues and of noticing and seeking to explain them. In some seaside communities, there is a belief that living beings, both human and animal, never die at flood tide. 'They always go out with the ebb,' it is said. Again, there is a general belief, which was shared by such an eminent scientist as Herschel, that the full moon in rising possesses some power of dispersing the clouds. Careful observations made at the Greenwich observatory have, however, shown conclusively that the moon has no such power as that supposed.[2]

Another circumstance to be considered in this connection is the inaccuracy and fallibility of ordinary memory. Every one must have noticed how rarely two persons agree completely in the report which they give of a conversation which they have heard, or of events which they have experienced. This is due in part to diversity of interest: each person remembers those circumstances in which for any reason he is most strongly interested. But, in addition, it is largely the result of the inevitable tendency of the mind to confuse what is actually observed, with inferences made from its observations. The inability to distinguish between what is really perceived, and what is inferred, is most strongly marked in uneducated

[1] *Logic*, Bk. V., Ch. IV.

[2] Cf. Jevons, *Principles of Science*, Ch. XVIII.

persons, who are not on their guard against this fallacy. An uneducated person is certain to relate, not what he actually saw or heard, but the impression which the events experienced made upon him. He therefore mixes up the facts perceived, with his own conclusions drawn from them, and with statements of his own feelings in the circumstances. A lawyer who has to cross-examine a witness is usually well aware of this tendency, and may take advantage of it to discredit the testimony. The experienced physician knows how worthless is the description of symptoms given by the ordinary patient, or by sympathetic friends, or by an inexperienced nurse. The more one's sympathies and interests are aroused in such a case, the more difficult it is to limit oneself to an exact statement of actual occurrences.

But this tendency is not confined to persons deficient in knowledge and ordinary culture. It usually requires special training to make one a good observer in any particular field. It is by no means so easy as it may appear to describe exactly what one has seen in an experiment. If we know, or think that we know, the explanation of the fact, there is an almost inevitable tendency to substitute this interpretation for the account of what has been actually observed. Recent psychological investigation, aided by exact experimental methods, has done much to disentangle the data of perception from inferences regarding these data. As every one knows who has practised psychological introspection, it is only with the utmost difficulty, and after long training, that one can distinguish the actual psychological processes present to consciousness, from the associative and logical elements which are bound up with them in our ordinary experience. The following passage from Mill deals with this question: —

"The universality of the confusion between perceptions and the inferences drawn from them, and the rarity of the power to discriminate the one from the other, ceases to surprise us when we consider that in the far greater number of instances the actual perceptions of our senses are of no importance or interest to us except as marks from which we infer something beyond them. It is not the colour and superficial extension perceived by the eye that are important to us, but the object of which these visible appearances testify the presence; and where the sensation itself is indifferent, as it generally is, we have no motive to attend particularly to it, but acquire a habit of passing it over without distinct consciousness, and going on at once to the inference. So that to know what the sensation actually was is a study in itself, to which painters, for example, have to train themselves by long-continued study and application. In things further removed from the dominion of the outward senses, no one who has not had great experience in psychological analysis is competent to break this intense association; and when such analytic habits do not exist in the requisite degree, it is hardly possible to mention any of the habitual judgments of mankind on subjects of a high degree of abstraction, from the being of God and the immortality of the soul down to the multiplication table, which are not, or have not been, considered as matter of direct intuition."[1]

(1) In pointing out the evils arising from confusing fact and theory, it is not forgotten that what are taken as 'facts' are the results of earlier theorizings and interpretations (cf. § 53). But the results of past processes of combination and comparison become embodied or fixed in more or less definite form in the course of experience. Moreover, they are fixed in language — whether in the language of common life or in the technical terminology of the different sciences. There always is a kind of convention conveyed, both by the lan-

[1] *Logic*, Bk. V., Ch. IV., § 5.

guage of ordinary life and by that of the sciences as to what may be taken as a fact in that court circle, — *i.e.* taken for granted as a datum or starting-point for further construction. What is a fact in science may, of course, be an inference from the standpoint of popular knowledge, or *vice versa*.

Now, the fallacy against which warning is here given, arises from not understanding clearly what, in any given circumstance, may properly be taken as 'fact.' If there is confusion as to the starting-point, there is no proper basis on which to construct a theory. Moreover, without some certain starting-point, some well-ascertained datum, there is no means of testing and criticising our theories.

§ 74. **Mistakes in Reasoning.** —The problem of the inductive processes of reasoning is to ascertain what facts are necessarily and essentially connected; and to explain this connection. Now, in order to distinguish between chance conjunctions of phenomena, and real causal connections, careful and extensive observation, aided whenever possible by experiment, must be employed. In short, to establish a real law of connection between phenomena, it is necessary to use one or more of the inductive methods described in Chapters XVI. and XVII. But to do this implies, in many cases, long processes of analysis; the performance of intellectual work, which ordinary minds, at least, have the tendency to shirk whenever possible. It is much easier to allow associations to control our thoughts, and to assume, (1) that events which happen together in our experience a number of times are causally connected; or, (2) that things that are in some way alike are causally connected, or of the same kind. We are led to such a conclusion by a natural psychological tendency, without taking any

thought about the matter, while logical analysis and discrimination require a distinct conscious effort.

The general name used to describe the first class of fallacies which are due to this particular form of mental sluggishness is *post hoc, ergo propter hoc.* Two events occur in close conjunction with each other, and it is then assumed without further investigation that they are related to each other as cause and effect. Many popular superstitions are examples of this fallacy. Some project begun on Friday turns out disastrously, and it is inferred that some causal relation existed between the fate of the enterprise, and the day on which it was begun. Or thirteen persons sit down to dinner together, and some one dies before the year is out. It is to be noticed that such beliefs are supported by the tendency, to which we referred in the last section, to observe only the instances in which the supposed effect follows, and to neglect the negative cases, or cases of failure. 'Fortune favours fools,' we exclaim when we hear of any piece of good luck happening to any one not noted for his wisdom. But we fail to take account of the more usual fate of the weak-minded. The belief that the full moon in rising disperses the clouds, which was also quoted earlier, is a good example of *post hoc, propter hoc.* In fact, all the fallacies treated in this chapter, except those due to language, might quite properly be included under this heading. The tendency to neglect negative instances was given by Bacon as the most striking example of the 'Idols of the Tribe' (*Idola tribus*), *i.e.* of the species of fallacies to which the whole tribe or race of men are subject.

A special case of this fallacy, to which attention may be called separately, arises from hasty generalization, or generalization on an insufficient basis of fact. The term 'generaliza-

tion' is often used in logic to denote the whole inductive movement of thought from particular facts to general principles and laws. But the fallacy to which reference is here made usually concerns a special stage in that process—the stage where a first generalization is made from instances. We are said to generalize when, after a more or less extended and careful set of observations, we take the instances observed as typical of all phenomena of the same field, or of the same general character. When due care has not been exercised in making the observations or when the observations are few in number, or all drawn from a limited part of the whole field, we speak of 'hasty generalization.' Thus it is not unusual to hear a traveler declare, on the basis of a very limited experience, that 'the hotels of some city or country are thoroughly bad.' The generalizations which are so frequently made regarding the peculiar characteristics of Americans, or Englishmen, or Frenchmen are usually of the same sort. What is exceptional tends to attract the attention more than what is usual and normal; hence the tendency *to take the exceptional for the typical.* Even scientific books are not always free from this error. In a recently published psychological study of the first year of the life of a child, by the mother, it was explained why a baby always sucks its thumb rather than its fingers. The explanation was that the thumb, being on the outside and projecting outwards, got oftenest into the baby's mouth, and so the habit was formed. The mother assumed what she had observed in her own child to be true universally. Other parents declare that their babies never put the thumb into the mouth, but always the fingers or the whole hand.

Another fallacy belonging to this group arises from the

uncritical use of Analogy. False Analogy is closely connected with the fallacies of figurative language. Indeed, the latter type of fallacies, almost without exception, arise from a loose use of Analogy. It has been pointed out (§ 66), that the value of an inference from Analogy depends upon the 'depth' or 'importance' of the resemblances upon which it is based. False inferences arise in every field from taking some striking or surface resemblance as the basis of a conclusion. Nothing is easier than to be led uncritically by vague resemblances, or even to imagine them where they do not exist. Vague or fancied analogies are the foundation of many popular superstitions regarding omens, illness, cures, etc., and also play an important part in many of the sympathetic and imitative practices of Magic.

§ 75. **Fallacies due to Individual Prepossessions.**—Bacon named this class of fallacy "The Idols of the Cave." Each individual, as he represents the matter, is shut up in his own cave or den; that is, he judges of things from his own individual point of view. In the first place, one's inclinations and passions, likes and dislikes, pervert one's judgment. It is exceedingly difficult, as we all know, to be fair to a person we dislike, or to refrain from judging too leniently the shortcomings of those to whom we are warmly attached. Again, it is not easy to put oneself in the position of an impartial spectator when one's interests are at stake. "The understanding of men," says Bacon, "resembles not a dry light, but admits some tincture of the passions and will." Furthermore, each individual has a certain personal bias as a result of his natural disposition and previous training. Thus it is almost impossible for an individual to free himself from national prejudices, or from the standpoint of the political party, or the

church in which he was brought up. Or, if a person does give up his old views, he not infrequently is carried to the opposite extreme, and can see no good in what he formerly believed. Even education and the pursuit of special lines of investigation may beget prejudices in favour of particular subjects. When a man has been engaged exclusively for a long time in a particular field, employing a particular set of conceptions, it is almost inevitable that he should look at everything with which he has to do in the same light. The mathematician's view of the world is almost sure to be different from that of the historian, or that of the student of æsthetics. It is very difficult for the physicist to conceive of any natural process except in terms of molecules and vibrations. It is inevitable that each man should be blinded to some extent by his own presuppositions. But to recognize one's limitations in this respect, is to pass, to some extent at least, beyond them.

(1) Moreover, each age, as well as each individual, may be regarded as governed largely by current presuppositions and prejudices. Bacon does not, however, classify the errors into which one may be led by the spirit of the time (*Zeitgeist*), or the beliefs derived from the past, with the 'Idols of the Cave,' but speaks of them rather as "Idols of the Theatre." (*Idola theatri*). He draws his examples of this from the influence which the traditions of the Schoolmen still continued to exert in his own day. Throughout the Middle Ages, theological doctrines and opinions controlled almost absolutely the opinions and beliefs of mankind. This influence, doubtless, still makes itself felt, but people are now pretty generally awake to the dangers from this source. On the other hand, it is more difficult to realize at the present time that it is not impossible for prejudices and prepossessions

to grow out of scientific work. The success of modern scientific methods has sometimes led investigators to despise and belittle the work of those who do not carry on their investigations in laboratories, or do not weigh and measure everything. But conceptions and methods which prove useful in one science cannot always be employed profitably in another. A conception, or mode of regarding things, which has proved serviceable in one field is almost certain to dominate a whole age, and to be used as an almost universal principle of explanation. The eighteenth century, for example, was greatly under the influence of mechanical ideas. Newton's discovery made it possible to regard the world as a great machine, the parts of which were all fitted together according to the laws of mechanics. This view led to such a vast extension of knowledge in the realm of physics and astronomy, that the conceptions upon which it is based were applied in every possible field — in psychology, in ethics, in political science. The world itself, as well as religious creeds and political and social institutions, were supposed to have been deliberately made and fashioned by some agent. Again, at the present time we are dominated by the idea of evolution. The biological notion of an organism which grows or develops has been applied in every possible field. We speak, for example, of the world as an organism rather than as a machine, of the state and of society as organic. And the same conception has been found useful in explaining the nature of human intelligence. It is easy for us to realize the limitations and insufficiency of the notion of mechanism as employed by the thinkers of the eighteenth century. But it is not improbable that a future century may be able to see more clearly than we are able to do, the weaknesses and limitations of the conception which has proved so fruitful in this generation.

REFERENCES

Bacon, *Novum Organum*, Aph. XXXVIII.–LXVIII.

Locke, *Essay Concerning Human Understanding*, Bk. III., Chs. X and XI.

J. S. Mill, *Logic*, Bk. V.

A. Bain, *Logic*, Pt. II., Induction, Bk. VI.

J. Fowler, *Inductive Logic*, Ch. VI.

J. G. Hibben, *Inductive Logic*, Ch. XVII.

A. Sidgwick, *Fallacies* [Int. Scient. Series].

PART III.—THE NATURE OF THOUGHT

CHAPTER XXI

JUDGMENT AS THE ELEMENTARY PROCESS OF THOUGHT

§ 76. **Thinking the Process by which Knowledge grows or Develops.** —Logic was defined (§ 1) as the science of thinking, and we have seen that the business of thought is to furnish the mind with truth or knowledge. Under what general conception, now, shall we bring thinking, and what method shall we adopt to aid us in its investigation? It is at once clear that thinking, the conscious process by which knowledge is built up, does not resemble mechanical processes like pressure, or attraction and repulsion. It is more nearly related to something which has life, like a plant or an animal, and which grows or develops from within, in accordance with the laws of its own nature. Thinking must be regarded rather as a living process, than as a dead thing, though it is necessary also to remember that it is conscious as well as living.

When the thinking process is regarded in this way, moreover, a method of procedure at once suggests itself. In these days we have become familiar with the notion of evolution or development, and the application of this notion has proved of the greatest service to science, and particularly to those sciences which deal with the phenomena of life. What is characteristic of this manner of regarding things

is the fact that it does not consider the various phenomena with which it deals as fixed, unchangeable things, each with a ready-made nature of its own. But each thing is simply a stage of a process, a step on the way to something else. And the relations of the various phenomena to each other, their connection and unity as parts of the one process, come out more clearly when viewed in this way. In other words, by taking a survey of the genesis and growth of things, or the way in which they come to be, we gain a truer idea of their nature and relations than would be possible in any other way. The past history of any phenomenon, the story of how it came to be what it is, is of the greatest possible service in throwing light upon its real nature. Now, one cannot doubt that this conception will also prove serviceable in the study of logic. That is to say, it will assist us in gaining a clearer idea of the nature of thinking, to conceive it as a conscious function, or mode of acting, which unfolds or develops in accordance with the general laws of organic evolution. And this process may be supposed to go on both in the individual, as his thought develops and his knowledge expands, and in the race, as shown by its history. By adopting this notion, we may hope to show also that there is no fundamental difference in kind between the various intellectual operations. Judgment and Inference, for example, will appear as stages in the one intellectual process, and the relation between Induction and Deduction, as each having its own work to do, will become evident.

§ 77. **The Law of Evolution and its Application to Logic.** —The most striking characteristic of any organism at a low stage of development is its almost complete lack of structure. An amœba, for example, can scarcely be said

to have any structure; it is composed of protoplasm which is almost homogeneous, or of the same character throughout. When we compare an amœba, however, with an animal much higher in the scale of life, *e.g.* a vertebrate, a great difference is at once evident. Instead of the simple, homogeneous protoplasm, the organism is composed of parts which are unlike, or heterogeneous, such as bones, muscles, tendons, nerves, blood-vessels, etc. In Mr. Spencer's language, there has been a change from a state of homogeneity, to one of heterogeneity. The process of evolution from the lower organism to the higher has brought with it a differentiation of structure. That is, in the amœba there are no special organs of sight, or hearing, or digestion, but all of these acts seem to be performed by any part of the organism indifferently. In the vertebrate, on the other hand, there is division of labour, and a separate organ for each of these functions. One may also notice that the same change is observable when the *acts or functions* performed by a lower organism are compared with those of a higher. The life of the amœba seems to be limited almost entirely to assimilation and reproduction; while, when we advance from the lower animals to the higher, and from the higher animals to man, there is an ever-increasing complexity and diversity in the character of the actions performed. We thus see how the process of evolution involves differentiation both of *structure* and of *function*, in passing from the homogeneous to the heterogeneous.

But differentiation, or increase in diversity, is only one side of the process of evolution. As we pass from a lower to a higher stage, the various parts of an organism are seen to become more essential to one another. If certain plants or

low animal organisms are divided into several parts, each part will go on living. Its connection with the other parts does not seem to have been at all necessary to it. But when we are dealing with higher forms of life, each part is seen to have its own particular function, and to be essential to the other parts, and to the organism as a whole. In other words, the parts now become members, and the whole is not simply an aggregation of parts or pieces, but is constituted by the necessary relation of the members to one another. The more highly evolved the whole with which we are dealing, the more closely connected and essential to one another are the various parts seen to be. It becomes increasingly true that if one member suffers, all the other members suffer along with it. The same principle is illustrated by the relation of classes and individuals in modern society. In spite of the conflicts between capital and labour, between rich and poor, it is becoming increasingly evident that the unity of society is more fundamental than its differences and antagonisms.

Evolution, then, not only exhibits a constant process of differentiation, and a constant increase in the diversity of parts and organs, but there goes along with this what might be called a process of unification, whereby the parts are brought into ever closer and more essential relation to one another. In this way, a *real or organic whole*, as opposed to a mere *aggregate*, is formed. This is what Mr. Spencer calls the process of **integration**; and it accompanies, as we have seen, what the same writer calls **differentiation.**

The application of this general law of evolution to the development of the thinking process is not difficult. We shall expect to find that thinking, in its first beginnings,

both in the individual and in the race, will be much less complex and differentiated than at a higher stage. That is, the earliest or simplest thinking tends to take things in a lump, without making any distinctions. The infant, for example, does not distinguish one person from another, or perhaps does not distinguish even the parts of its own body from surrounding objects. Now, it is clear that intellectual development, growth in knowledge, must in the first place involve differentiation. What is complex must be analyzed or separated into its various parts. Things which are different must be distinguished and clearly marked off from one another. The development of thought implies, then, as one of its moments, discrimination or analysis — what we previously called differentiation.

The other moment of the law of evolution, integration, also finds a place in the development of thought, and goes hand in hand with the former. The child and the uneducated man not only often fail to make distinctions where these really exist, but the parts of their knowledge are fragmentary, and have little or no relation to one another. The various pieces of their knowledge are like the parts of the amœba — they may be increased or diminished without themselves undergoing any change. But, in order to pass from a lower to a higher intellectual point of view, — to become better educated, in a word, — it is necessary to see the way in which the various pieces of our knowledge are connected and dependent upon one another. It is not enough to analyze and keep separate things which are distinct, but it is also necessary to understand how the various parts of our knowledge are inter-related and essentially dependent on one another. In other words, we may say that it is character-

istic of our intelligence to endeavour to put things together so as to form a whole, or system of interconnected parts. And the more completely it is able to do this (provided that the process of differentiation has also made a corresponding advance), the higher is the stage of development which has been attained. The ideal of knowledge, or of complete intellectual development, would be to understand the oneness and relation of everything which exists, even of all those things which seem now to be entirely different in kind. A knowledge of any one fact would then carry with it a knowledge of every other fact. Or, rather, our knowledge would be so completely unified, that each part would show the nature of the whole or system to which it belongs; just as a leaf of a plant, or a tooth of an animal, may be sufficient to tell the naturalist of the wholes to which they belong.

This, of course, will always remain an ideal; but it is in this direction that thinking actually develops. It is a step in advance to discover the reasons for any fact which one previously knew as a mere fact. For, to discover the reasons for a fact, is to bring it into connection with other facts, to see them no longer as isolated and independent, but as belonging together to one group or system of facts. And the further the process of explanation goes on, the more completely is our knowledge unified and related.

There is, however, another fact implied in the very nature of evolution, of which logic, as well as the other sciences, may take advantage. We have assumed that the more complete and difficult kinds of thinking have grown or developed from simpler types of the same process, and not from something different in kind. It will therefore follow, that the essential characteristics of the thinking process may be

discovered in its simplest and most elementary form. It is found that all the essential functions of the fully developed organism are discharged by the primitive cell. And because it is easier to study what is simple than what is complex, the cell is taken as the starting-point in biology. Similarly, there will be an advantage in beginning with the simplest and most elementary forms of thinking. What is found true of these simple types of thought, may be assumed to be essential to the thinking process as such.

§ 78. **Judgment as the Starting-point.** — What, then, is the simplest form of thinking? What shall we take as a starting-point, which will correspond to the cell in biology, or the elementary process in psychology? To answer this question, it is not necessary first to decide where in the scale of animal life that which we are entitled to call thinking actually begins. We shall not be obliged to discuss the much-debated question, whether or not dogs think. Wherever thinking may be found, it is essentially an activity of the mind. When it is present, that is, there is always intellectual work done, something interpreted or put together, and a conclusion reached. One may perhaps say that thinking is simply the way in which the mind puts two and two together and sees what the result is. It implies that the mind has waked up to the significance of things, and has interpreted them for itself. Suppose that one were sitting in one's room very much engaged with some study, or wrapped up in an interesting book, and suppose that at the same time the sound of a drum should fall upon one's ears. Now, the sound sensations might be present to consciousness without calling forth any reaction on the part of the mind. That is, we might be so intent on our book that we should not

wake up, as we have been saying, to the meaning or significance of the drum-taps; or perhaps not even to the fact that they were drum-taps at all. But if the mind did react upon the sound sensations, it would try to interpret them, or put them together so as to give them a meaning. As a result, some conclusion would be reached, as, for example, 'the drum is beating'; or sufficient intellectual work may have been done to give as a conclusion, 'that is the Salvation Army marching up the street.' In any case, it is of the greatest importance to notice that the conclusion does not come into our minds from without, but that it is the product of the mind's own activity, as has been described. It is not true, in other words, that knowledge passes into our minds through the senses; it is only when the mind wakes up to the meaning of sensations, and is able to put them together and interpret them, that it gains any knowledge.

Now, the simplest form of such an act of thought is called a judgment. Judgment, we may say, is a single intellectual act of the kind we have described; and its conclusion is expressed by means of a Proposition; as, for example, 'the grass is green,' 'the band is playing.' In accordance with general usage, however, we may use the term 'Judgment' for both the act itself and its result. And the word 'Proposition' will then denote the external expression in speech or writing of the product of an act of judgment.

In our investigation of the nature of thought, then, we must begin with Judgment. There are three things which we shall have to do: (1) To endeavour to discover the fundamental characteristics of this simple type of thinking; (2) To show the various forms which it assumes, or to describe the different kinds of Judgment; and (3) To trace the process

by which Judgment expands into the more complete logical form of Inference. Before any of these questions are considered, however, it is necessary to meet a very serious objection to our whole procedure of beginning with Judgment as the elementary process of thinking.

§ 79. **Concepts and Judgment.** — In the last section, we endeavoured to show that Judgment is the elementary process of thought, and that with it all knowledge begins. The same position was also maintained in an earlier chapter (§ 11). This view, however, may seem to be contradicted by the treatment of Judgment usually found in logical text-books.

Judgment, it is said, is expressed by a proposition; and a proposition is made up of three parts, subject, predicate, and copula. Thus in the proposition 'iron is a metal,' 'iron' is the subject, 'a metal' the predicate, and the two terms are joined or united by means of the copula 'is.' A Judgment is therefore defined as an act of joining together, or, in negative judgments, of separating, two concepts or ideas. If this account be accepted, it follows that the ideas of which the judgment is composed (iron and metal, in the example given above) are pieces of knowledge which precede the judgment itself. And the act by which these logical ideas (or, as they are usually called, concepts) are formed must also be earlier and more fundamental than the act of judging. It is therefore held that logic should begin with concepts, which are the elements out of which judgments are compounded, and that the first logical act consists in the conception or simple apprehension of the ideas or concepts.

It is necessary to examine this position very carefully. What is maintained is that a process of forming concepts,

or logical ideas, presumably quite distinct from the activity of judgment, necessarily precedes the latter. Before it is possible to judge that 'iron is a metal,' for instance, one must have gained, by means of Conception or Apprehension, the ideas denoted by the subject and predicate of this proposition. Judgments, that is, are made or compounded out of something different from themselves.

It may be well to begin the defence of our own position by noting what is undoubtedly true in what has just been stated. In making a judgment like 'iron is a metal,' it is, of course, necessary to have the concept 'iron,' and the concept 'metal.' But what is implied in having a concept of anything? Let us suppose that a person is making the above-mentioned judgment for the first time — that is, really drawing a conclusion for himself, and not merely repeating words. He would begin, we may say, with the concept 'iron.' But if this concept is more than a mere word, if it really means anything, it must have been formed by a number of judgments. The concept 'iron,' if it has any significance for the person using it, means a definite way of judging about some substance — that it is hard, malleable, tough, etc. The greater the number of judgments which the concept represents, the more meaning or significance it has; apart from the judgment, it is a mere word, and not a thought at all.

To admit, then, that in judging we always start from some concept, does not imply that there is a different form of intellectual activity prior to judgment, which furnishes the latter with ready-made material for its use. But, as we have seen, in ordinary judgments like the example with which we have been dealing, the new judgment is a further

expansion or development of a previous set of judgments which are represented by the concept. The concept, then, stands for the series of judgments which have already been made. Language comes to the aid of thought, and makes it possible to gather up such a set of judgments and represent them by a single expression — often by a single word. Every word which is the name of some logical concept represents intellectual work — the activity of judgment — in its formation. In learning our own language, we inherit the word without doing the work. But it must never be forgotten that the word in itself is not the concept. To make the thought our own, to gain the real concept, it is necessary to draw out or realize to ourselves the actual set of judgments for which the word is but the shorthand expression.

The view which regards the judgment as a compound of two parts — subject and predicate — rests upon the substitution of words for thoughts. It analyzes the *proposition* (the verbal or written expression of the judgment), instead of the judgment itself. In the proposition, the parts do exist independently of each other. The subject usually stands first, and is followed by the predicate. But there is no such order of parts in a judgment. When one judges, 'it is raining,' or, 'that is a drum,' the piece of knowledge is one and indivisible. And the act by which this knowledge is gained is not an external process of joining one part to another, but is an intellectual reaction by which we recognize that something, not previously understood, has a certain meaning or significance.

Again, it is only when concepts are identified with the words which make up the parts of the proposition, that they can be regarded as ready-made existences which are quite independ-

ent of their connection in a judgment. The terms 'iron' and 'metal' are separable parts of the proposition and exist independently of their connection with it. The conclusion has been therefore drawn that concepts had a like independence of judgments, but might enter into the latter and form a part of them without affecting their own nature in any way. But, as we have already seen, the concept has no meaning apart from the series of judgments which it represents. And, as thinking goes on, and new judgments are made, its nature is constantly changing. In short, concepts are not dead *things*, but living *thoughts* which are in constant process of development.

The objection, then, which urges that conception is a logical process that is prior to judgment, turns out, when rightly understood, to be no objection at all. For, in the light of what has been already said, it only amounts to this: In making new judgments regarding anything, we must set out from what we already know of it, as represented by the judgments already made. That is, the starting-point for a new judgment is the concept or series of judgments which represents the present state of our knowledge. The progress of knowledge is not from the unknown to the known, but from a state of partial and incomplete knowledge to one of greater perfection. Thus the judgment 'gold is malleable' (supposing it to be a genuine judgment made for the first time) adds to, or develops farther, our existing knowledge of gold, as represented by a series of judgments previously made regarding it.

It may be urged, however, that not every judgment can grow out of previous judgments in this way. For, if we go back far enough, we must reach some judgment which is absolutely first, and which presupposes no antecedent judgment. This is like the paradox

regarding the origin of life. If all judgments are derived from antecedent judgments, how was it possible for the first one to arise? It will, perhaps, be sufficient answer to deny the existence of the paradox. Consciousness must be regarded as having from the first the form of a judgment. No matter how far one goes back in the history of consciousness, one will always find, so long as consciousness is present at all, some reaction, however feeble, upon the content, and something like knowledge resulting. Even the consciousness of the newly born infant reacts, or vaguely judges, in this way. These primitive judgments are, of course, very weak and confused, but they serve as starting-points in the process of intellectual development. Growth in knowledge is simply the process by means of which these vague and inarticulate judgments are developed and transformed into a completer and more coherent experience.

REFERENCES

W. S. Jevons, *Elementary Lessons in Logic*, pp. 9–16.
F. H. Bradley, *The Principles of Logic*, Bk. I., Ch. I.
B. Bosanquet, *Logic*, Vol. I., Ch. I., §§ 1–6.
H. Lotze, *Logic* (Eng. trans.), Vol. I., pp. 13–61.
C. Sigwart, *Logic*, §§ 40–42.
L. T. Hobhouse, *The Theory of Knowledge*, Pt. I., Chs. I. and II.

CHAPTER XXII

THE MAIN CHARACTERISTICS OF JUDGMENT

§ 80. **The Universality of Judgments.** — We have now to examine the nature of Judgment a little more closely than has been done hitherto. In the first place, we note that all judgments claim **universality.** There are, however, several kinds of universality, and more than one sense in which a judgment may be said to be universal. We speak of a universal judgment (more properly of a universal proposition), when the subject is a general term, or is qualified by some such word as 'all,' or 'the whole.' And we distinguish from it the particular judgment, where the subject is only the part of some whole, and is usually preceded by 'some,' or by other partitive words. But here we have no such distinction in mind; we are speaking of the universality which belongs to the very nature of Judgment as such, and which is shared in by judgments of every kind.

When we say that judgments are universal, in the sense in which the word is now used, we mean that the conclusions which they reach claim to be true for every one. No matter what the subject and the predicate may be, a judgment, *e.g.* 'man is mortal,' comes forward as a fact for all minds. We have shown in the last chapter that it is by judging, or putting things together for itself, that the human mind gains knowledge. Now, the assumption upon which this process is based is that the result thus reached — knowledge — is not

something merely individual and momentary in character. When I judge that 'two and two are four,' or that 'iron has magnetic properties,' the judgment is not merely a statement of what is going on in my individual consciousness; but it claims to express something which is true for other persons as well as for me. It professes to deal with facts which are true, and in a sense independent of any individual mind. The judgments by which such conclusions are reached are universal, then, in the sense that they are asserted as true for every one and at all times. The word 'objective' has essentially the same meaning. Although each man reaches truth only by actually judging for himself, yet truth is objective, out there beyond his individual or 'subjective' thought, shared in by all rational beings. The assumption upon which all argument proceeds is that there is an objective standard, and that if people can be made to think they will arrive at it. Thought is in essence a process of self-criticism; for it has in itself its own standard of truth, which comes to light in and through the process of development.

(1) The only alternative to this position is scepticism, or pure individualism. If Judgment is not universal in the sense that it reaches propositions which are true for everybody, it is of course impossible to find any standard of truth at all. The judgments of any individual in that case would simply have reference to what seemed true to him at the moment, but could not be taken to represent any fixed, or permanent truth. Indeed, if one regards Judgment as dealing merely with particular processes in an individual mind, the ordinary meanings of truth and falsehood are completely lost, and it becomes necessary to give a new definition of the words. This was the position of the Sophists at the time of Socrates (cf. § 5). Each individual man was declared to be the measure of what is true and

false, as well as of what is good and bad. There is thus no other standard of truth or value than the momentary judgment (or caprice) of the individual. This is, in a way, the *reductio ad absurdum* of scepticism.

The common nature of truth, as something in which all can share, presupposes, then, a common mode of thinking or judging on the part of all rational beings. And it is this universal type or form of knowing with which logic deals. The question as to whose thought is investigated, or in what individual mind the thought takes place, is in itself of no importance. The consciousness of a savage differs very greatly from that of an educated man; it is much less complex and less highly developed. But yet, in spite of the enormous differences, there exists in both an intelligence, or way of thinking, which shows the same essential character, and operates according to the same fundamental laws.

§ 81. **The Necessity of Judgments.** — The second characteristic which we note as belonging to Judgment is **necessity.** By this we mean that when a person judges, he is not free to reach this or that conclusion at will. As an intellectual being, he feels bound to judge in a certain way. This is sometimes expressed by saying that we cannot believe what we choose; we must believe what we can.

In many of the ordinary judgments of everyday life, which are made without any clear consciousness of their grounds, logical necessity is implicitly present as an immediate feeling of certainty. In cases of this kind, we simply identify ourselves with the judgment, and *feel* that it is impossible that it can be false. But, of course, no judgment can claim to be necessary in its own right. Its necessity comes from its connection with other facts which are known to be true. Or, in logical terms, we may say that it comes from reasons or prem-

ises which support it. And one should always be ready to show the grounds or reasons upon which one's feeling of necessity rests. But in ordinary life, as we have said, it is not unusual to regard a conclusion as necessary, without clearly realizing the nature of the reasons by which it is supported. An uneducated man is rarely able to go back and discover the reasons for his belief in some other statement of which he is convinced. If you question his assertion, he feels that you are reflecting upon his veracity, and consequently grows angry. In the feeling of immediate necessity or conviction, he identifies himself with the judgment, and does not see that the criticism is not directed against the latter, but against the grounds by which it is supported.

In this distinction between necessity that is merely *felt*, and the necessity that is *conscious* of its own grounds, we see the direction in which judgment must develop. In the evolution of thought, we gradually become conscious of the grounds upon which our judgments are made. That is, the simple judgment, which seems to stand in isolation, is seen to expand so as to include its reasons as an organic part of itself. By itself, it is only a fragment of a more complete and widely embracing thought. The feeling of necessity is an evidence of its dependence and connection, though this dependence and connection upon other facts may not be clearly understood. But what is implicit must be made explicit; the necessity which is merely *felt* to belong to the simple judgment must be justified, by showing the grounds or reasons upon which it rests. And, for this purpose, the simple judgment has to be brought into relation with other facts and judgments which are outside of it, yet constitute its reasons, or are necessary to support it. In other words, it

must develop into an inference. As a matter of fact, the same form of words as used by different persons, or by the same person at different times, may express either a judgment or an inference. Thus, 'the price of wheat rose after the war began' might express either a simple historical fact, which is accepted from experience or from hearsay, or it might, in the mouth of a person acquainted with the laws of supply and demand, be the necessary conclusion of a number of premises. Again, a child might read that, 'the travelers found great difficulty in breathing when they reached the top of the mountain,' accepting this as a simple statement of fact. If he were to read this same statement some years later, however, he would probably connect it at once with other facts regarding the nature of the atmosphere, and the action of gravity, and so perceive at once its inferential necessity.

(1) According to the view which has just been stated, necessity is not a property which belongs to any judgment in itself, but something which arises through its dependence upon other judgments. In other words, necessity is always mediate, not immediate. This view, however, differs from a theory that was once generally received, and has some adherents, even at the present time, especially among thinkers who belong to the Scottish or 'common-sense' school. In dealing with the facts of experience, we always explain one fact by referring it to a second, and that second by showing its dependence upon some third fact, and so on. Thus the movement of the piston-rod in an engine is explained by the pressure of steam, and this is due to the expansive power of heat, and heat is caused by combustion of fuel, etc. We are thus referred back in our explanations from one fact or principle to another, without ever reaching anything that does not require in its turn to be explained.

Now, it is said that this process cannot go on forever; for if it did there could be no final or complete knowledge; the whole system would be left hanging in the air. There must, therefore, it is argued, be some ultimate facts which furnish the support for the world of our experience, some principle or principles which are themselves necessary and do not require any proof. That is, there must be certain propositions which are *immediately* necessary, and which serve as the final explanation for everything else. Now, it is clear that such propositions must be entirely different in character from the ordinary facts of experience, since their necessity belongs to their own nature, and is not derived from any other source. It had to be supposed, therefore, that they stood upon a different plane, and were not derived from experience. To explain the superior kind of certainty which they were assumed to possess, it was supposed that they were present in the mind at birth, or were innate. They have also been called *necessary truths*, *a priori truths*, and *fundamental first principles*, in order to emphasize their supposed distinction from facts which are derived from experience.

When one regards knowledge as an internal process of growth or development, however, where each element plays its part, as do the members of a living body, the inadequacy of any view which looks for a mechanical basis for knowledge is apparent. What is present in experience is a moving system of functions, not a structure of fixed mechanical parts, such as exist, for example, in a building.

§ 82. **Judgment involves both Analysis and Synthesis.** — The business of our thought is to understand the ways in which the various parts of the real world are related. And a judgment, as we have already seen, is just a single act of thought, — one step in the process of understanding the world. Now we ask: How does Judgment accomplish its task? Does it proceed altogether by analysis, by pointing

out the parts of which things are composed, or does it also employ synthesis in order to show how various parts combine in such a way as to form a whole? Or is it possible for both these processes to be united in one and the same act of judgment?

Suppose that one actually makes the judgment for oneself (and does not merely repeat the words of the proposition), 'the rose has pinnate leaves.' What has taken place? We notice, firstly, that a new property of the rose has been brought to light; a distinction, or mark, has been discovered in the content 'rose,' which was not seen to belong to it before the judgment was made. So far, then, the process is one of analysis, of discovering the parts or distinctions of something which is at first taken, as it were, in a lump. And this is a most essential element in all thinking. In order to know, it is absolutely necessary that the differences between the parts of things should be clearly apprehended, that we should not confuse things which are unlike, or fail to make proper distinctions. If we examine a number of instances where a real judgment is made, we shall find that this moment of analysis, or discrimination, is always present. Sometimes, indeed, analysis may not seem to be the main purpose of the judgment; but if one looks closely, one will always find in a judgment that elements which are unlike are held apart or discriminated.

But let us look again at the same judgment, 'the rose has pinnate leaves.' It is not difficult to see that the discovery of something new in itself is only one part of what the judgment has accomplished. The judgment also affirms the union of this new discovery with the properties of what we call the rose. It is, therefore, from this point of view, an act of

synthesis. It asserts that the prickly branches, fragrant flowers, feather-like leaves, and other distinctions are united in the one content which we call the rose. It does not stop with the mere assertion, 'there is a mark or distinction,' but it affirms that it is a mark of something, *i.e.* that it is united with other marks or properties to form a concrete whole. In other words, we may say that every judgment affirms the *unity* of the different parts, or aspects, of a thing; and this is, of course, synthesis. From this point of view, then, Judgment can be defined as a process of synthesis, just as we defined it above as one of analysis.

But how, it may be asked, is it possible for a judgment to be both analytic and synthetic? Are not these processes directly opposed to each other? It is true that there can be no doubt that this is the case when we are dealing with material things: pulling things to pieces is the opposite of putting them together. When we are doing the one we cannot also be doing the other. But there is no such opposition between these processes when they go on in our minds. An illustration may make this clear. Suppose that one is trying to understand some piece of mechanism, say a watch; in order to be able to see how it goes, or judge correctly regarding it, two things are necessary. First, one must notice all the parts of which it is composed — the wheels of various sizes, springs, pins, etc. But, in the second place, one would not understand the watch until one saw how all the parts were united, how one part fits into another, and all combine together into one whole. We do not mean that these are two steps which take place in succession; as a matter of fact, the detection of the various parts, and the perception of their connection, go hand in hand. In the process of understand-

ing the watch, we have both taken it to pieces and put it together again at one and the same time. Not really, of course, but in our thought. In the world of material things, as we have said, only one of these processes could go on at a time; but in every act of thinking, in every judgment, analysis and synthesis go hand in hand, and one has no meaning except with reference to the other.

But the two moments or factors of analysis and synthesis, although present in every judgment, are not always equally prominent. The main purpose of the judgment usually falls on one side or the other. In a judgment like, 'water can be divided into hydrogen and oxygen,' the main emphasis seems to be on the parts, and the assertion that these elements are *parts of a whole*, though present, is only implied. But when one asserts, 'these springs and wheels together make up a watch,' it is the nature of the whole upon which the emphasis is laid, and the separation or discrimination of the parts is, as it were, secondary. It is not difficult to see, however, that the two moments of Judgment are present in both of these cases. The difference consists in the fact that at one time analysis, and at the other synthesis, is made the main purpose.

It was at one time supposed that analytic and synthetic judgments were entirely different in kind from each other. An analytic judgment, it was said, is one in which the predicate is obtained by analyzing, or bringing to light, what is contained in the subject. Thus the judgment, 'all material bodies fill space,' is analytic; for the predicate (space-filling) is contained in the very notion, or idea, of a material body. All that is necessary in order to obtain the judgment is to comprehend the meaning of the subject. An analytic judg-

ment, then, adds nothing to our knowledge. It merely enables us to bring to light and express what is contained in the ideas we already possess. A synthetic proposition, on the contrary, was defined as one in which the predicate was not already contained in the subject, but which added a new element or idea to it. 'This body weighs ten pounds,' for example, is a synthetic proposition, for one cannot obtain the predicate by analyzing the subject. The predicate adds a new fact which must have been derived from experience.

(1) This view is, of course, fundamentally different from the account of Judgment which we have just given. The absolute distinction between analytic and synthetic judgments, like the theory that thought begins with concepts, arises, I think, from a substitution of the spoken or written proposition for the judgment itself. In the proposition the subject seems to be the starting-point. We have a word or term which appears to be independent and capable of standing alone. The question is, then, where shall we find the predicate? For example, in the proposition, 'iron is an element,' the subject stands first, and the predicate comes later. It seems possible then to say that we have first the subject 'iron,' and then join on to it the predicate 'element,' which has been obtained either by analyzing the subject, or from some previous experience. But the proposition, as a collection of words, must not be substituted for the act of judgment. Judgment, as we have already seen, is a single act of intelligence, which at once discriminates and brings into relation different aspects of the whole with which it is dealing. A mere subject by itself has not any intelligible meaning. If one hears the word 'iron,' for example, the word may call up certain mental images; but by itself it is not a complete thought or fact in which we can rest. 'Well, what of it?' we say. The mind at once goes on to form some judgment like, 'this is iron,' or 'iron is heavy.' We cannot *think* a term without thinking something *of* it. In short,

although the words which form the subject of a *proposition* are relatively independent, and can be used without the words which make up the predicate, in a *judgment*, on the other hand, a subject is only a subject *through* its relation to a predicate. The proposition may be divided into parts, but the judgment is a single thought-activity, and cannot be divided (cf. § 79).

§ 83. **Judgment as Constructing a System of Knowledge.** — In this section we have not to take account of any new characteristic of Judgment, but rather to emphasize the part it plays in building up knowledge. As we have seen, Judgment works both analytically and synthetically: it discovers new parts and distinctions, and at the same time brings the parts into relation and thus builds up a whole. That is the law according to which thinking develops, and is just what we called differentiation and integration in a previous section (§ 77).

It is necessary here, however, to dwell upon the fact that each judgment may be regarded as a step in the process of building up a *system* of knowledge. The emphatic word here is 'system,' and we must be perfectly clear about its meaning. A system is a whole which is composed of various parts. But it is not the same thing as an aggregate or heap. In an aggregate or heap, no essential relation exists between the units of which it is composed. In a heap of grain, or pile of stones, one may take away any part without the other parts being at all affected thereby. But in a system, each part has a fixed and necessary relation to the whole and to all the other parts. For this reason we may say that a building, or a piece of mechanism, is a system. Each stone in the building, each wheel in the watch, plays a part, and is essential to the whole. In things which are the result

of growth, the essential relations in which the parts stand is even more clearly evident. The various parts of a plant or an animal have their own functions, but at the same time they are so necessary to one another that an injury to one is an injury to all. We express this relation in the case of living things by saying that the parts are *organic* to one another. And, in the same way, it is not unusual to speak of society as an organism, in order to express the fact that the various individuals of which it is composed are not independent units, but stand in necessary relations to one another, and are all mutually helpful or hurtful.

We have said that Judgment constructs a *system* of knowledge. This implies, then, that it is not merely a process of adding one fact to another, as we might add one stone to another to form a heap. Judgment combines the new facts with which it deals, with what is already known, in such a way as to give to each its own proper place in relation to and interdependence with the others. Different facts are not only brought together, but they are arranged, related, systematized. No fact is allowed to stand by itself, but has to take its place as a member of a larger system of facts, and receive its value and meaning from this connection. Of course, a single judgment is not sufficient to bring a large number of facts into relation in this way. But each judgment contributes something to this end, and brings some new fact into relation to what is already known. Even in a simple judgment like, 'that was the twelve o'clock whistle,' the constructive or systematizing work accomplished is evident. The auditory sensation, which in itself, as a mere sound, was not a piece of knowledge at all, is interpreted in such a way as to find a place in the system of experience.

One may appreciate what part the judgment really plays by remembering how the sound appeared before one was able to judge. There may have been at first a moment of bewilderment —' What does this mean?' one asks. In the next moment the judgment is made: 'It is the twelve o'clock whistle.' That is, our thinking has constructed a meaning for it, and brought it into relation with the rest of our knowledge.

(1) Every new experience is thus brought into relation with the facts which we already know, and is tested by them. It has to find its place in the system of knowledge — to join itself to what is already known. If this is impossible, if what claims to be a fact is entirely opposed to what we already know on the same subject, it is usually declared to be false. Thus, we would refuse to believe that some person whom we know well and respect was guilty of theft; for it would be impossible to connect such conduct with what we already know of his character. And, similarly, we find it impossible to believe, even although we have the evidence of our senses, that the conjurer has actually performed what he professes; for to do so would often be to reverse entirely our conception of natural laws. It must not be forgotten, however, that the existing system of knowledge, which seems to serve as the standard and test of new facts, is itself undergoing constant modification through the influence of these facts. As new experiences are brought into connection with the existing body of our knowledge, there is a constant rearrangement and readjustment of the latter going on. Usually this adjustment is slight, and takes place almost imperceptibly. But, in some cases, a single fact may be so significant as completely to transform what seemed to be the accumulated knowledge of years. The experiment which Galileo made by dropping balls of different weight from the tower of Pisa, made it impossible to hold any longer the old theory — which seemed as certain as anything well could be

— that the velocity with which bodies fall is proportional to their weight. Again, if theft were actually proved against the man we respect, that single fact might be sufficient to force us to give up everything which we supposed that we knew about his character.

(2) We have said that judgment is the process by which knowledge grows into a system. It is by judging or thinking that we attempt to bring the various parts of our experience into relation with one another. The degree to which this has been done is the measure of our intellectual development. The knowledge of the uneducated and unthinking man, like that of the child, is largely composed of unrelated fragments. It is an aggregation, not a system of facts. The facts which go to make it up may quite well be contradictory, but this contradiction is not seen because no attempt is made to unite them. There is, of course, no human experience which is entirely systematic, or which has been completely unified. Even those who have thought most deeply find it impossible to fit together exactly knowledge gained from different fields, and from different sciences. The facts of one science, for example, may seem to stand by themselves, and not to have any relation to the facts derived from another science. Or there may appear to be a conflict between the results of physical sciences, and the truths of moral philosophy and religion. But the ideal always remains, that truth is one and indivisible, and that it must be possible ultimately to harmonize all facts in one all-embracing system of judgments (cf. Ch. XXVI.).

REFERENCES

B. Bosanquet, *The Essentials of Logic*, Lecture II.
" " *Logic*, Vol. I., pp. 97–103.
C. Sigwart, *Logic*, § 18.

CHAPTER XXIII

THE LAWS OF THOUGHT

§ 84. **The Law of Identity.** — We found (§ 78) that Judgment is the simplest form of thinking. And, in the last chapter, we were engaged in studying its main characteristics, and becoming acquainted with its mode of operation. The essential nature of the thinking process, therefore, has already been stated, though we have not traced the mode of its development, or shown its application to the various problems of experience. But, before undertaking this, it is necessary to turn aside to consider another problem. In nearly all books dealing with logic one finds a statement of three fundamental laws of thought which differ greatly, in form at least, from what we have so far learned regarding the nature of Judgment. These laws are so well known by name, and yet so ambiguous in their mode of statement, that it seems well to try to decide what meaning to apply to them. For their interpretation will be found to furnish further illustration of the nature of Judgment, and will thus throw light on the discussions of the last chapter. The laws of Thought are usually regarded as axioms, or propositions which require no proof, rather than as laws descriptive of the nature of thought in any special circumstance. In this sense, they are supposed to be the foundation of all logic, since they are presupposed in all thinking.

The first of these laws, or axiomatic principles, is that of Identity. 'Whatever is, is;' 'Everything remains identical

with itself;' 'A is A.' These are some of the forms in which the law is usually stated. What is meant by these statements is, that in all argument, we necessarily assume, if we are to reason at all, that each thing possesses a permanent character, and does not pass now into this, now into that *at random.* If any knowledge is to be possible at all, the character of things must remain fixed. Socrates is always to be Socrates, and iron, iron. Things are also constantly undergoing changes. The law of Identity, of course, does not deny this, or declare that the changes are unreal. It rather presupposes the changes; but goes on to affirm that there is an *identity persisting in and through the difference.* Identity means *identity in difference:* it is this which all our judgments assert. Socrates changes, or is different from day to day and from year to year. But he also remains identical with himself; he is in his old age the same Socrates who talked with Parmenides in his youth and fought at Potidæa when in middle life. Identity, then, does not affirm the static and unchangeable character of things and thoughts; but that there is *continuity* in change, in virtue of which things maintain themselves and are capable of being known as parts of a coherent system. Every one assumes as much as this in every judgment he makes, though he may not himself be conscious of it (cf. § 9).

Another interpretation of this principle was, however, offered by Boole and Jevons, who developed what is known as the Equational or Symbolic logic. According to these writers, the law of Identity expresses the fundamental nature of Judgment, and is to be interpreted as a statement of an exact and bare identity. That is to say, every judgment is the expression of an identity between the subject and the

predicate. The judgment, 'New York is the largest city in America,' is simply a case of *a* is *a*. It expresses the fact, that is, that New York and the largest city in America are identical. 'Iron is a metal,' is another example of the same principle. It may be written: iron = metal. And, since the copula may often be ambiguous, it will be better to discard it in working out arguments, and adopt, in its place, the sign of equality.

Judgment, from this point of view, is thus simply an equation, and may be written as such. Furthermore, the conclusion of a series of logical premises may be obtained by a process similar to that employed in working algebraic equations. That is, we can substitute for any term in a judgment, its equivalent, or the value which it has in another judgment. This method Jevons calls 'the substitution of similars,' which he maintains is the fundamental principle of all reasoning.

If, now, we employ letters to symbolize the terms of the propositions, it is claimed that we can work out any argument by the equational method. Take the argument,

> All metals are elements,
> Iron is a metal,
> Therefore iron is an element.

Now represent metal by M, iron by I, and element by E. Then the argument in equational form will be,

$$M = E \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

$$I = M \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

and by the substitution in (1) of the value of M in (2) we get I = E, the required conclusion.

Or, we may illustrate this method by a somewhat more complex example which is also taken from Jevons: 'Common

salt is sodium chloride, which is a substance that crystallizes in cubical form; but what crystallizes in cubical form does not possess the power of double refraction.' The conclusion of this argument may be found by letting A = Common Salt, B = Sodium Chloride, C = something which crystallizes in cubical form, and D = something which possesses the power of double refraction. The negative of any of these terms will be expressed by the corresponding small letters. The argument may now be expressed: —

$$A = B \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$
$$B = C \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$
$$C = d \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

By substitution of the value of C in (2) we get,

$$B = d \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

And substituting here the value of B in (1),

$$A = d.$$

Giving to these symbols their meanings, we get the result 'common salt does not possess the power of double refraction,' which is the conclusion of the argument.

Of course, in simple arguments like those we have been examining, there is nothing gained by the use of symbols, and the representation of arguments in this form. But when the various terms employed are much longer and more complex, simplification may be attained in this way. Various other symbols have also been used to express the relation of the various terms to one another, and a symbolic logic has been developed which follows very closely the procedure of algebra. By following closely the methods of mathematics, but seeking to obtain a more general form of expressing the relations than mathematics employs, results have

been obtained that are of much interest and which may prove valuable.[1]

It is, however, as a *theory* of the meaning of Judgment that we are interested in this mode of interpreting the law of Identity. We have seen that it works fairly well in practice, and therefore cannot be wholly false. But there are certain forms of reasoning in which it will not work. We cannot get the conclusion by the equational method in an example like the following: 'B is greater than A, C is greater than B, therefore C is still greater than A.'

This practical objection being left out of account, we have to ask whether an equation represents fairly the nature of Judgment. Does a judgment express merely the identity of subject and predicate? And if so, what *kind* of identity is referred to? In mathematical reasoning, the sign of equality expresses the identity of quantitative units. When one says, $2 + 3 = 5$, the meaning is that the number of units on each side of the equation is identical. And, similarly, the assertion that a parallelogram = 2 triangles with the same base and of the same altitude as itself, expresses the fact that, in the two cases, the number of units of area, square feet, square yards, etc., is the same. In mathematics, the equation declares that the quantitative relations of its two sides are identical. It does not assert that the two things compared — the triangle and one-half the parallelogram, for example — have the same qualities, or are exactly the same in all respects. Now, if we extend the use of the sign

[1] The clearest statement of the aims and methods of the Equational Logic may perhaps be obtained from Jevons, *The Principles of Science*, Introduction. Cf. also G. Boole, *An Investigation of the Laws of Thought*, London, 1854; and A. T. Shearman, *The Development of Symbolic Logic*, London, 1906.

of equality, it must take on a new meaning. It is clear that in a judgment like 'iron = metal,' there is no reference at all to quantitative relations. We are not asserting that the number of units in the two terms is identical. What, then, does the sign of equality express in such a case?

The answer is not difficult, say those who hold this theory. The sign of equality in such cases expresses *absolute identity;* the entire and complete sameness of subject and predicate. The proposition, 'mammals = vertebrates,' asserts that mammals and vertebrates are one and the same thing. But that statement in its present form is not true: the class mammal does not completely correspond with the class vertebrate. To make it exact, reply those who uphold the equational form, one must qualify or limit the predicate and write the proposition, 'mammals = some vertebrates.' But, even so, we may urge, the form of the judgment is still defective. In the first place, it does not correspond to the model $a = a$. For one side, 'mammal,' is clearly marked off, while the other is indefinite and vague. And, secondly, just because of its vagueness, it is not a satisfactory piece of knowledge. To obviate these objections, one must go further and write, mammals = mammalian vertebrates. At last the judgment seems to correspond to the type, $a = a$. But a new difficulty arises. Has not the judgment lost all its original meaning and become a mere tautology? There seems to be no escape from the following dilemma: *either* there is some difference between subject and predicate, and the judgment is therefore not in the form $a = a$, *or* the judgment is tautologous and expresses nothing. The view of the equational logic that Judgment affirms the entire identity of subject and predicate refutes itself. The form $a = a$

cannot be regarded as the type to which all judgments conform.

But there must be some kind of identity between the parts of a judgment. In one sense, we do seem to declare that the subject and predicate are identical when we say, 'iron is a metal.' As we have seen, however, if these terms are merely identical and nothing more, the judgment loses all meaning. We are forced to the conclusion that every judgment affirms both identity and difference, or that there is identity running through and underlying the diversity. But is not this a paradoxical statement? When we affirm identity, does not this imply the absence of all difference? If *a* is *a*, how can it at the same time be something different from itself?

And yet this is just what every judgment which has any meaning affirms. 'Iron is fusible.' 'This table is made of oak.' 'The sword is rusty with age.' In all these judgments there is an assertion of the unity of different properties or parts in one whole. A is B, and yet does not cease to be A, is rather the type of judgment than *a* is merely or abstractly *a*. It is worth noticing that this view of the matter corresponds with the account of Judgment already given. We saw that Judgment constructs a *system* of knowledge by showing that various things, which seem at first unrelated, are yet connected by an underlying unity. Knowledge is always the synthesis or union of different parts or different properties in a common identity. And each judgment, as an element of knowledge, displays the same essential structure which belongs to knowledge as a whole. It involves, as was shown in § 82, both analysis and synthesis, and declares the oneness or identity of a number of properties or parts, without at the same time losing sight of their distinctness.

Let us now sum up our discussion of the law of Identity. When rightly understood, as we have seen, it does not affirm that *a* can only be bare *a*, that the subject and predicate are absolutely identical. As a law of thought, it expresses the fact that Judgment brings together differences, *i.e.*, different things and qualities, and shows that they are parts of one whole or unity. That is, judgment reveals the underlying unity or identity which is present in the midst of variety. This law also states another characteristic of Judgment which we have already emphasized. This is what we have called the universality of Judgment (§ 80). It is to judgments, and not to concepts or terms, as has sometimes been supposed, that the law of Identity properly applies. What it affirms in this connection is simply that Judgment claims to be true, and hence is identical at all times and for all persons. It cannot be true for you and false for me that, 'iron is a metal,' and the judgment must at bottom mean the same for all men. Truth is not a matter of individual taste, but every judgment which is true has a permanent character or identity of meaning belonging to it.

§ 85. **The Law of Contradiction.** — The law of Contradiction is the second of the so-called laws of thought. It is usually stated as follows: it is impossible for the same thing both to be *a*, and not to be *a*, or, *a* is not *not-a*. It is evident that this law states in a negative form the same characteristics of thought as the law of identity. Indeed, it was in this form that the principle was first laid down by Aristotle. "It is impossible," he says, "that the same predicate can both belong and not belong to the same subject at the same time, and in the same sense."[1] We cannot assert

2.

[1] *Metaphysics*, Bk. III., Ch. IV. See also the remaining chapters of the

that Socrates is both wise, and not wise. Truth is not, as the Sophists supposed, a matter of taste or convenience, but must be consistent with itself. If a judgment affirms that 'iron is a metal,' it at the same time excludes the assertion that it is not a metal. There is a fixity and permanence about judgments which prevents them from changing into anything else. And it is just this permanence which we have already called the universality of Judgment, which the law of Contradiction expresses in a negative form.

The law of Contradiction has, however, sometimes been interpreted in such a way as to make it equivalent to the assertion of abstract or bare identity which we found in the Equational logic. That is, the statement that it is impossible for any judgment to unite *a* and *not-a* may be taken to mean that it is impossible to assert the unity of *a and anything different from a*. But, as we have seen, this is exactly what we do in every judgment which is more than a tautology. The law, then, does not forbid the union of *differences* in one judgment, but of *contradictories*, or of what would destroy the integrity of the judgment and render it unmeaning. If the law is to hold true of Judgment, *not-a* must not be taken as equivalent to anything which is different from *a*, but as signifying what is opposed or contradictory to *a*.

It is not by any means easy to decide what things are merely different, and therefore compatible with one another, and what contradictory or opposed. Logic can give no rule which may be applied in every case. If experience shows that two things, or two properties, are at any time united, we say that they are merely different from each other; if they have never been found in conjunction and

same book for Aristotle's demonstration that all thought presupposes such a principle.

we are not able to conceive how their union could take place, we call them opposites or contradictories. It is worth noticing, too, that no terms are *in themselves* contradictory, except those which are in the form *a* and *not-a*, wise and not-wise. But they *become* contradictory and exclude each other when they claim to occupy the same place in some particular system of facts. Thus 'maple' and 'oak' denote trees of a different variety, which are, however, so little opposed that they may exist side by side. If both these terms were applied to the same tree, however, they would become contradictory. By claiming to stand in the same relations, these terms become rivals, as it were, and exclude each other. But a knowledge of the particular facts involved is always necessary in order to determine whether or not two assertions are really incompatible.

§ 86. **The Law of Excluded Middle.** — The third law is a corollary from what has just been said in the last section. There is no middle ground, it declares, between contradictories. A is *either b or not-b.* To affirm the one is to deny the other. When we have real contradictories, — *i.e.*, when *not-b* is not merely something different from *b*, but something which excludes it, — every judgment is double-edged, and both affirms and denies at the same time. To deny that the throw of a penny has given heads, is to assert that it has fallen tails. As we have seen, however, logic affords no rules for deciding when things do thus stand in the relation of mutual exclusion. The law of Excluded Middle states only that *where* this relation does exist, every proposition has a double value, and both affirms and denies at the same time. It requires special knowledge of the particular facts in each case to enable us to decide what things are thus opposed to one another. There is no logical law by

means of which things may be divided into two contradictory or exclusive groups or classes.

It is important to notice that all of the judgments which we use in everyday life are to some extent double-edged. That is, they contain, besides what is directly affirmed, some implication or counter statement. For example, to say, 'that object is red,' is implicitly to deny that it is blue, or any other colour. The statement, 'A never looks at a book,' carries with it certain implications which may perhaps be held in mind as a series of hypotheses: 'Is he then too busy, or sick, or simply indifferent?' In almost any field where we have any systematic knowledge, we can limit pretty definitely the number of possibilities — *a* must be either *b*, or *c*, or *d*. In such cases, to affirm that *a* is *b*, is of course to deny implicitly *c* and *d;* and conversely, the denial of any one possibility, as *c*, enables one to assert that *a* is *b* or *d*. In ordinary conversation, misunderstandings and misconceptions frequently arise because neither party is fully aware of all the possible cases and the relation between them. It is very difficult, however, to make a statement which will have no counter implications. If one says, 'this railway system does not employ steam power,' the proposition seems to justify the question: 'Does it then use electricity or compressed air?' We should feel that it was a mere quibble if the person who made the statement should reply: 'I did not say it employed any kind of power.' 'There are some small errors in this paper,' would ordinarily be taken to imply the counter proposition, 'the paper contains no serious errors.' It is clear that it is only when one's knowledge becomes *systematic,* — *i.e.*, when one knows the relations in which all the facts in the field under consideration stand to one an-

other, — that one can be fully aware of what is really implied in each assertion or denial (cf. §§ 26, 83). It is, however, of fundamental importance to understand that in its work of defining the nature of things thought works with a double-edged tool. *Omnis definitio est negatio*, wrote Spinoza: to define is to exclude or eliminate. But as we have shown, the process of elimination is not merely positive but yields positive results.

These so-called Laws of Thought, when read in relation to one another, may then be interpreted as expressing the universal Postulate of our intelligence, that experience shall be capable of being organized as a system. If there were nothing but Identity — if everything were identical with everything else — there could be no universe and no knowledge. Nor would any knowledge be possible if things were merely different: if there were no common space and time, no common natures and laws of relationship, the world would be nothing but a disorganized chaos, without form and void. Finally, experience would not be possible as a coherent system if each fact had not some particular place or bearing, in such a way that one affirmation or denial carried others with it. Reality exists as a system of mutual implications and exclusions. It must so exist if it is to be knowable. That Reality is knowable by Intelligence, may, then, be regarded as the ultimate postulate of knowledge, and this, as we have seen, is the final interpretation to be given of the Laws of Thought.

REFERENCES

F. H. Bradley, *The Principles of Logic*, pp. 131–154, 343–360.

B. Bosanquet, *Logic*, Vol. II., pp. 207–212.

W. S. Jevons, *The Principles of Science*, Introduction.

CHAPTER XXIV

TYPES OF JUDGMENT

§ 87. **Judgments of Quality.** — We have hitherto been considering the nature of Judgment in general, and have learned something regarding its main characteristics. It is now necessary to examine briefly some of the more important forms or types of Judgment. In § 51, we spoke of the different forms or conceptions in terms of which things are brought into relation as 'Categories.' This chapter might therefore have been entitled, 'The Main Categories of Thought,' as it is with certain typical ways in which things are related that we are here concerned. We shall begin with very simple and elementary ways of judging, and afterwards consider some of the more complex types. In this way, we shall see the nature and structure of Judgment illustrated at different levels of thought. We also hope to show, by this review of types, that there are no arbitrary divisions in the process of thinking, but that the lower forms of Judgment gradually develop into the higher in accordance with the general law of evolution. It is, of course, impossible to carry out at present this plan in detail, for that would be to give a complete history of the development of thought. It will be necessary for us to take long steps, and content ourselves with a general view of the relation of the various stages in the development of Judgment.

The first efforts of intelligence to understand the world take the form of judgments of Quality. At a low stage of

mental development, it is the simple qualities of things which force themselves on attention. The young child, for example, takes notice only of the most striking qualities of things. His judgments are very vague and indefinite, and take account only of some prominent quality of things. That is, there is in them no discrimination of the various parts and relations of the objects, but they express merely a general impression based upon some striking quality. Thus it has often been noticed that the child calls every man 'papa,' and any light, of whatever size, the moon. A little boy, known to the author, used to call Sisters of Charity crows, on account of the colour of their dresses. The objects as he apprehended them were simply black, and nothing more. His intelligence rested in the qualitative total impression: the various parts, with their diverse relations, which he afterwards learned to know and distinguish, did not at that time exist for him.

It is perhaps impossible to find in the experience of an adult any judgments which deal entirely with simple qualities, and which take no account of the numbers, or even to some extent of the relations, of the parts. But we can find examples of judgment where the qualitative aspect is much the most prominent — where indeed the quantitative and more complex relations are scarcely noticed at all. 'This is green,' 'that is a strange odour,' 'there is something a long way off,' — all these seem to be judgments of quality or general impression, and to involve scarcely any other element. This is, also, the easiest kind of judgment to make, the judgment which involves least mental effort, and which notices only the most evident, and, as may be seen, the most superficial, aspect of things. It is evident that such judgments belong to a lower

stage of thinking than those which imply analysis and perception of quantitative relations. Compare, for example, 'this is very large,' with, 'this tree is made up of roots, trunk, branches, and leaves'; or 'this is green,' with, 'this leaf is divided into two parts by a rib running through the centre.' The first judgment in each pair obviously involves much less intellectual work than the latter. The judgment of simple quality accordingly is, as we have said, the starting-point of thought. It is with this kind of thinking that the knowledge of the child begins. And, before the savage learns to count, *i.e.* to distinguish and enumerate the parts of the objects with which he deals, his judgments must necessarily belong to this same type.

It must never be forgotten, however, that simple judgments of quality are really judgments; that is, they are not given to the mind from any external source, but are the products of its own activity. A judgment, as we have already pointed out (§ 78), implies a reaction on the part of the mind on what is presented to consciousness through the senses. It distinguishes and puts together the material which sense presents in such a way as to perceive its significance — what it really amounts to — as a piece of knowledge. This act of interpretative intelligence has gone, however, but a little way in the type of judgment with which we are dealing. But even in a vague qualitative judgment like, 'there is something black,' the essential characteristics of Judgment can be already distinguished. For it presupposes at least some analysis or discrimination of the black object from the rest of the environment, and of the black colour from other colours. And the judgment, 'something is black,' has made at the same time a beginning in constructing this vague some-

thing into a system of qualities, or into a thing that is known The other qualities and relations are as yet wrapped up in the indefiniteness of the 'something.' In spite of its indefiniteness, however, the latter plays the part of a permanent centre or identity. It is the whole from which the quality of blackness has been separated out, and to which it is again attached.

Our thought, however, is not satisfied with a knowledge of the general qualities of things, but pushes farther its work of analysis and construction. In this way, it begins to distinguish the various parts of objects, and to compare one with another. We not only judge that 'the grass is green,' but go further and say 'this piece is dark green, and that light green.' The indefinite judgment, 'this cane is heavy,' is no longer satisfactory, and is replaced by, 'this end of the cane is much heavier than that.' And when this stage is reached, judgments of Quality are already passing into the next higher type, judgments of Quantity. For the element of comparison, which is already contained in these judgments, is the basis of counting, measuring, and all quantitative determination. In advancing from the simple apprehension of quality, to the stage where it takes note of, and compares, the *degree* or *intensity* which the same quality manifests in different instances, intelligence has entered upon a path which leads directly to judgments of quantity. To distinguish parts, to regard things as degrees or instances of a common quality, is at once to suggest the quantitative process of counting and measurement.

§ 88. **Judgments of Quantity.** — It is very difficult, as we have seen, to draw a hard and fast line between quality and quantity. Indefinite judgments of general impression which do not imply any comparison, seem always to be qualitative

rather than quantitative in character. This is true, I think, of judgments like, 'this object is very large,' 'there was a great flock of sheep in the field.' In such cases, the interest does not seem to be quantitative at all; *i.e.* there is no effort made to determine *how many* units or parts there are in the whole about which the judgment is made. But the general impression of size or number is apprehended and judged of at the same level of intelligence, and in the same vague way, as the simple qualities with which we dealt in the last section. It is by means of such a general qualitative impression that the savage who cannot count beyond five, is able to distinguish between six and some larger number. And we cannot imagine that the shepherd's dog learns that some of the sheep are missing by any process of counting. We must suppose that the general qualitative impression made by the smaller flock is different from that made by the larger, and that there has been no real counting or estimation of number in the case.

But quantitative judgments proper belong to a higher stage of intelligence than do those which have just been described. Indefinite judgments, like 'this is very large,' or, 'there are a great many stars in that group,' are not satisfactory pieces of knowledge. We accordingly set ourselves to get more exact information about the parts which compose the wholes, or to analyze and distinguish. The first step in this process leads to *Judgments of Enumeration*. If the whole which is analyzed is composed of homogeneous parts, the judgments of enumeration take the form of simple counting. 'There are one, two, three, . . . twenty men in this company.' Where the parts are not of the same kind, however, a separate name may have to be given to each. 'This plant is composed of root, stalk, leaves, and flower.'

But exact quantitative knowledge requires us to do more than enumerate the parts of which a whole is composed. We must go on and *weigh* or *measure* them. There is of course no essential difference between weighing and measuring, so that we may call all judgments which express the result of this process *Judgments of Measure.* It is worth noting that judgments of this class are not so simple and direct as may appear at first sight. When we measure, we express the relation of the parts with which we are dealing to some common unit or standard. The judgment, 'this tower is 200 feet high,' means that if the tower is compared with a foot-rule, it will be found to be 200 times as long. It really, then, involves a proportion, and might be expressed:—tower : foot-rule = 200 : 1.

The point which it is important to notice is that all measurement is the result of comparison. In the first place, some unit is more or less arbitrarily selected. Then the judgment states simply the relation between this unit and the object measured: one is contained in the other once, or twice, or ten times. The quantitative determination thus obtained is accordingly merely relative. That is, it does not belong absolutely, and in its own right to the object measured, *but indicates the relation of that object to something else.*

For this reason, it may seem that quantitative relations tell us nothing regarding the real nature of objects, and that to discover what the latter are *in themselves,* we shall have to return to the point of view of quality. But we have seen that simple judgments of quality yield a very vague and unsatisfactory kind of knowledge. Moreover, we should discover, by thinking the matter out, that even qualities always imply a reference to one another, and are no more absolute than quantities.

In order to obtain more satisfactory knowledge regarding things, we shall have to go forward to a higher type of judgment, rather than backward to quality. But the importance of quantitative determination for exact knowledge must not be overlooked. By means of measurement, things are reduced to common terms, as it were, and thus a basis of comparison is afforded where it would otherwise be impossible. To reduce everything to such a common measure is the business of the physico-mathematical sciences. Everything has a quantitative value, and can be expressed mathematically in terms of some unit or standard, as, for example, the unit of heat, or of pressure, or the electrical unit. It was this tendency to count and measure and weigh things which established the body of exact knowledge which we call science. And in almost every field, knowledge increases greatly, both in extent and exactness, as soon as it is found possible to reduce the phenomena under investigation to a common measure, and to express their relations by means of mathematical formulas.

It is a great step in advance to be able to compare things as quantities, and to express their relations in terms of number. But judgments of quantity are not entirely satisfactory; they are, as has already been noticed, merely *relative* in character. Moreover, from a quantitative point of view, each thing is equivalent to the sum of its parts. When the parts have been enumerated and measured, the value of the whole is obtained by addition. But it is scarcely ever possible to represent adequately the nature of a whole in this way. So long as we are dealing with a piece of inorganic matter, the method of regarding the sum of the parts as equivalent to the thing, generally gives good results and leads to no difficulty. But it is quite different when the whole question belongs to something which has life and consciousness. In such cases, we have what has

already been called an organic whole (§ 83). Now, it is clear that the principle of quantity, which can only add and subtract, is insufficient to represent completely the nature of an object of this kind. It has no means of representing the individuality or real whole, which rather constitutes the parts, than is constituted by them. That is, to understand such objects, we shall have to take a new point of view, and begin with the whole rather than with the parts. From the point of view of quantity, the nature of the whole is discovered by adding together the parts; while in objects which possess an individuality of their own, there seems to be a central principle to which the parts are subordinated, and in relation to which alone they can be understood. The type of judgments which deal with such objects we shall have to discuss in § 90.

§ 89. **Judgments of Causal Connection.** — Another class of judgments used in building up knowledge, may be called judgments of Causal Connection. They undertake to show how the various changes which go on in things are connected causally with other things or events. This type of judgment — leading as it does beyond the particular object to a knowledge of the ways in which objects are connected — seems to belong to a higher stage of mental development than those which merely take note of quality and quantity. This does not mean that we never look for causes until the qualities and quantities of things have been discovered. Nor is it true that any causal judgment, however vague and unsatisfactory, is higher than any judgment of quality or quantity whatsoever. But, in the beginnings of knowledge, one may say, thought does not travel outside the particular object to show the connections of the latter with anything else. And, beginning in this way, it seizes first upon quality and quantity; which seem to belong to things in themselves. We have seen, however, that as a matter of fact judgments of quantity involve com-

parison, and so a reference of one thing to another, though that reference is not usually made consciously or explicitly. In this form of judgment, the reference does not seem to imply any objective relations of the things compared. If, for example, I say that this desk is twice as long as my arm, this relation appears quite external and accidental: the nature of the one remains independent of that of the other. But, when we judge that one thing is causally connected with another, the accidental relation expressed in quantity has become essential and objective, indicating a closer relationship between things than is expressed in a quantitative comparison of the judgment.

The word 'cause' has been used in a great many senses, and its various meanings have given rise to a great deal of discussion. That every event must have a cause, was formerly regarded as an innate truth, or *a priori* proposition. We have seen, however, that we do not come into the world with any ready-made stock of knowledge. All knowledge, we have often repeated, is the result of the mind's own judging activity. The so-called law of causation (every event must have a cause) must therefore express the fact that thought does connect things as causes and effects. Intelligence is not satisfied to take things in isolation; it tries to gain an insight into the ways in which they are connected, to discover what one has to do with another. And this is just the characteristic of thought which was emphasized in § 83. Judgment, it was there said, is a process of constructing a *system*, of showing how the various parts of knowledge fit into one another, and are mutually dependent upon one another. The tendency of thought to connect things causally, then, is simply one of the fundamental forms in which its tendency towards a system

expresses itself. In employing the causal category, judgment has become more explicit and conscious of itself than it was in quality and quantity.

It is interesting to note some of the more important changes which take place in the principle of causal explanation at different stages in the development of knowledge. The child and the savage regard all changes and events which take place in the natural world, as due to the agency of living beings. These beings are represented as more or less similar to men, and as endowed with human passions and emotions. Thus we say that the earliest kind of explanation is essentially anthropomorphic. This word is derived from ἄνθρωπος, a man, and μορφή, shape or form, and hence is used to describe the way of representing either a spiritual being, as, for example, the Deity, or natural forces like fire, wind, etc., in human form. It is probably true that at a very early stage in the development of both the individual and the race, every object is supposed to have life. Or, perhaps, it would be truer to say that the young child (and the same would be true for the savage on a low plane of intelligence) has not yet made the distinction between animate and inanimate objects, but vaguely regards everything as like himself. This first stage is usually known as *animism*, because each object is supposed to be endowed with a spirit, or *anima*.

Gradually, however, the distinction between animate and inanimate objects becomes clear. Accordingly, we find that at a somewhat more advanced stage the mode of explanation takes a different form, though it is still anthropomorphic. Physical objects are no longer regarded as having life in themselves; the changes in them are supposed to be due to the action of spirits, who are separate from the objects, but

who use them to accomplish their purposes. These invisible spiritual agents, to whom all natural events are referred, have been variously named. It is clear, however, that the gods of mythology belong here, as well as the fairies, elves, ghosts and witches of the popular folk stories. It was a great advance when a Greek thinker, named Thales, came to the conclusion that it does not in any way explain natural events to refer them to the action of the gods. For, in the first place, to say that the gods cause this or that event, is to state something which we have no means of proving. And, even if the assertion were true, it would not really explain anything. For it would not enable us to understand *how* the changes in question came about. It would tell nothing whatever regarding the actual steps in the process itself. Thales saw this, and tried to give a *natural* explanation of the world, and all that goes on in it. He tried to build up a real system of knowledge by attempting to show how everything which has happened in the world has been connected with some natural cause. We know very little about the actual explanation of the world which Thales gave, except that he tried to derive everything from water. It is on account of the method which he adopted, rather than of what he actually performed, that he is regarded as the founder of science. Thales first showed, one may say, that knowledge means an insight into the ways in which the actual phenomena of the world are connected with one another. We cannot unite into a system things so different in kind as spirits and natural phenomena. Or we may say that real explanation demands that there shall be some likeness, or ground of similarity, between the cause and the effect. An event which happens in the world of objects must be explained by showing its connection with some other event, of a similar character, on which it depends.

The development of this conception of scientific explanation also influenced still further the notion of causality. We have seen that in the beginnings of knowledge every event was supposed to be due to the action of some living agent, or spiritual being. Even after this mythological mode of explanation is discarded, and natural causes put in the place of spirits, it is still difficult to rid oneself entirely of the old anthropomorphism. The popular mind still tends to regard the cause as an *agent* which *produces* the effect, through some power or efficiency which it possesses. It is not necessary to raise the question at present whether there are any grounds for this belief. To discuss this problem would carry us beyond logic into metaphysics. What we wish to notice is that science has gradually abandoned the notion that the cause *does something* to the effect. That, as we have seen, is a remnant of the old pre-scientific idea, and a notion which does not aid at all in explaining phenomena. It is the business of science to show *how* the things and events which make up our experiences are necessarily connected with one another. Science has to discover what things invariably go along with one another, and necessarily presuppose one another. And, when it is found that some particular thing or event, A, is invariably necessary for the appearance of another particular occurrence, B, the former is regarded as the cause, and the latter as the effect. In order to eliminate as far as possible the notion of agency or efficiency which attaches to the word cause, the terms 'antecedent' and 'consequent' are often used to indicate this relation. For science, the cause is not an active agent, but the invariable and necessary antecedent of something else which simply follows it. The cause does not explain the effect by assigning an agent which brings the

latter about through its personal efforts; but it explains, because it reveals another necessary step in the process, and gives us a new fact which joins on or can be connected with the one from which we start.

We conclude then that the cause of any event is its invariable and necessary antecedent. It has been already explained, however, (p. 237) that by antecedent is not meant merely that which is prior to the effect in time. The word must be understood as signifying the essential condition or what is 'logically' prior. Temporal priority is often taken practically as an indication of logical priority, but the two relations cannot be identified. In another part of this book (Chs. XVI., XVII.), it is shown what tests it is necessary to apply in order to determine whether two phenomena are merely accidentally conjoined, or whether the connection is essential and real. It is necessary now to take one more step in tracing the various ways in which the idea of causality has been used. As a result of a famous scientific discovery, which was made about the middle of the preceding century, a new element has been added to the notion of cause in its application to physical phenomena. The law of the Conservation of Energy states that the amount of energy, or power of doing work, possessed by any set of bodies, regarded as a closed mechanical system, remains constant. Any change in a material body is the result of a transformation of energy from one form to another. The same notion is applied to the world as a whole: it is assumed that the total amount of energy which it contains remains constant. All changes which take place in the physical universe — motion into heat, or electricity into motion — are regarded as simply different forms, or manifestations, of the one world-energy.

As a result of this law, the effect always represents the same amount of energy, or power of doing work, as the cause. Since no energy is ever lost, the one must be equal to the other. And, as a matter of fact, the quantitative equivalence of many of the various forms of energy has been proved by actual measurement. In working out this law, for example, Joule showed that "the energy stored up in the 1-lb. weight which had been pulled up 772 feet was gradually transformed, as soon as the weight was released, into an amount of heat capable of raising the temperature of a pound of water 1° Fahr.; while Hirn showed, on the other hand, that exactly this amount of heat would, if it could be turned back again into energy, raise the 1-lb. weight to the height of 772 feet at which it stood before."[1]

The new element which this law adds to the idea of cause as a necessary and invariable antecedent, is that of the *quantitative identity of cause and effect.* Taking the phenomena which are connected in this way to represent simply certain quantities of energy, we say that the one is equivalent to the other. The energy which the cause represents has been transformed without loss, and reappears in the effect. If what seems to be the total effect is not equal to the cause, part of the energy of the latter must have been transformed into something else as yet perhaps unnoticed. No energy can have been lost.

It becomes, therefore, the task of the physical sciences to show that this relation of quantitative identity exists between phenomena which are causally connected when these are regarded by the science as constituting a closed mechanical system. The ideal of physical science is to prove that two

[1] Buckley, *Short History of Natural Science*, p. 339.

groups of phenomena are connected as cause and effect, by showing that both represent the same quantity of energy. For this purpose, measurement and calculation are necessary. The physical sciences, as was pointed out in the last section, deal largely with judgments of quantity, and devote themselves to showing by measurement that the same amount of energy persists through the various changes which phenomena undergo. In establishing causal connections, therefore, the physical sciences find it necessary to use the principles of measurement and calculation.

It will be evident, from what has been already stated, that this relation of cause and effect should, in theory, apply to all phenomena whose energy is capable of being measured and represented in quantitative terms. As a matter of fact, however, the law has been proved only in physics and chemistry. From the very nature of the case, it is extremely difficult to measure exactly the relations of cause and effect in the sciences which deal with organic life. But even in those sciences, the law of the Conservation of Energy is assumed to hold true. For example, the amount of energy which a plant contains, is assumed to be exactly the same as that represented by the various elements or forces — water, sunlight, mineral substances, etc. — which were instrumental in composing it. In the same way, we suppose that the same relation holds of the changes which go on in the brain, though we are, of course, unable to prove this by actual measurement. We may accordingly speak of the law of the Conservation of Energy as the working postulate of these sciences.

It is difficult, however, to see how this law can have any application to mental phenomena. We can indeed measure the intensity and duration of sensations. But neither feelings nor complex processes of mind seem to be capable of measurement in fixed and unambiguous units. Moreover, it is never possible to measure

the energy, or power of doing work, which states of consciousness possess, and to equate one with another in this respect. And this being so, the law of the Conservation of Energy cannot, of course, apply to psychical causes and effects. In the mental sciences, then, we cannot claim that the notion of Causality contains the element of quantitative identity between cause and effect which has been found to exist in the physical sciences.[1]

§ 90. **Judgments of Individuality.** — By Judgments of Individuality, we mean judgments which regard some complex object as a real whole with a definite nature of its own. Judgments of this kind are also frequently called **Judgments of Purpose, or Teleology.** We have already had occasion (§ 83) to distinguish a mere aggregate or sum of parts, like a heap of stones, from a true whole which possesses a certain character and individuality of its own. It is as aggregates rather than as true wholes that judgments of quantity and of causal connection regard objects. For these types of judgments are concerned with the parts — the former to measure them, and the latter to show their causal connection. It requires a new form of judgment to represent adequately the nature of a complex object which possesses individuality. This form gives expression to the organic unity and wholeness of things, and emphasizes the way in which the parts coöperate for a common purpose or end. Thus we regard the parts of a plant as a unity coöperating in a common purpose, and a man as a conscious system of ends. The question as to whether it is allowable to employ any other category, or form of explanation, in science than that of causality, is of great importance. In biology, for example, it is usual to explain certain structures of plants and animals

[1] Cf. Wundt, *Ethik* (1st ed.), pp. 398 f.; Sigwart, *Logic*, § 97 *a*, 7.

as purposive. How far, now, is it allowable to go in substituting this teleological form of explanation for explanation in causal terms? This question is too large to be discussed here, but it is suggested as of fundamental importance both for science and philosophy.

(1) We have seen that judgments of causal connection relate phenomena as causes and effects. A change in an object is explained by showing that some other change or event invariably precedes it. But this change, in its turn, demands explanation, and has to be accounted for by the discovery of a new cause. This type of judgment shows that one phenomenon is connected with a second, a second with a third, and so on indefinitely. The view of the world which it presents is that of a never-ending series of causes and effects. It is never possible to find a cause which is not itself the effect of something else. No phenomenon possesses any independence of its own, but is simply a link in a series, or a piece of a whole that is never completed. We say, therefore, that causal explanation leads to an infinite regress. The notion of a 'first cause' is then contradictory, if 'cause' be defined in the scientific sense, as a phenomenon existing in time and space.

In the last section, it was stated that causal judgments connect one part of our knowledge with another, and, in this way, aid in uniting the parts of our experience in a systematic way. Now it is undoubtedly true that it would be impossible to have any genuine knowledge of anything as a whole, or an individual, without knowing the way in which the parts are related, and mutually depend upon each other. In that sense, judgments of causal relation are indispensable to a knowledge of a true whole. But this form of judgment itself resolutely goes on connecting part with part — one phenomenon with another — and refuses to regard any group of parts as possessed of an independent character or individuality. From this point of view, everything is externally determined; its

cause, or principle of explanation, lies outside of it in something else. The mark of individuality, on the other hand, is the power of origination, or self-determination. If, then, there exist any genuine individuals, they are something more than causally determined phenomena.

(2) Psychology, at least modern structural psychology, adopts the standpoint of Causal Connection; while Ethics, assuming that men as moral beings are responsible for their actions takes to some extent at least the standpoint of Individuality. The former science regards mind as a *sum* of mental processes, and undertakes to show how its various parts are connected. Every state of consciousness is supposed to be determined by something external to itself — some antecedent mental state, or some bodily process. The interest, as was previously said, is centered in the parts, and it is very rarely that the psychologist stops to look at the mind as a whole. Ethics, on the other hand, has to begin with the individual. It does not regard mind as a thing or substance (that is the naïve point of view against which psychology rightly warns us), but as a self-conscious system of ideas, purposes, and feelings, which possesses the power of initiating action, and of determining itself in accordance with some purpose. The judgment of Individuality, as a more concrete form, must use the results of judgments of Causal Connection. What it really does, is to interpret what for the psychologist is a *sum* of mental processes in terms of a *system* which has a real unity of its own. For it is only when a person is regarded as a self-conscious and self-acting individual, that he can be supposed capable of conduct to which the terms 'moral' and 'immoral' can properly be applied.

REFERENCES

Hegel, *Logic*, Pt. II., The Doctrine of Essence (Wallace's trans. 2d ed.), pp. 206–286.

B. Bosanquet, *Logic*, Vol. I., Chs. II.–V.

J. S. Mill, *Logic*, Bk. III., Ch. V.

C. Sigwart, *Logic*, § 73.

CHAPTER XXV

THE NATURE OF INFERENCE. — INDUCTION AND DEDUCTION

§ 91. **Judgment and Inference.** — It must not be forgotten that our object in these chapters is to obtain as definite a conception as possible regarding the nature of thought. To attain this end, we agreed (§ 76) that it would be advantageous to begin with the simplest or most elementary form of thinking. That form we found to be Judgment. We have now endeavoured to show what Judgment is, and what part it plays in building up knowledge. And, in the last chapter, we have attempted to see some of the steps in the evolution of Judgment, as it passes from simple judgments of Quality to judgments of Individuality. This account being completed, it remains now to discuss the nature of Reasoning, or Inference, as the process in which judgment occurs.

We shall probably get the clearest idea of the nature of Inference by regarding it as a completely developed judgment. As thinking develops from the form of simple judgment to that of Inference, it displays progressive differentiation and integration. In accordance with this law, we can say, (1) that Inference is more complex than Judgment. The latter process, in its simplest form, can scarcely be said to have any parts: it represents a single act or pulsation of intelligence. Inference, on the other hand, seems to imply steps or stages in thinking — a passage of the mind from one

fact to another. Moreover, (2) Inference differs from Judgment in exhibiting the grounds upon which its statement rests. The simple judgment makes a declaration on the basis of sense-perception, as, for example, 'the mail-train has just gone down'; 'it rained yesterday.' Each of these statements stands alone, as it were; it does not attempt to gain support by pointing out the connection of the asserted fact with other facts. To infer, however, is just to show the necessary connection of facts — that from the presence or absence of certain things, the presence or absence of certain other things necessarily follows. It is not necessary for Inference that the conclusion reached should be a fact which was not hitherto known. We often do reach new truths by reasoning from necessary connections. Thus we might *infer* that the mail-train has just gone down, from the fact that this train is always on time, and that it is now five minutes past the hour. Or, we might prove, to a person who doubted the correctness of our memory, that it rained yesterday, by pointing to other facts with which rain is necessarily connected. We might point to the muddy condition of the roads, the swollen streams, or, perhaps, might remind the person who questions the statement, that it was yesterday that A was out driving, and came home soaking. In this way, one tries to exhibit the *necessity* of the fact under consideration; and to do this is to infer.

But in the actual process of knowledge, we more frequently go from a fact to its reasons, than in the opposite direction. The intelligence begins by accepting all the connections as true and universal which it meets with in ordinary experience, or which are suggested to it in any way. It does not trouble itself at all about the grounds of its judgments, and

thus the insufficient basis on which many of these stand is at first not evident. The child, for example, believes everything which it is told by its mother or nurse, or, it may be, all the pleasant things which it imagines. Very often, too, the judgments of older persons are determined by their own wishes. The man of sanguine temperament is quite sure that his project cannot fail to succeed. Another principle upon which both children and adults quite unconsciously proceed, is that the future must always resemble the past. The child assumes that the order of events each day will be the same, — that there will always be games after dinner, and visitors in the afternoon, because that has happened a number of times in the past. And one may have no better reason for believing that the sun will rise to-morrow, than the fact that it rose yesterday and to-day.

In these early, unreflective judgments, the ground or principle upon which they are based is, of course, not conscious at all. Each judgment is accepted by itself, and no questions are raised as to how it is known. But the development of intelligence may be regarded as a process of becoming conscious of the reasons which show the falsity of certain of our beliefs and the necessity of others. The original judgment is not in reality so isolated and unrelated as it appeared; it contains implicitly its own reasons. But the validity of its procedure cannot be made manifest, until the reasons for the statement made by the judgment are brought to light. In the development of knowledge, the judgment must expand so as to show the reasons which it necessarily presupposes. In itself, it is only a fragment of the complete statement, and it tries to complete itself by making clear the nature of the whole which it involves, or to which it really belongs.

It is not until the implicit reasons which every judgment contains are thus brought to consciousness, that it can be either proved or disproved. Taking the mere judgment by itself, it is only possible to place one man's assertion against another's denial. But proof or disproof of a proposition implies that reasons are given for or against it. If its connection with some fact, or set of facts, known to be true, becomes evident on reflection, the *felt* necessity which the judgment possesses (§ 81) is transformed into a logical necessity. But, if no such connection can be found, or, if the judgment in question is seen to presuppose propositions which are themselves false, we must, of course, cease to regard it as valid.

When a judgment develops so as to become conscious of its reasons, it has already taken on the form of Inference. And, as we have already seen, this is the usual procedure of knowledge. We begin by believing without reason, or we assume that certain things are true, and try to find reasons for our belief. The conclusion, which is, of course, logically last, is usually first for us, and we set out from it to find the grounds, or the premises.

This way, however, of proceeding from conclusion to premises, or from a judgment to its reasons, implies that the mind is already aware of the distinction between false knowledge and true, and therefore that the work of criticising and testing knowledge has already begun. The criticism of knowledge is probably forced upon the mind at first by the practical consequences of false judgments. So long as false judgments lead to no unpleasant results, they are likely to pass unnoticed, without any question being raised regarding the grounds by means of which they are supported. The child usually

believes all that he is told, until he discovers that his credulity is making him a laughing stock, or has led to the loss of some pleasure which he values. Sooner or later he learns that the ground upon which he has been unconsciously proceeding — somebody told me — is insufficient. In the same way, the natural tendency to regard all connections which we happen to find existing between events as universal and necessary, becomes more critical and discriminating. The child soon learns that the events of one day do not necessarily follow in the order of the day before, and that it is not always rainy on Fridays, and fine on Sundays. But, in order to discriminate between what is true and what is false, he is obliged to go beyond the facts themselves, and to become more or less clearly aware of the grounds assumed in each type of judgment. He is forced to include ~~in the judgment~~ the reasons by which it is supported. And, in this way, the distinction between valid and invalid principles of connection is gradually learned. Through experience, which is more or less dearly bought, we learn that we cannot depend upon hearsay, and also that many of the most obvious connections between events are not essential, and have no claim to be regarded as universal laws. It becomes evident that it is necessary, in order to reach true principles of connection, to take a wider survey of the facts, and to push the process of analysis further than is done by our ordinary judgments of sense-perception. For example, we may at one time have supposed it to be a universal law that hot water will break glasses when poured into them. But as soon as we have experience of any instance or instances to the contrary, we see that there is no essential connection between hot water and broken glasses. It is necessary then

to go behind the obvious facts of the case, in order to discover what is the real antecedent in the two cases. The two instances — where the glasses break, and where they do not — *seem* to be the same; and yet, since the result is different, there must be a difference which further analysis will bring to light, such as the greater thickness of the glasses which break. It is by penetrating beneath the point of view of ordinary knowledge, that science endeavours to show how phenomena are really and essentially connected.

The judgments of ordinary adult life usually involve some consciousness of their grounds, and are therefore so far inferences. But in many cases of this kind it would be difficult for the individual to state explicitly the reasons for his judgment. The connection which he asserts may be guaranteed to his mind by some complex set of circumstances very difficult to formulate. Or it may rest upon some general similarity or analogy, which is so obviously insufficient that he hesitates to acknowledge that it is the only ground he has for judging. Thus one may be vaguely conscious that one's only reason for l king A is his resemblance to B. It may be impossible to say exactly in what points A resembles B; one may proceed on a vague general similarity. Or one may hesitate to make clear, even to oneself, that the only reason for disliking A is because of some external resemblance — in name, or dress, or figure — to C, whom one dislikes.

§ 92. **The Nature of Inference.** — We have seen that it is difficult to draw any hard and fast line between Judgment and Inference. In general, however, we may be said to reason when we do not simply accept a fact on the basis of sense-perception or memory, but show that it necessarily follows from some other known fact or facts. Inference, then, requires (1) that certain data or premises should be

accepted as already known; and (2) it implies an insight into the necessary connection of some new fact or set of facts with what we already know. Thus one is said to infer B when one sees that it necessarily follows from some fact which is already known. It is not necessary for an inference that B should never have been in consciousness before. As we have seen in the last section, what we very often do in inference is to show the reasons or necessity of some fact which we have previously accepted without knowing why. No matter whether we go from premises to conclusion (from the reasons to the fact), or in the opposite direction, from the conclusion to the premises, we are said to infer whenever we find the ground for the existence of one fact in the nature of another fact. In the former case, we use words like 'therefore' and 'consequently,' to indicate the connection; or, when the reasons are stated last, we use 'for' and 'because.' Whenever these conjunctions are used correctly, an inference has been made, and it is always useful in following a course of reasoning to make clear to ourselves precisely on what grounds it has been made.

Although Inference seems very simple and very natural, its procedure is much more puzzling, when looked at closely, than one would at first imagine. As we have seen, there is no Inference unless the result reached is different from the starting-point. But how are we ever justified in passing from a knowledge of one fact to another different from it? How can we ever pass from the known to the unknown? The Greeks, who loved to bring to light the paradoxes which so often underlie familiar facts, used to discuss this question. How is it possible for that which is unknown — external to the mind — to pass into the mind and get itself known? It

was to solve this puzzle that Plato propounded the doctrine that all knowing is remembering.[1] Knowledge, he declared, is not increased by learning that of which we were altogether ignorant, but by a process of calling to mind or recollecting the knowledge which the soul possessed in a previous state of existence, but which was forgotten when it entered upon the conditions of the present life. It was therefore not necessary to suppose, according to Plato, that the mind performed the impossible feat of knowing what is external to itself, or that things previously unknown pass bodily into our minds, and thus become known.

Plato was undoubtedly right in protesting against the popular view that knowledge is received into the mind in mechanical fashion, as food is received into the stomach. Knowledge, as we have frequently seen, is built up from within, and not put in from without. But the apparent paradox of knowledge may be explained without adopting Plato's poetical notion of a previous state of existence. We may admit that the process of inference would be quite inexplicable, if it proceeded from one fact, A, to a knowledge of a second fact, B, which is totally different from the former. When we examine cases of inference, however, we find that there is always a certain amount of identity between the two ends of the process. The conclusion is always different, and yet not entirely different from the premises. Thus, from the propositions, 'all metals are elementary substances,' and 'gold is a metal,' one can infer that gold is an elementary substance. It is possible to connect 'gold' and 'elementary.' Here the identical link — what is called in formal logic the middle

[1] This is the theory upon which Wordsworth based his "Ode on the Intimations of Immortality."

term — is 'metal.' It is possible to connect gold and elementary substance, because the former is at the same time a metal, which in its turn is an element. Of course, these conceptions — gold, metal, element — are not absolutely identical; it was pointed out in § 84 that propositions cannot be regarded as expressing mere identity without difference. But we can say that there is a common thread or element running through these notions, which furnishes the principle of connection. Where we cannot discover such a common nature, no inference can be made. Thus, for example, it would be impossible to draw any conclusion from the statements that 'it rained yesterday' and 'gold has been discovered in Alaska,' because there is no common element or connecting thread present which would lead us beyond the premises.

In formal arguments the middle term, or connecting link, is usually explicitly stated; but in the actual process of reasoning things out, it is frequently necessary to go in search of it. We may notice, for example, that the fire in a stove burns more slowly when the damper is shut. In order to understand the fact, we have to find out some fact which is common to 'closed-damper' and 'slow-burning,' some link of identity, as it were, which enables us to pass from the one to the other. Such a connecting link is afforded, of course, in this case by the supply of oxygen. Darwin was noted for his keenness in detecting connections which escape the ordinary eye, as well as for his skill in giving explanations of them. On one occasion, he observed that in the part of the country where he lived, clover was abundant in those fields which were situated near villages, while the outlying fields were almost destitute of it. What now, he asked himself, is the

connecting link between these facts? Some investigation of the matter convinced him that the three agencies which produced this result were humble-bees, mice, and cats. The bees fertilize the clover flowers, and thus make the plant abundant, the field mice destroy the bees' nests, but the cats go out from the villages into the fields near by and kill the mice.

We have seen that the passage from one fact to another in inference does not involve a transition to something wholly different from the starting-point. There is always some aspect or feature in which the premises are identical with the conclusion. And it is on the strength of this identity that a passage can be made from one to the other. The same fact may be expressed differently by saying that all inference takes place within a system, 'where the parts are so held together by a common nature that you can judge from some of them what the nature of the others must be.' Suppose you were given the leaf of a plant. If you had some systematic botanical knowledge, it might be possible to infer the species of plant to which the leaf belonged. That is, from the nature of a part, the nature of the whole to which it belongs could be determined. The part represents the whole — in some sense contains it implicitly. It is said that the great naturalist Cuvier could determine by examining a single tooth the nature of the animal to which it belonged. Let us suppose that the tooth were that of a ruminant animal. Now a zoölogist, who knows the characteristics of such an animal, could draw various inferences regarding the possessor of the tooth. He could conclude, for example, that the animal to which it once belonged must also have had cloven hoofs. A single piece or part, that is, would

enable one who knows accurately the system or common nature to which all the parts belong, to judge what the other parts are like.

The examples just given have referred to the possibility of an inference from one part of an organism to another. But, as we have already seen, the systematic connection which here exists between the parts is more or less completely present whenever it is possible to infer at all. Inference pushes further the work of constructing a system begun by Judgment (§ 83). If each thing were known by itself, if the parts of our knowledge did not fall together into systems where each part to some extent determines the nature of the other parts, no inference would be possible. It is because the various pieces of our knowledge are never independent of one another, but form an organic whole, like the members of a living organism, that certain facts follow, as we say, from certain other facts. Otherwise we could only guess, or infer vaguely on the expectation that the future will resemble the past. Even this expectation, however, has no rational basis, unless the world does form some kind of a coherent system. It is, of course, true that practically a great deal of the knowledge of every one is unsystematic, being composed of facts and theories which have never been brought into relation. But knowledge is not to be described in terms of such defects in the case of individuals. To understand it, we must take it at its best and in its most complete form. It is obvious that, as our knowledge in any field becomes more completely and exactly organized, it it will be increasingly possible to use it as a basis for inference. The better we are able to put together in a systematic way the various facts which we have learned about geology, or astronomy, or the weather,

the more *significant* each fact becomes. The geologist may be able to tell from the appearance of the cliffs what has taken place in a locality thousands of years ago. And, similarly, for the fisherman, the temperature, direction of the wind, its rising or falling, etc., are all *signs* from which he is able to infer, more or less correctly, the kind of weather which may be expected. A person who had no systematic knowledge in either of these fields would, however, see nothing in the scarred rocks, or in the sudden changes of the wind; he might notice the facts, but would not be able to use them as a basis of inference.

It is important to notice that what has just been said goes to confirm our previous statements regarding the increasing degree of integration which knowledge shows in the course of its development. The knowledge of the scientist differs from that of the ordinary man, not only in the greater number of facts which the former contains, but also, as we have seen, in the degree of integration or coherence which these facts possess. Inference, then, is simply a deep insight, based on definite knowledge, into the necessary connection of things. It is an act of thought which discovers the essential relations between things which at first sight appear to have no connection with one another. As has already been said, it is a reasoned judgment; *i.e.* a judgment which has become conscious of the reasons for the connections which it affirms.

§ 93. **Induction and Deduction.** — It has been already pointed out that there are two directions in which inference or reasoning may proceed. We may begin with certain facts or principles which are already known, or are assumed to be true, and proceed to show that some result necessarily follows from them. Thus we might infer, from our know-

ledge of chemical principles, that if the draughts of a stove are closed so that the supply of oxygen is lessened, the fire will burn slowly; or from the relative positions and revolutions of the planets, astronomical reasoning might lead to the conclusion that an eclipse of the sun will take place on a specified day and hour. This method of reasoning is known as Deduction. It proceeds, as we have seen, from premises to conclusion. In the first part of this book, this form of reasoning has been treated at some length and its rules of procedure stated. At present, we need only notice that in deductive reasoning, the particular case is always brought under some general law or principle, which is already known or assumed as true. Socrates is known to be mortal, because as a man he falls under the general law that all men are mortal; the closing of the draughts is a case of lessened supply of oxygen, and, therefore, in accordance with the general law, a case of slow burning. A deductive inference shows what are the results of the application of a general law to particular facts or instances. It proceeds downwards, as it were, from the general law to its consequences.

In Induction, on the contrary, the procedure is just the opposite of this. We begin with particular phenomena, and try to discover from them the law or principle which unites them. Certain facts are observed to happen together, and the problem is to find the ground or explanation of this connection. Inductive inference is thus a process of reading the general law out of the particular facts, of transforming the hypothetical answer to the problem into a systematic principle or theory. It is an insight into the nature of the whole or system, based upon a careful examination of the parts. 'Yesterday the smoke tended to fall to the ground, and it

rained in the afternoon.' These two facts may simply be observed a number of times without any thought of their connection. But intelligence asks: Why should they happen in conjunction? And to answer this question, we must begin by analyzing the facts in our possession. When the smoke falls to the ground, the atmosphere must be lighter than usual; this is the case when it contains a great deal of moisture; but when the atmosphere is in this condition, it usually tends to discharge its moisture in the form of rain: therefore we have the general law which enables us to show that the behaviour of the smoke and the rain yesterday were not only accidentally *conjoined*, but essentially *connected*.

Deduction and Induction, then, are both forms of inference, but the starting-point and mode of procedure of the one is different from that of the other. Consequently, it is not unusual to speak of them as two *kinds* of reasoning which are quite distinct and independent of each other. It is, however, important to avoid this popular error, and to remember that the real process of inference is in each case the same. The essence of inference, as has been shown, consists in the fact that it exhibits the manner in which particular facts are connected together into a system or whole. And this end is achieved by both Deduction and Induction. In the former case, the general law of connection — what we may call the nature of the system within which the particulars fall — is known, and we argue from this as to the nature and relations of the various parts which fall within it. We have the common thread which unites the various facts in our hand, and following it out are able to show its application in determining the nature of events which have not yet come within the range of our experience. Knowing the law of gravity, for example,

one could infer deductively what momentum a ball weighing one pound must necessarily have after falling one hundred feet. It would not be necessary actually to measure the momentum of the falling body in this particular case, but it could be shown to be the necessary result of the general law. What the deductive inference shows us is the way in which a general principle or law of connection runs through a group of facts, and constitutes them a real or organic whole. The same insight is reached by inductive inference, although the starting-point is entirely different. As we have already seen, induction begins by observing that certain phenomena are frequently conjoined, and attempts to discover some law or principle which will make the fact of their connection intelligible.

It is usual to say that in induction we go from the particular facts to the general law. The following, however, would be a more correct form of statement: Before the inference, we observe that a number of phenomena occur together, but do not know whether this conjunction is necessary or not; or, if we assume that it is necessary, we do not understand why it should be so. As a result of the inductive inference, we gain an insight into the necessary connection of the observed phenomena, and also understand the principle according to which the latter are united. What we really obtain through an inductive inference is not only a general law, but also a perception of its concrete application to particular phenomena. This being so, it is clear that Induction and Deduction are not two different kinds of inference. Inference always implies an effort on the part of the mind to see how phenomena are necessarily connected according to some general principle. And, in carrying out

this purpose, the mind must begin with the knowledge which it already possesses. When the general law of connection is known, and the object is to discover the nature of some particular fact, the method of procedure is deductive. But, when the problem by which we are confronted is to read out of the facts of sense-perception the general law of their connection, the method of inference which must be employed is that of Induction. But, from whatever point we set out, and whatever may be the immediate object of the inference, the result is always the same — an insight into the necessary connection of facts according to some general principle.

It is not unusual to hear the remark made that modern science has been built up by the employment of the inductive method. This must not, however, be interpreted to mean that deductive inferences are not also used in the discovery of scientific truth. Science (which is simply another name for systematic knowledge) is the product of thinking; and thought, as we have seen, is not limited to any one mode of procedure. Thought aims at extending knowledge, and so long as it can find any link of connection, or guiding thread, it is not limited to any one direction, or to any fixed mode of working. It is, of course, to be admitted — and this is the truth in the statement which we have quoted — that general laws cannot be discovered without an examination of particular facts, and that their validity must always be tested by comparison with the facts. But as soon as a general law is discovered in any field, it is always used as a principle from which to deduce new results. When it is possible to employ mathematics in the calculation of these results, it is usually possible to extend our knowledge of the subject much more rapidly than before. Thus physics and astronomy owe their rapid development to

the application of mathematics. It must be remembered, however, that this presupposes a certain stage of advancement — a certain inductive stage, as it were — on the part of the science. But even in this earlier stage we are constantly employing deduction, always reasoning out the results of certain guesses or suggestions to see if they hold true (cf. § 48). Both in ordinary life and in scientific procedure Induction and Deduction are constantly employed together as mutually supplementing each other in the work of organizing experience.

REFERENCES

B. Bosanquet, *Logic*, Vol. II., Ch. I.
F. H. Bradley, *The Principles of Logic*, pp. 430–468.
W. James, *The Principles of Psychology*, Vol. II., Ch. XXII.
J. G. Hibben, *Inductive Logic*, Chs. I. and II.

CHAPTER XXVI

THE UNIFICATION OF KNOWLEDGE

§ 94. **Science and Philosophy.** — Throughout the preceding chapters thinking has been described as the function through which the organization of experience is achieved, or as a process of building up a system of knowledge. It has become clear that the development of thinking involves a continuous increase in both differentiation and integration, and that these two moments or aspects of thought are organically related to each other. An advance in knowledge implies at once new facts and distinctions, and also the perception of new connections and relations among facts. The ideal of completed knowledge, accordingly, would be a system of truths in which the place and meaning of every fact would be completely defined, and where, at the same time, the complete relation of every fact and every group of facts to every other would be fully exhibited. Nothing would then be indefinite for knowledge, and nothing would be isolated; to know things in this completely systematic way would be to see the world steadily and to see it whole.

Like all ideals, this conception is never completely realized in experience as we know it. This, however, does not render it idle or without practical significance. In the first place, it has importance as indicating the direction in which the further development of knowledge must proceed. And, secondly, it is only by reading our actual knowledge in the

light of the end towards which it is progressing that we are able to understand its nature. That is, as stated in the first section of this book, thinking has to be defined as the function, or system of functions, whose end and goal is *knowledge.* Now knowledge is only attained in so far as unification and system are attained: the essence of knowledge is not found in its lack of system and definiteness — these are its defects and privations — but the cognitive experience of any individual has a right to the title of knowledge just in so far as these conditions are realized.

The problem of how a more complete unity of knowledge than that realized in the results of the special sciences is to be attained, thus becomes of the highest importance. We may use the term Science to denote the entire work of discovery and systematization of facts which is carried on by the various civilized nations through successive generations and centuries. In this inclusive sense, Science is undoubtedly one of the greatest achievements of the human race, and one of the highest objects of endeavour for the individual. Within this one body of knowledge, however, it is possible to make various distinctions between different sciences and groups of sciences. The various sciences might be classified, for example, as more or less abstract, or as more or less inclusive in character. Or again, the sciences of nature might be distinguished from the humanistic sciences, which deal with the distinctive products of man's life and thought, as shown, for example, in religious, social, or political institutions, or in art, science, and philosophy. But the division of the complete body of knowledge (Science, *Wissenschaft*) with which we are here directly concerned, is that between the sciences and philosophy. For philos-

ophy is the name given to the endeavour to reach some rational unification of the knowledge derived from the various forms of experience, and especially from the various sciences. "Knowledge of the lowest kind," said Herbert Spencer, "is un-unified knowledge; science is partially-unified knowledge; philosophy is completely-unified knowledge."[1] We may accept this statement with the understanding that of course no knowledge is entirely un-unified, and that, on the other hand, no actually existing system of philosophy can claim to have achieved a complete and satisfactory unification of knowledge.

At the present time the systematic interpretation of the nature of the real world has been divided into various fields of investigation. Each science takes as its subject-matter a definite field, or group, of phenomena and endeavours to describe and explain, as accurately as possible, the facts that fall within that field. Thus, for example, astronomy studies the heavenly bodies with the purpose of making clear and comprehensible their changing phases and relations; botany deals with the various forms and functions of plant life; history describes the significant events which have occurred during the past life of man in society. It is, however, not true that the sciences can be distinguished merely with reference to the nature of the particular field which they occupy. The same body of facts may be dealt with by a number of sciences; or, rather, there are certain more general or fundamental sciences whose principles and results have to be employed in the work of the more special fields of inquiry. In botany, for example, physical and chemical facts and laws are cited in order to render the behaviour of the plant

[1] *First Principles*, § 37.

intelligible. In political economy, in like manner, one has to make constant use of history in the investigations which one undertakes. Nevertheless, even where two or more sciences seem to occupy the same field, it will be found that each has its own special way of reading the facts, so that strictly speaking, the same phenomena are never studied in the same way or with the same purpose in view.

The question to be considered here, however, is the question of the relation of the special sciences to philosophy. It might appear at first sight as if the whole field of reality were occupied — or soon to be occupied — by the various sciences and that no problem were therefore left for philosophy. But the very fact that each science is obliged, in order to render its investigations definite and fruitful, to limit the field of its inquiry, makes necessary some attempt to bring the results derived from the different fields into relation. And, as will appear more clearly in the last section of this chapter, to correlate the results of these different scientific inquiries, which are gathered with various purposes, and often by the employment of quite different hypotheses, is not merely to set them side by side. The work that philosophy is called upon to undertake is to interpret these results in such a way as to render them coherent and thinkable. Philosophy aims at unifying knowledge by finding a conception or set of conceptions which will enable us to think the world as some kind of a consistent system. It seeks to satisfy our demand for a world-view, a *Weltanschauung*. When we take the widest and most accurate survey within our power of the facts of experience, what conclusions are we warranted in drawing regarding the whole system of things of which we are a part?

In attempting to find an answer to this most practical question, it is of course necessary to take account of every well-authenticated form of experience, and to give to each its proper place and value. It is obvious, too, that the problem is the final problem of knowledge, and one that cannot be finally and fully solved by any individual or by any generation. But that is not a reason for abandoning it as insoluble. In the first place, it is a problem to which human reason from its very nature can never be permanently indifferent. It is only the animals, Hegel remarks, who are not metaphysicians. It is true that the majority of men never apply themselves directly to the solution of ultimate philosophical questions; but every one holds, more or less consciously, and in more or less definite form, some conception regarding the nature of the world and his own place in it. It is perhaps most frequently from theology or from literature that men derive their world-view, and they hold this, not as a reasoned system of knowledge, but rather through belief in authority, or on emotional or æsthetic grounds. As distinguished from constructions of this character, philosophy aims at a reasoned system. Like the sciences, it discards both emotion and tradition as guides, and proceeding by means of careful analysis and definition, it subjects all hypotheses to rational criticism. Its postulate is that there is nothing irrational, or from its very nature incomprehensible, in the nature of the world. It is true that science and philosophy will never complete the work they are carrying on: the results arrived at are never final, but only starting-points for new investigations. But in the one case as in the other, the road is never barred; progress is always possible if the problem is formulated in an intelligible way.

Two considerations, which are frequently overlooked, follow from the conception of philosophy as a construction that is being continuously achieved by the human race in the course of its history. The first is, that it would be idle for any individual to begin the work anew on his own account, refusing to learn or to profit by the labours of the past. And secondly, it is obvious that from the nature of the case there will be, as long as the human race endures, no ultimate or finally complete system of philosophy. When it is remembered that philosophy is the completion of the sciences, that the philosophical problem is the final problem of knowledge, the fact that neither the foundations nor its outlines are yet finally determined will not appear either strange or discouraging.

§ 95. **Science as Philosophy.** — In this connection, the question arises whether the conceptions employed by the sciences are not themselves capable of effecting a final unification of knowledge. Why is it necessary to turn to philosophy, or if the name of philosophy is still used to denote the most comprehensive science, why should not the ultimate account of reality be given in the same terms as the descriptions of the special sciences? Why, in short, not accept as philosophy the general standpoint and results of the sciences?

As a matter of fact, this is often done. During the last two centuries — and more particularly during the last century — the greatest advances in knowledge have been attained in the field of the natural sciences. As a consequence, it has been natural to assume that the same success may be attained everywhere by employing the same unifying conceptions in the solution of all kinds of problems.

Now the fundamental categories with which natural science operates are those of Quantity and Causality. The latter conception, when used exactly or scientifically, includes the former, as has appeared in our study of Induction (cf. pp. 255 ff.). The assumption on which the causal category proceeds is that reality is composed of phenomena which are external to one another and at the same time dependent on one another. Every phenomenon is at once a cause and an effect. There obtain everywhere unvarying laws of connection between all events so that nothing can be thought of as happening except in one determinate and fixed way. For every phenomenon a cause must be sought in some other phenomenon, or group of phenomena, and thus everything is determined or conditioned, both as to its existence and nature, by something external to itself. External determination, or conditioning through something external to that which is to be explained, is thus the form of relation employed by natural science in its work of unifying knowledge.

Now in attempting to interpret the entire world as a series of phenomena which are everywhere related in terms of cause and effect, one might be either more or less thoroughgoing and consistent. On the one hand, one might assume that the complete unification of knowledge demands that there shall be only a single series of causes and effects. This would imply that all the phenomena of which the world is composed are at bottom reducible to the same terms, and are all manifestations of some one material, or one principle. Or, on the other hand, it might be assumed that there is more than one series and more than one fundamental principle involved in the nature of things. The first view would be Monistic and the second Pluralistic. I have said

that the first is more thorough-going, because Pluralism still has to face the problem as to the relation of the different forms of existence which it assumes. What is it that unites the plural forms of existence into a single world, a universe?

Without entering into the arguments in support of either Monism or Pluralism, however, we may illustrate the application of the causal point of view when employed under either assumption. Let us first assume that everything in the universe, without exception, can be reduced to some physical principle. It is indifferent for our illustration whether that elementary term be regarded as matter or energy, so long as it can be measured and its exact results calculated. The place of philosophy and of all the sciences would then be filled by a universal system of physics which would be able to describe and explain all forms of existence and all changes in terms of its own principle. Not only that, but since it would deal with a strictly determined and calculable series of events, it would, theoretically at least, be able to predict all occurrences of the future, both mental and physical alike, down to the smallest detail. More than a century ago Laplace wrote: "We ought then to regard the present state of the universe as the effect of its antecedent state, and as the cause of the state that is to follow. An intelligence who for a single instant should be acquainted with all the forces by which nature is animated, and with the several positions of the beings composing it, if further his intellect were vast enough to submit these data to analysis, would be able to include in one and the same formula the movements of the largest bodies in the universe, and those of the lightest atom. Nothing would be uncertain for him;

the future as well as the past would be present to his eyes.' If now, as we are assuming, all phenomena are in the last resort reducible to some physical principle, Laplace's hypothetical calculator would also be omniscient with respect to all the contents of every mind that ever existed or will exist.

The assumption that mental phenomena are at bottom physical in character — special forms of matter or energy — may, however, appear to us untenable. It may appear that experience compels us to take a Dualistic hypothesis, and to assume that mental events form an independent series. Nevertheless, since we are still at the standpoint of natural science, we shall find in this field also causes and effects. This, indeed, is what we must assume if the natural science methods of description and explanation are to be employed in psychology. So long as psychology sets itself the task of explaining mind as made up of a causally connected series of events, so long must every mental state be regarded as capable of explanation in terms of some antecedent process or processes. There can be no state that is not determined or conditioned by something outside itself.

Now there is, perhaps, at first sight nothing repugnant in a philosophy which interprets the external world as a strictly mechanical series of causes and effects. Further consideration might, indeed, make it apparent that, if this is the ultimate truth regarding the physical world, the mental life, through its close and necessary connection with the physical, cannot possess any real freedom. It is, however, with the natural science account of the mental life itself that discontent first arises. The physical world, we are likely to feel, may be mechanical: indeed, the mechanical

view at first sight seems fairly adequate to state what we know regarding its behaviour. But the psychological interpretation of mind as made up of phenomena which are all conditioned externally, conflicts directly with our ordinary beliefs regarding our own conscious life, and that of our fellow-men. If the causal account of mind is ultimate, there can, of course, be no freedom or self-determination on the part of the individual: the mind is simply the consciousness of a succession of states which are strictly determined in the order and mode of their appearance. It seems extremely difficult to reconcile this interpretation of the mental life with what we demand from ourselves and others in the life of society, as well as with the sympathy and interest that we have with motives and acts of historical individuals. The scientific view of mind, as made up of elements which are conditioned in a purely mechanical way in their mode of combination, necessitates a fundamentally different view of human conduct and of human responsibility from that usually entertained: it requires us to regard our own conduct and that of our fellow-men, not as subjects of praise or blame, but simply as phenomena to be understood. This is the philosophy of mind and of human action at which we arrive when the scientific point of view is regarded as the final interpretation.

96. **The Assumptions of the Sciences.** — The possibility of reaching a different interpretation of the world and of experience from that afforded by natural science has been more than once suggested in the preceding paragraphs. It has never been shown, however, that any other interpretation is possible. If the account given by the sciences is true, how is any other theory possible? Does not a phi-

losophy lose all title to respect which begins by proposing to discredit the established results of the sciences?

The reply to these objections is that it is not proposed to question the competency of science in its own field; but merely to show that, from the very conditions under which it is formulated, it cannot supply an answer to the problems of philosophy. In the first place, the inquiries of the special sciences are not directed primarily towards the discovery of the ultimate relations of things. Their object is rather to discover some method of describing certain groups of phenomena in such a way as to enable others to apprehend them readily and clearly. With this, there is also usually connected the practical purpose of determining how the phenomena can be produced most conveniently, or modified in the directions we may desire. Each of the special sciences, in other words, takes the point of view and employs the conceptions which will enable it to describe most conveniently, in accordance with its own purpose, the group of phenomena which constitutes its subject-matter. The various conceptions employed have sometimes been compared to instruments or tools which enable the sciences to attain the results at which they aim; namely, a systematic description and correlation of facts. The test of these conceptions from the scientific point of view is found simply in their efficiency, or in their capacity to afford a basis for clear description and for practical manipulation of the phenomena under investigation. "All physical ideas and principles," says Mach, "are succinct directions, frequently involving subordinate directions, for the employment of economically classified experiences, ready for use. Their conciseness, as also the fact that their contents are rarely exhibited in

full, often invests them with the semblance of independent existence."[1] Moreover, what is here said regarding the ideas and principles of physics applies equally to the other sciences: in no case are the conclusions derived by employing the methods and assumptions which a special science finds adequate for its purpose to be accepted without modification or interpretation, as a direct description of the nature of reality.

The matter may be put in the following way. All thinking proceeds on the basis of certain assumptions. The most general form of these assumptions is expressed by the so-called laws of thought as a postulate that the various facts that make up our world of experience are to be related in a coherent and systematic way. Now, there are various ways, more or less adequate, and more or less final, of thinking the relations of things. Although each of the natural sciences makes the special assumptions which enable it to deal most effectively with the facts in its own field, so that its form of explanation always differs in some respect from every other, yet all the natural sciences make certain assumptions in common, and therefore may be at first considered together in our discussion. The general nature of these assumptions we found expressed in the law of causality with its corollary, the conception of the uniformity of nature. It is plain, therefore, that the conclusions of all these sciences is in a sense hypothetical, rather than categorical. What they assert is, that *if* the field of reality is defined as composed of phenomena external to one another, but standing in strict causal relations, *then* these laws and conceptions appear to express, more adequately than any other, the relations of the facts when read from that point of

[1] *Popular Scientific Lectures*, p. 204.

view. Statements of this nature are obviously not intended to be absolute, or to exclude alternative ways of interpreting the facts. The character of the results is evidently conditioned by the initial assumptions of the whole group of natural sciences.

This point of view may be further illustrated by considering its application to some of the special sciences. Mathematics does not belong to the causal group of sciences whose assumptions we have considered; but its hypothetical and abstract character is not difficult to realize. The subject-matter of mathematics, we say, is not any actually existing set of phenomena, but certain ideally simplified forms or relations of the real world. The straight line, or the triangle, for example, as defined by geometry, are ideal conceptions, or hypotheses, from which the science proceeds to deduce the consequences. These consequences are not taken as direct descriptions of the physical world, though they illustrate certain phases or aspects of that world. If now we turn to physics, we find that the character of the results is determined in the same way, though not to the same degree, by the initial definitions and hypotheses of the science. In order to be able to deal with the changing and almost infinite variety of the physical world, it is necessary to adopt and carefully define certain conceptions, such as space, time, energy, atom, ether, etc. It then becomes the problem of physics to represent all the manifold phenomena of the external world as determinate relations between these conceptions. The choice of these conceptions is determined by their capacity to correlate facts, and to serve as instruments of investigation. In the progress of the science, a constant working over and redefining

of the working hypotheses goes on, the attempt being always to reach conceptions that will be more effective as instruments of investigation, and at the same time permit of the description of phenomena in more concrete terms.

It follows, then, that the working conceptions of physics, no more than the working conceptions of mathematics, are exact descriptions of concretely existing things. They are ideally simplified conceptions, adopted and defined as effective instruments for dealing with the physical world for certain purposes. These conclusions are at the present time recognized by physicists who are interested in the logical interpretation of their results as well as by philosophers. A parallel to the following passage from Mach may readily be found in the writings of many other writers: "When a geometer wishes to understand the form of a curve, he first resolves it into small rectilinear elements. In doing this, however, he is fully aware that these elements are only provisional and arbitrary devices for comprehending in parts what he cannot comprehend as a whole. . . . Similarly, it would not be right for physical science to regard its self-created, changeable, economical tools, — the molecules and atoms — as realities behind phenomena . . . the atom must be regarded as a tool for representing phenomena, like the functions of mathematics. Gradually, however, as the intellect, by contact with its subject-matter, grows in discipline, physical science will give up its mosaic play with stones and will seek out the boundaries and forms of the bed in which the living stream of phenomena flows. The goal which it has set itself is the simplest and most abstract expression of facts."[1]

[1] *Op. cit.*, p. 206.

In the opening chapter of this book (p. 9) it was stated that at the present time an important difference of opinion exists as to the proper standpoint and working conceptions of the science of psychology. Functional psychology attempts to employ the principle of purpose or adaptation as its principle of analysis and explanation. Structural psychology, on the other hand, following more strictly the methods of the other natural sciences, describes and explains mental life in terms of causally related elements. It is this latter view of mind, when accepted without modification as a philosophical theory, to which attention is called in the last section. When we reflect on the meaning of the results obtained by the natural science method of procedure in psychology, it becomes evident that these cannot be regarded as furnishing a final or categorical account of the real character of the mental life. For, as in the case of physics, their form is due to the nature of the assumptions adopted by the science. The concrete mental life, as we know it in our experience, is a life directed more or less consciously, and more or less consistently, to the attainment of certain ends. To live as conscious beings means to have purposes, to will certain results, and employ ourselves in such a way as to bring about their attainment. Without the conception of the mind as a system of functions, engaged in realizing certain ends, mental life appears unmeaning, both from the standpoint of ordinary experience, and also from that of sciences like history, ethics, and logic. Now, psychology undertakes, in the interest of exact description, to exhibit this mental life as a series of causally conditioned phenomena, possessing certain definitely ascertainable characteristics, and taking place in accordance

with certain laws. In looking at the mental life as made up merely of a series of states to be described, psychology necessarily has to abstract from the function or work of mind. It deals only with a certain phase of mind; or it may be said that its results are true only of one side or aspect of the total mental life. As science, its results are true and satisfactory if they adequately fulfil the purposes of the psychologist. It is only when they are mistaken for philosophy that they become false and misleading. Psychology, as Professor Muensterberg has remarked, "must not be transformed into Psychologism. In the preface to his book entitled *Psychology and Life* the same author writes: "Popular ideas about psychology suggest that the psychological description and explanation of mental facts expresses the reality of our inner experience. It is a natural consequence of such a view that our ethical and æsthetical, our practical and educational, our social and historical views are subordinated to the doctrines of psychology. These papers endeavour to show that psychology is not at all an expression of reality, but a complicated transformation of it, worked out for special logical purposes in the service of our life. Psychology is thus a special abstract construction which has a right to consider everything from its own important standpoint, but which has nothing to assert in regard to the interpretation and appreciation of our real freedom and duty, our real values and ideals."

§ 97. **Philosophy as the Interpretation of the Sciences.** — The work of philosophy is, however, not fulfilled in simply showing that there is no finality in the conclusions of the special sciences: there is still demanded, as we have seen, such an interpretation of the various facts of experience as

will render possible some coherent view of the nature of the world as a whole. Now into this construction the results of the special sciences must, in some way, enter. These results, as has been shown, are hypothetical, abstract, and incomplete in character, but they are not arbitrary or capricious. Although they cannot be taken as directly or categorically descriptive of concrete things, scientific propositions do illustrate certain general phases or aspects of both physical and mental experience, and are therefore significant for philosophy. To understand what they really assert, then, it is essential to comprehend the limitations and conditions which the postulates of the field from which they are derived impose upon them. It is only by making clear their assumptions that their true import and significance can be brought to light.

When this has been done, the further problem will remain as to what category or conception is most adequate to express the relations of all of the various parts of the world of experience. What is the highest or final category of thought which will prove adequate to the complete unification of knowledge? It is clear that the conception which philosophy aims to define would only be adequate, if it included within itself, as relative or partial truths, the results obtained by the investigations of the other sciences. Or, in other words, while each of the special sciences, limiting itself, as it must, to the investigation of a particular part of the world, is never able to obtain the full and final truth about that part, its results are never without significance for an ultimate synthesis. Indeed, it is only by making use of the work of the sciences that philosophy is able to advance to a more comprehensive interpretation. On the other hand,

the sciences are aided in their work of interpreting special groups of phenomena by philosophical conceptions regarding the meaning and bearing of their special results. The ultimate postulate of our thought being that the universe is systematic and coherent, the part can only be fully comprehended when it is seen in the light of the whole.

The history of philosophy may be read as an account of the attempts made by the human race to find a conception, or category, adequate to unify all the facts of experience. Beginning with the childlike idea that everything must be composed of the same kind of stuff or matter, philosophical thought quickly advanced to more rational statements of its problem. At the present time, one may perhaps say that the fundamental question in philosophy is whether it is possible to employ the category of Teleology or Purposiveness as an explanation of the universe and of our own experience; and, if so, what content is to be given to this conception. We have noted the fact that an explanation in causal terms leads necessarily to an infinite regress (cf. p. 371), as well as the other difficulties that arise when this category is taken as ultimate. The question then is: Are we justified in advancing to a different form of judgment, to judgments of Teleology or Individuality? (Cf. p. 370.) If this question be answered in the affirmative, it is above all essential to remember that a change of category is no excuse for indefiniteness. Philosophical analysis and interpretation are necessarily different from those of science, but philosophical procedure must not be less strict than that of the sciences, or its conceptions less carefully defined.

REFERENCES

H. Muensterberg, *Psychology and Life*, 1899.

" " *The Eternal Values*, 1909.

James Ward, *Naturalism and Agnosticism*, 2d edition, 1903, especially Lectures II.–VI. and XIX.

E. Mach, *Popular Scientific Lectures*.

J. E. Creighton, "Methodology and Truth," *Philos. Review*, Vol. X., pp. 45 ff.

E. Albee, "The Significance of Methodological Principles," *Philos. Review*, Vol. XV., pp. 267 ff.; "Descriptive and Normative Sciences," *Ibid.*, Vol. XVI., pp. 40 ff.

QUESTIONS AND EXERCISES

INTRODUCTION

Chapter I. — *The Standpoint and Problem of Logic*

1. What are some of the main characteristics of thought or thinking? Explain the distinction between a subjective and an objective account of thought.

2. Explain the use of the verb *to think* in each of the following sentences: 'I do not know, but I think so'; 'If you think the matter over, you will come to the same conclusion'; 'I was not thinking.'

3. 'Words and phrases are often repeated without reflection, and their very familiarity is likely to prevent us from attempting to understand exactly what ideas they represent.' Give illustrations of this fact.

4. What do you mean by science? How does 'scientific' knowledge differ from the knowledge of ordinary life?

5. What is the meaning of the word 'law' in the phrase 'a law of thought'? Compare with this the use of the word in such expressions as 'laws of nature,' 'the laws of the land.'

6. Discuss the ways in which logic and psychology respectively deal with thought.

7. Illustrate by means of examples not used in the text the relation in which science and art, or theory and practice, stand to each other.

8. In what sense does the study of Logic have 'practical' value? Discuss the case of the man who has not studied Logic and yet reasons correctly.

9. In what sense is Logic a 'normative' science? Name other sciences that are 'normative.'

10. 'A great library contains the diary of the human race.' How does this statement suggest the connection between Logic and all other branches of knowledge?

11. What parts do the study of history and introspection play respectively in the study of Logic?

Chapter II. — *Stages in the Development of Logic*

1. What form did the questions concerning the nature of knowledge first take? Under what conditions did these first receive definite formulation?

2. 'The sciences have arisen in response to the practical needs of mankind.' Is this statement confirmed by the history of the origin and development of Logic?

3. 'Since each individual sees things from his own point of view, there is therefore nothing really true in itself, or good in itself.' Give some illustrations of the former part of this statement. What term would you use to describe the theory which the sentence expresses?

4. Explain what is meant by the statement that Socrates and Plato found a standard of truth and of conduct in the Concept.

5. Why was it not possible for Aristotle to lay down a complete theory of Inductive Reasoning?

6. Describe the attitude toward Logic during the Middle Ages. How can this be accounted for?

7. What is meant by Bacon's 'method'? In what does its value consist?

8. Describe Mill's services to Logic, also the defects in his view of experience.

9. Describe the standpoint of Modern Logic.

PART I — THE SYLLOGISM

CHAPTER III. — *The Syllogism and its Parts*

1. Describe the general purpose and nature of the syllogism.

2. What is the principle upon which syllogistic reasoning depends? Why is it impossible to reason if this principle be violated?

3. What is meant by calling Logic a 'formal' study?

4. Explain the distinction between a Percept and a Concept.

5. What is meant by the transforming and the conserving functions of thought? What part does language play in the process of thinking?

6. Arrange the following sentences as logical propositions, pointing out the logical Subject and the Predicate in each case: —

(*a*) Error cannot be defended but by error.
(*b*) The wise learn many things from their foes.
(*c*) Time and tide wait for no man.
(*d*) Some to the fascination of a name surrender judgment hoodwinked.

7. In the following examples, the student is required (1) to state the arguments in syllogistic form, rearranging them if necessary in the order of Major Premise, Minor Premise, and Conclusion; (2) to supply missing premises or conclusions, or to condense, where several statements really constitute one proposition; (3) to state whether the argument seems to be valid: —

(1) He is not indifferent to money; for he is a sensible man, and no sensible man despises money.
(2) All human productions are liable to error, and therefore all books, being human productions, are liable to error.

(3) It is not true that wealth is the only thing that affords satisfaction, for a good conscience is not wealth.

(4) All bodies which move round the sun are planets; therefore the earth is a planet.

(5) Platinum is a metal, and therefore combines with oxygen.

(6) Every honest man attends to his business; this person attends to his business; therefore this person is an honest man.

(7) Rational beings are accountable for their actions; brutes, not being rational, are therefore exempt from responsibility.

(8) A system of state pensions in old age is an evil because it discourages thrift and independence.

(9) Of course he defends State Rights, for he is a Southerner.

(10) The poor must be oppressed, for the rich are accumulating millions.

(11) These men are traitors, for they oppose the President.

(12) It cannot be said that no impractical man is a politician, for some politicians are idealists, and no idealist is practical.

(13) Phenomena attributed by savages to the existence of spirits are fully capable of explanation by science without this hypothesis, therefore the hypothesis ought to be entirely discarded.

(14) Neglect of pleasure is the best way to secure it; for the more we aim at pleasure, the less likely we are to get it.

(15) Restless nations are not progressive, for we see that the civilized nations are all progressive, while all the uncivilized nations are restless.

(16) Any citizen may rightly resist a law of which his

reason disapproves, for every man is in duty bound to follow the dictates of his reason.

(17) More books are written directly after a war for freedom than at any other time because people are then vitalized by a more active contact with the minds of others.

(18) Every one desires money, because every one desires power.

(19) We must not give in to him, for if you give him an inch, he will take an ell.

(20) Covetous men are not happy, seeing that they are always in fear.

(21) The example of Virgil shows that a great poet may be led into some faults by the practice of imitation.

(22) You must have met him, for you were at the university at the same time.

(23) No strike but injures trade, and consequently impoverishes the country. But this is to diminish the means of happiness. And as all that is detrimental to happiness is to be condemned, we must absolutely condemn strikes.

(24) This document cannot be genuine, or it would have been referred to by the supposed author's contemporaries.

(25) This is too good to be true.

(26) It is inconceivable that the material world should be perceived, since we can only perceive that which lies within consciousness.

(27) A great chess-player is not a great man, for he leaves the world as he found it.

(28) It is a mistake to improve the economic condition of the inefficient, so we ought not to assist the destitute.

(29) B is so bad a marksman that the safest place to stand is directly in front of the bull's-eye. (12–29 St. Andrew's.)

(30) He is already rich and powerful, so that he cannot be guilty of usury and extortion.

(31) Years bring wisdom, but you are still young. (30–31 Glasgow.)

(32) The man I don't like is the man I don't know.

(33) No human being in this country can exercise any kind of public authority which is not conferred by law; and under the law of the United States it must be given by the express words of a written statute. Whatever is not so given is withheld, and the exercise of it is positively prohibited. Courts-martial in the army and navy are authorized; they are legal institutions; their jurisdiction is limited, and their whole code of procedure is regulated by act of Congress. Upon the civil courts all the jurisdiction they have or can have is bestowed by law, and if one of them goes beyond what is written, its action is *ultra vires* and void. But a military commission is not a court-martial, and it is not a civil court. It is not governed by the law which is made for either, and has no law of its own. . . . So these commissions have no legal origin and no legal name by which they are known among the children of men; no law applies to them, and they exercise all power for the paradoxical reason that none belongs to them rightly. (J. S. Black.)

Chapter IV. — *Terms*

1. Distinguish in the following list the terms which are usually (1) Singular, (2) General, and (3) Collective. If any

term may belong to more than one class, explain and illustrate its various uses.

the country club	organism	humanity
law	school board	Candlemas
window	the summer solstice	society
fruit	the river Dart	a straight line
the army of occupation		

2. Explain and illustrate the ambiguity in the use of the word 'all.'

3. In what two ways are the words Abstract and Concrete used? In what sense, if at all, can we say that Psychology and Logic are 'abstract' sciences?

4. What do you think that Hegel meant when he said that "it is the uneducated man who thinks abstractly"?

5. Distinguish carefully between Contradictory and Contrary terms.

6. What are Correlative terms? Give at least three examples.

7. Mention the synonyms for Intension and Extension.

8. Explain the Extensional and Intensional use of the following terms: —

person	book	idea
mountain	force	friend
university	the Renaissance	the judicature
government	state	church
nationality		

9. Criticise the statement that 'Extension and Intension stand in inverse ratio to each other.' What truth does it contain?

Chapter V. — *Definition and Division*

1. Discuss the function of definition as a remedy for ambiguity.

2. What is the distinction between extensive and intensive definition? What is a verbal definition?

3. In what two ways may we conceive the problem of Definition?

4. What do you understand by the Socratic Dialectic? Explain its purpose and mode of procedure.

5. Explain the terms: —

genus	differentia	infima species
species	summum genus	sui generis

6. What various methods or kinds of definition can you distinguish? What is it which determines which method shall be used in any particular case? What is *genetic definition?*

7. Criticise the following definitions, pointing out what rules, if any, are violated by them, and distinguishing genus and differentia, if possible, in each: —

(1) Memory is the tablet of the mind.

(2) Sodium is an element exhibiting line D in the spectrum.

(3) Tin is a metal lighter than gold.

(4) A utilitarian is one who does not believe in an intuitional basis of morals.

(5) Justice is the health of the soul.

(6) Injustice is the not keeping of covenant.

(7) Duration is a temporal slab of nature.

(8) A moral man is one who does not lie or steal or live intemperately.

(9) A star is a stellar body seen in the heavens at night.

(10) Evolution is to be defined as a continuous change from indefinite incoherent homogeneity to definite coherent heterogeneity of structure and function, through successive differentiations and integrations. (Spencer.)

(11) Oats is a grain which in England is generally given to horses, but in Scotland supports the people.

(12) Tickling may be defined as an intensely vivid complex of unsteady, ill-localized, and ill-analyzed sensation,

with attention distributed over the immediate sensory contents and the concomitant sensations reflexly aroused.

(13) *Panmixia* is the fact that "natural selection is required to preserve an organ in an active condition as well as to produce it, and if this action is withdrawn, the organs will degenerate from promiscuous breeding."

(14) Belief is the consequence of an indissoluble association of ideas.

(15) Wisdom is the avoidance of folly.

(16) Reverence is the feeling produced by the recognition of worth or superiority in others.

(17) Religion consists in the feeling of absolute dependence. (Schleiermacher.)

(18) Religion is the sentiment aroused by regarding duty as based on a divine command. (Kant.)

(19) Religion, in its lowest terms, is the belief in spiritual beings. (Tylor.)

(20) Humor is an imitation of the natural or acquired absurdities of mankind, or of the ludicrous in accident, situation and character. (Hazlitt.)

(21) Humor is wit and love. (Thackeray.)

(22) Humor is the oil and wine of merry meeting. (Irving.)

(23) Rent is what is paid for the license to gather the produce of the land. (Smith.)

(24) Rent is that portion of the produce of the earth which is paid by the farmer to the landlord for the use of the natural and inherent powers of the soil. (M'Culloch.)

(25) Rent is the difference between the return made to the most productive, and that which is made to the least productive, portion of capital employed on the land. (Mill.)

(26) Rent is the income, derived from the ownership of land and other free gifts of nature. (Marshall.)

(27) Vestigial characters in animals are the remnants of past adaptations.

8. Give examples of terms which are indefinable, and explain why this is the case. What is the distinction between Description and logical Definition?

9. Discuss the possibility and logical desirability of framing a final and complete definition of anything.

10. Define the following terms by giving the genus and differentia: —

family	corporation	ether	sociology
ballad	robin	tariff	stalactite
parallel lines	tort	image	

11. Define the following terms in whatever way seems most suitable and satisfactory: —

mechanism	journal	garden	rhombus
parallax	function	primaries	spring-pin
anarchy	charity	cause	
dogma	edema	orthodoxy	

Can any of these be defined in more than one way?

12. It is said that definitions deal only with words, which are merely conventional signs. Yet the construction of a definition may be a great step in science. Would this be possible if the first statement were strictly true?

13. Examine the following Divisions and point out which are logical and which are not: —

(1) Men into imaginative and unimaginative.

(2) Theories into true and false.

(3) Schools into technical, preparatory, professional, and scientific.

(4) Books into bound and unbound.

(5) Soldiers into artillery, cavalry, privates, and volunteers.

(6) Orchards into those that grow bush-fruit, tree-fruit, and vines.
(7) Experiences into pleasant and painful.
(8) The fine arts into sculpture, painting, drawing, architecture, poetry, and photography.
(9) Plants into poisonous, and non-poisonous.
(10) Poems into epics, sonnets, ballads, lyrics, and didactic poems.

Chapter VI. — *Propositions*

1. Distinguish between a logical Judgment, a logical Proposition, and a grammatical Sentence.

2. (*a*) Distinguish between Categorical and Conditional propositions. (*b*) Do categorical form and categorical import always coincide?

3. What is meant by (*a*) the Quality, and (*b*) the Quantity, of propositions?

4. Arrange the following sentences in the form of logical propositions, and indicate the Quality and Quantity of each categorical proposition by the use of the letters A, E, I, and O:—

(1) To be great is to be understood.
(2) There are people whom nothing would content.
(3) Socrates declared knowledge to be virtue.
(4) Phosphorus does not dissolve in water.
(5) Nearly all the troops have left the town.
(6) Only ignorant persons hold such opinions.
(7) Few, save the poor, feel for the poor.
(8) Over the mountains poured the barbarian horde.
(9) Fine words butter no parsnips.
(10) Logic is only common sense formulated.
(11) General notions are generally wrong.
(12) The fewer, the better.

5. How does formal logic interpret the relation between the subject and predicate of a categorical proposition? Does this view do full justice to the signification of propositions?

6. How would you represent by means of circles the proposition, 'gold is the most precious metal'?

7. What do you mean by the distribution of terms? Explain why negative propositions distribute the predicate, while affirmative propositions do not.

8. State precisely what is asserted by Proposition I. What forms may the diagrams which represent this proposition assume?

Chapter VII. — *The Interpretation of Propositions*

1. Does the interpretation of propositions involve the process of inference?

2. What is meant by the Opposition of propositions?

3. Explain the distinction between Contrary and Contradictory propositions.

4. If proposition O is false, what is known regarding the truth or falsity of A, E, and I?

5. What is the simplest proposition which must be established in order to disprove the following statements: (*a*) All men desire wealth. (*b*) No man is perfectly happy. (*c*) Some knowledge is not of any value. (*d*) Pain alone is evil. (*e*) All is not lost.

6. Give the contrary (or sub-contrary), and the contradictory of: (*a*) All metals are elements. (*b*) No coward need apply. (*c*) Socrates was the wisest man in Athens. (*d*) Not all men are brave. (*e*) No man but a traitor would have done this.

7. Give the Obverse and, in the cases where it is possible, the Inverse, of the following propositions: —

(1) He that makes haste to be rich is not innocent.

(2) Everything that is wise has been thought already.

(3) The patient who makes the doctor his heir is not likely to recover.
(4) No person who is not a great sculptor or painter can be an architect.
(5) That man sins charitably who damns none but himself.
(6) Many who speak much do not speak well.

8. Convert in at least one way: —
(1) All men are rational.
(2) In all forms of government the people is the true legislator.
(3) Some metals are readily fusible.
(4) Witty men are seldom humorous.
(5) Only philosophers fail to see the difference between a post and my idea of a post.
(6) All knowledge is only recollection.
(7) All men are not liars.
(8) Any one but an idiot would believe it.

9. Give the contrapositive of the following propositions: —
(1) All bullies are cowards.
(2) Not all who came to scoff remained to pray.
(3) Whatever is not metallic is not capable of magnetic influence.
(4) All acids are substances that turn blue litmus-paper red.
(5) A triangle is a figure bounded by three straight lines.

10. Describe the logical relations of the four following propositions: —
(1) All substances which are material possess gravity.
(2) No substances which possess gravity are immaterial.
(3) Some substances which are immaterial do not possess gravity.
(4) Some substances which do not possess gravity are immaterial. (Jevons.)

11. What is the Obverse of the Converse of, 'None of the planets shine by their own light'?

12. Can we logically conclude that because heat expands bodies, therefore cold contracts them? (Jevons.)

13. What is the logical relation, if any, between the two assertions in Proverbs xi. 1, 'A false balance is an abomination to the Lord; but a just weight is his delight'? (Jevons.)

Miscellaneous Exercises in Propositions

In the case of each of the single propositions following, it is suggested that the student first state it in strict logical form, classifying it as A, E, I, or O, and then give, in the order named, its Contrary (or Sub-contrary), Contradictory, Subaltern (or Superior), Converse, Obverse, and, in case it has any, Contrapositive and Inverse.

The other questions are self-explanatory.

1. All are not wise who read much.
2. A noisy man is ever in the right.
3. Brute facts exist only for the brute.
4. No scientific advance can take place without dialectic.
5. It is not given to every one to rise to the highest position in life.
6. No one who is truly disinterested pursues ambitious ends.
7. Democracy is distrust of greatness.
8. Some men are fleet of foot.
9. All men are at times actuated by unselfish motives.
10. Theologians are far from unanimity in their attitude toward religion.
11. All lawyers are not formalists.
12. Sorry her lot who loves too well.
13. Public Credit means the contracting of debts which a nation never can pay.

14. Examine the following argument: —

If proposition O be true, I may be true; if I may be true, A may be true: ∴ if O be true, A may be true. (St. Andrews.)

15. All probable events are possible.

16. All parallel lines are lines which do not meet.

17. No one who is not a taxpayer can vote in this election.

18. What can't be cured must be endured.

19. All bacteria are not harmful.

20. Whatever is, is right.

21. Some citizens are not eligible to the presidential office.

22. Four years of study is required for a degree.

23. A point has no magnitude.

24. Conscience is capable of becoming more than the handmaid of the law.

25. Philosophy bakes no bread.

26. Does the second of these propositions follow from the first, and, if so, what is the logical relation between them?

(*a*) Things equal to the same thing are equal to each other.

(*b*) Things not equal to each other are not equal to the same thing.

27. No wise man runs into danger needlessly.

28. All warm-blooded animals are air-breathers.

29. Some criminals are well-educated men.

30. No triangle has one side equal to the sum of two others.

31. No news is good news. (St. Andrews.)

32. Only those who have never felt a wound jest at scars.

33. Assuming that 'All monochromatic light is coloured,' what can you conclude as to the truth or falsity of the following propositions, *monochromatic* and *mixed*, and *coloured* and *white*, being contradictories?

(*a*) No mixed light is coloured.

(*b*) Some coloured light is not mixed.

(*c*) All coloured light is mixed.

(*d*) Some white light is monochromatic.

(*e*) Some mixed light is not white.

34. If 'All who are happy are wise,' does it follow that 'All who are foolish are unhappy'? (Glasgow.)

35. Few men can resist temptation.

36. A few are always true to their task.

37. Criticise the following:—

Granted that it is true that,

All wise men are mortal,

then, No wise men are immortal,

and, No immortal beings are wise men.

Hence it is false that,

Some immortal beings are wise men,

and that, Some immortal beings are not unwise men.

But if this is false, it must be true that,

All immortal beings are unwise men,

and that, Some unwise men are immortal beings.

38. If the failure of apes to use weapons of defence is indicative of lack of intelligence, may we fairly conclude that where weapons are used intelligence exists?

39. What *general* propositions are implied in the following?

(*a*) 'Johnson said of Somerville that he wrote very well for a gentleman.'

(*b*) 'A maiden of our century, yet most meek.'

40. 'No psychosis without neurosis; no neurosis without psychosis.' Does the truth of the first half of this statement involve that of the second?

41. Where an opinion is general, it is usually correct.

42. There is no power on earth which setteth up a throne or chair of state, in the spirits and souls of men, but knowledge and learning.

43. Give the contradictory of 'Few men reason'; the ob-

verse of 'I shall not all die'; and the partial and full contrapositives of 'Only social ends are ultimately reasonable.'

44. State the relation between the three propositions contained in the following sentence: 'The voluntary muscles are all striped, and the unstriped muscles are all involuntary, but a few of the involuntary muscles are striped.'

45. If it is true that there is 'No faith without works,' does it follow that the doing of works proves that faith is present?

46. State the relation between

(*a*) Good men are wise.

(*b*) Unwise men are not good.

(*c*) Some unwise men are good.

(*d*) No good men are unwise.

47. You cannot be just, if you are not humane.

48. Few of us are not in some way infirm.

49. All our mistakes are not borne by ourselves.

50. Only the young prefer bravado to experience.

51. Only the impartial reason.

52. All seeds do not contain albumen.

53. Few candidates were satisfactory.

54. The burnt child dreads the fire.

55. No one who presented himself failed to pass.

56. Only the wise are prudent.

57. A friend in need is a friend indeed.

58. Some victories are worse than defeats.

59. There is none virtuous, no, not one.

60. No one can be rich and happy unless he is also prudent and temperate, and not always then.

61. No child ever fails to be troublesome, if ill-taught and spoiled.

62. Many a rose is born to blush unseen.

63. All emotions are compound mental states.

64. It's an ill wind that blows good to nobody.

65. Some laws arise from custom.

66. Not all who are called are chosen.

67. He envies others' wealth who has none himself.

68. Only doctors understand this subject.

69. If it is true that 'Students who do their work faithfully should receive university credit,' does it follow that 'Students should receive university credit for a faithful effort to do their work'?

70. A few Greeks vanquished the vast army of Darius.

71. Only ignorant persons hold such opinions.

72. If it is true that 'There is no disgrace in losing when one has done one's best,' does it follow that 'Those who win deserve no particular glory'?

73. Since 'Hottentots are men,' can we say that a clever Hottentot is a clever man?

74. In the case of the proposition 'All wise acts are honest acts,' answer the following questions: (*a*) How is its converse related to its subaltern? (*b*) How is its converse related to the converse of its subaltern? (*c*) How is its subaltern related to its contradictory? (Jevons.)

75. Name the logical process by which we pass from each of the following propositions to the succeeding one: —

(*a*) All metals are elements.
(*b*) No metals are non-elements.
(*c*) No non-elements are metals.
(*d*) All non-elements are non-metals.
(*e*) All metals are elements.
(*f*) Some elements are metals.
(*g*) Some metals are elements. (Jevons.)

76. 'Ignorance leads to superstition.' State in a series of propositions all the implications of this assertion.

CHAPTER VIII. — *The Syllogism and its Rules*

1. What is the relation of the Proposition and the Syllogism?

2. (*a*) What is the function of the Middle Term in a Syllogism? (*b*) Explain and illustrate the meaning of Aristotle's saying, that the search of science is for the middle term.

3. How are the major and minor terms, and the major and minor premises of a Syllogism distinguished?

4. Prove the seventh and eighth canons of the Syllogism. (*a*) by means of the previous rules, and (*b*) by the use of circles.

5. Construct an argument to illustrate the fallacy of ambiguous middle term.

6. Arrange the following arguments in the regular logical order of major premise, minor premise, and conclusion, and examine them to see whether they conform to the canons of the Syllogism: —

(1) Gold is not a compound substance; for it is a metal, and none of the metals are compounds.

(2) A classical education is worthless, for we make no use of the ancient languages in later life.

(3) All cruel men are cowards, no college men are cruel, therefore no college men are cowards.

(4) Some useful metals are becoming rarer. Iron is a useful metal, and is therefore becoming rarer.

(5) This man shares his money with the poor, but no thief ever does this, therefore this man is not a thief.

(6) He who is content with what he has is truly rich. An envious man is not content with what he has; no envious man therefore is truly rich.

7. What does the Figure of an Argument depend upon? How do you distinguish the four figures?

CHAPTER IX. — *The Valid Moods and the Reduction of Figures*

1. Arrange the following arguments in logical order, and give the mood and figure in each case: —

(1) No P is M,
Some S is M,
Therefore some S is not P.

(2) All M is S,
Some M is P,
Therefore some S is P.

2. Name the premises from which valid conclusions may be drawn, no account being taken of figures: —

AA, EO, IA, IO, II, EE, EI, AE, EA, OO.

3. Prove the special canons of the fourth figure.

4. 'The middle term must be distributed once at least.' In what figures may it be distributed twice? What is the character of the conclusion when this occurs?

5. Prove generally that when the major term is predicate in its premise, the minor premise must be affirmative.

6. If the major term be distributed in its premise, but used undistributively in the conclusion, determine the mood and figure.

7. Explain why we can obtain only negative conclusions by means of the second figure and particular conclusions by means of the third figure.

8. What conclusions do AA, AE, and EA yield in the fourth figure? Explain.

9. Is it possible for both major and minor terms to be undistributed at the same time in the premises? If so, construct an argument where this is the case.

10. What do you understand by Reduction? Reduce the following argument to the first figure: —

No fixed stars are planets,
All planets are bright and shining,
Therefore some bright and shining bodies are not fixed stars.

11. If the major premise and the conclusion of a valid syllogism agree in quantity but differ in quality, determine by general reasoning the mood and figure.

Chapter X. — *Abbreviated and Irregular Arguments*

1. Complete the following arguments, determine their mood and figure, and examine them to see if they violate any of the rules of the syllogism: —

(1) He blushes; therefore he is guilty.

(2) Idiots cannot be men, for man is a rational being.

(3) Zoöphytes have no flowers; therefore they are not plants.

(4) None but material bodies gravitate; therefore air is a material body.

(5) He has been a politician for years, and is therefore not to be trusted.

(6) It is not true that wealth is the only thing that affords satisfaction, for a good conscience is not wealth.

2. Illustrate the difference between the Progressive or Synthetic, and the Regressive or Analytic, methods as employed in Mathematics and Psychology. May a science employ both methods at the same time?

3. Arrange the following Sorites as a series of syllogisms: One must take pains if one is to be a successful man; industry is necessary to taking pains; health is necessary to industry; a regulated diet and sufficient recreation and rest are necessary to health; and, therefore, a regulated diet and sufficient recreation and rest are necessary if one is to become a successful man.

4. Show generally why all the premises except the first in the Aristotelian Sorites must be universal.

5. Prove that in the Goclenian Sorites the first premise alone can be negative, and the last alone particular.

6. In the examples of arguments given on page 139, is there any middle term? If not, what serves as the standard of comparison?

7. What is the general principle on which all *a fortiori* arguments proceed? How can you tell when an argument is of this type?

8. State the argument implied in the following: —

'If a man love not his brother whom he hath seen, how shall he love God whom he hath not seen?'

Chapter XI. — *Hypothetical and Disjunctive Arguments*

1. Explain how a disjunctive proposition always involves conditions. Illustrate your answer.

2. What are the rules of the hypothetical syllogism?

3. Is it ever possible to obtain a valid conclusion by denying the antecedent or affirming the consequent?

4. Determine which of the following hypothetical arguments are valid and which invalid; then express the latter in the categorical form, pointing out what are the categorical fallacies which result: —

(1) If a country is prosperous the people will be loyal. The people of this country are loyal and therefore it must be prosperous.

(2) If our rulers could be trusted always to look to the best interests of their subjects, monarchy would be the best form of government; but they cannot be trusted; therefore monarchy is not the best form of government.

(3) If ye were Abraham's children, ye would do the works of Abraham.

(4) If man were not capable of progress, he would not differ from the brutes; but man does differ from the brutes, therefore he is capable of progress.

(5) If he had studied his lesson, he would have been able to recite; but he was able to recite, and therefore must have studied his lesson.

(6) If it becomes colder to-night, the pond will be frozen over; but it will not become colder to-night, therefore the pond will not be frozen over.

5. What aspects of thinking are emphasized by the categorical and hypothetical forms of reasoning respectively?

6. How far may the disjunctive proposition be regarded as an expression of ignorance, and what is the justification for the statement that it involves systematic knowledge?

7. Show how the following argument illustrates the dangers of popular disjunctive inference: —

A has either been badly taught or has been himself lazy and indifferent. But as we know that his teacher is not a man of any learning or ability, we may conclude that A is not to be blamed for his failure.

8. How would you criticise the dilemmatic arguments given on page 158?

9. State the following fully as a dilemma: —

'There are two kinds of things which we ought not to fret about; what we can help, and what we cannot.' (Whately.)

10. A clever man would see his mistake and a candid man would acknowledge it. But he does neither. State the above fully as a dilemma, and construct a counter-dilemma in rebuttal.

Chapter XII. — *Fallacies of Deductive Reasoning*

1. What is the distinction between errors of interpretation and fallacies in reasoning?

2. Why is the detection of material fallacies a proper subject of logic?

3. If it is false that 'every man has his price,' is it true that

'no man can be bought'? What relation do the two statements have to each other?

4. Can we proceed logically from the proposition, 'all good citizens vote at elections,' to 'all who vote at elections are good citizens'?

5. 'No priests are saints. But some priests are martyrs, and there are no martyrs who are not saints.' Can these three propositions be all true at once?

6. Mention the fallacies of Equivocation, and explain what is common to them all.

7. Explain the terms: *Petitio Principii*, *Circulus in probando*, *Argumentum ad hominem*, *Argumentum ad populum*.

8. Examine the following reasoning: 'The argument from design must be regarded as without value; for it has been rejected by Spinoza, Kant, Spencer, and Darwin.'

9. Point out and name the fallacy or fallacies in the following: —

(1) I infer that this archaic statue must have been made about 525 B.C. because of the shape of the letters on the base of the pedestal. The inscription on the statue belongs to the period about 525 B.C.; for the statue shows the peculiar characteristics of that period.

(2) This is a dangerous doctrine, for we find it upheld by men who avow their disbelief in Revelation. (Jevons.)

(3) He must be a Mahometan, for all Mahometans hold these opinions. (Edinburgh.)

(4) It is not right for you to devote all your time to archæological research, for if all men did so, the business of the world could not go on.

(5) Every member of a jury is liable to error; therefore we can place no confidence in the decision of the jury.

(6) Great men have been derided, and I am derided; which proves that my theory ought to be adopted. (De Morgan.)

(7) All adults should be given the franchise; for the franchise is for the good of the state, that is, of the citizens; and the exercise of the franchise will make any adult a better citizen.

Miscellaneous Examples of Deductive Arguments

Arrange the following arguments whenever possible in regular logical order, supplying premise or conclusion where either is lacking, or condensing when several sentences are used to state one proposition; determine whether or not the arguments are valid; give the mood and figure of the valid categorical arguments; if any argument is invalid, point out and name the fallacy involved: —

1. Notes that produce beats are not harmonious. The fourth and fifth produce beats. Therefore they are not harmonious.

2. Every one desires happiness; virtue is happiness; therefore every one desires virtue.

3. All patrons of the arts and sciences are public benefactors. No poor men are patrons of the arts and sciences. Therefore no poor man is a public benefactor.

4. That which will bring a man peace at the last is to be sought after. A life of piety and virtue will bring a man peace at the last. Therefore a life of piety and virtue is to be sought after.

5. God is beneficial. Good is also beneficial. It would seem, then, that where the essence of God is, there, too, is the essence of Good. (Epictetus.)

6. He must be a Democrat; for all the Democrats believe in Free Trade.

7. The receiver of stolen property should be punished; you have received stolen property, and should therefore be punished. (Glasgow.)

8. Whoever believes this is a heretic; so that you are no heretic, for you do not believe this. (Glasgow.)

9. Good men write good books; this is a good book, and therefore its writer was a good man. (Glasgow.)

10. No man desires pain, and without pain your friend's cure is impossible; therefore he will not desire to be cured. (Glasgow.)

11. If this man were wise, he would not speak irreverently of the Scripture in jest; and if he were good he would not do so in earnest; but he does it either in jest or in earnest; therefore, he is either not wise, or not good.

12. Only material bodies gravitate; ether does not gravitate.

13. In reply to the gentleman's arguments, I need only say that two years ago he advocated the very measure which he now opposes.

14. Haste makes waste, and waste makes want; therefore a man never loses by delay. (Glasgow.)

15. C is not D, for A is B; and I know that whenever A is not B, C is D. (Glasgow.)

16. The existence of sensations consists in being perceived; all objects are really collections of sensations; therefore, their existence consists in being perceived. (Glasgow.)

17. If he claims that he did not steal the goods, why, I ask, did he hide them, as no thief ever fails to do?

18. If it be true, as Mr. Spencer thinks, that the past experience of the race has produced innate ideas and feelings, Weismann's denial of Use-inheritance would be refuted. Certainly, but it is just possible that Mr. Spencer's theory is not true.

19. Englishmen admire all who are successful; they must, therefore, admire some persons who are politically dangerous,

for assuredly there are some successful persons who are politically dangerous.

20. There should be no restriction of debate in the United States Senate, because freedom of speech is one of our most sacred privileges.

21. If only the ignorant despise knowledge, this man cannot be ignorant; for he praises it.

22. Every book is liable to error; every book is a human production; therefore all human productions are liable to error.

23. Of university professors, some are zealous investigators, and some good teachers. A is an excellent teacher, and we may therefore conclude that he is not a zealous investigator.

24. Seeing that abundance of work is a sure sign of industrial prosperity, it follows that fire and hurricane benefit industry, because they undoubtedly create work. (St. Andrews.)

25. I will have no more doctors; I see that all of those who have died this winter have had doctors. (St. Andrews.)

26. If a man is educated, he does not want to work with his hands; consequently, if education is universal, industry will cease. (London.)

27. Show why IE is an impossible mood in all the figures of the syllogism, while EI is possible in all of them. (Glasgow.)

28. If acquired variations are transmitted, there must be some unknown principle of heredity; if they are not transmitted, there must be some unknown factor of evolution. (Osborn.)

29. Some plant-products harmful to insects are not a protective development; for all tannin is harmful to insects, and most certainly not all tannin is a protective development. (St. Andrews.)

30. The spectra of compound bodies become less complex with heat; but the spectra of the elements do not, since they are not the spectra of compound bodies. (St. Andrews.)

31. What can you tell about a valid syllogism if you know: — (1) that only the middle term is distributed; (2) that only the middle and minor terms are distributed; (3) that all three terms are distributed. (Glasgow.)

32. Ease and pleasure quake to hear of death. But my life desireth to be dissolved. (It follows that) my life is full of cares and miseries.

33. Lord Chancellor Thurlow said that the corporations have neither bodies to be punished nor souls to be damned; they therefore do as they like.

34. No pauper has a vote: AB is not a pauper, therefore he has a vote. (St. Andrews.)

35. The love of nature is never found either in the stupid or the immoral man, therefore stupidity and virtue are incompatible. (Edinburgh.)

36. The figure of Tell cannot be historic, else he must have been mentioned by early historians, or his personality would be necessary to explain known facts of history. (St. Andrews.)

37. Carbon, which is one of the main sources of the nourishment of plants, cannot be dissolved in water in its simple form, and cannot therefore be absorbed in that form by plants, since the cells absorb only dissolved substances. All the carbon found in plants must consequently have entered them in a form soluble in water, and this we find in carbonic acid.

38. To improve is to change, and to be perfect is to have changed often. What hope can we entertain of those who oppose change?

39. Why should you think that I should woo in scorn?
Scorn and derision never come in tears:
Look, when I vow, I weep; and vows so born,
In their nativity all truth appears.

40. If a statesman who sees his former opinions to be wrong does not alter his course, he is guilty of deceit; and if he does

alter his course, he is open to a charge of inconsistency; but either he does not alter his course or he does; therefore, he is either guilty of deceit, or he is open to a charge of inconsistency.

41. No punishment should be allowed for the sake of the good that may come of it; for all punishment is an evil, and we are not justified in doing evil that good may come of it. (Edinburgh.)

42. As against the proposition that the formation of public libraries prevents private individuals from purchasing, and so decreases the sale of books, a writer urges that whatever encourages the reading of books encourages the buying of books. It is a library's purpose to encourage reading, and hence the net result is rather to increase than to lessen purchases.

43. The express train alone does not stop at this station, and, as the last train did not stop, it must have been the express train. (Glasgow.)

44. Arrange the following so as to show the difference between the Aristotelian and the Goclenian sorites: —

'The cost of labor depends upon the efficiency of labor; the rate of profits depends on the cost of labor; the investment of capital depends on the rate of profits; wages depend on the investment of capital; therefore wages depend on the efficiency of labor.'

45. No man should be punished if he is innocent; this man should not be punished; therefore he is innocent.

46. Partners in the same enterprise have the same interest. How then can there be antagonism between my workmen and me?

47. The philosophy of Naturalism, if regarded from the practical side, is insufficient; if from the speculative side, it is incoherent; therefore it fails to justify itself.

48. If the power to lead is in you, other men will follow; if it is not in you, nothing will make them follow.

49. Some very good people are zealous reformers. But zealous reformers often lack imagination and fairmindedness.

50. Logic and mathematics both furnish excellent formal discipline; therefore the latter may be regarded as a branch of the former.

51. "The truth is, that luxury produces much good. A man gives half a guinea for a dish of green peas; how much gardening does this occasion?" (Dr. Johnson.)

52. Protective duties should be abolished; for they are injurious if they produce scarcity, and they are useless if they do not. (Oxford.)

53. Animals only are sentient beings; all plants are insentient. (St. Andrews.)

54. Only native-born citizens are eligible to this office; but as you have this qualification, you need not hesitate to run for it. (St. Andrews.)

55. A primary election law is necessary, for at present the people have no voice in the nomination of candidates for office.

56. I do not see how Mr. Rhodes can escape censure. If he knew of Dr. Jameson's raid, he was guilty of complicity; if he did not, of negligence. (St. Andrews.)

57. Business enterprises are most successful when managed by those who have a direct interest in them; therefore enterprises carried on by the State are not likely to succeed.

58. All P is M; All S is M; therefore Some not-S is not-P. (Glasgow.)

59. Wherever ideas have become indissolubly associated, it is beyond our power to represent them separately; our attitude is that of belief. Belief then may be defined as the consequence of an indissoluble association of ideas. (Glasgow.)

60. No reason, however, can be given why the general happiness is desirable, except that each person, so far as he believes it to be attainable, desires his own happiness. This, however,

being a fact, we have not only all the proof which the case admits of, but all which it is possible to require, that happiness is a good, that each person's happiness is a good to that person, and the general happiness, therefore, a good to the aggregate of all persons. (Mill's *Utilitarianism.*)

61. This man is a Protestant; for he exercises the right of private judgment.

62. If the orbit of a comet is diminished, either the comet passes through a resisting medium, or the law of gravitation is partially suspended. But the second alternative is inadmissible. Hence if the orbit of a comet is diminished, there is present a resisting medium.

63. How do we know that our intuitive beliefs concerning the world are invariably true? Either it must be from experience establishing the harmony, or an intuitive belief must certify the correctness. Now experience cannot warrant such harmony except in so far as it has been perceived. Still more futile is it to make one instinctive belief the cause of another. Thus we cannot know that any intuitive belief is universally valid. (Bain.)

64. Which of the following are real inferences? (1) 'This weighs that down, therefore it is heavier'; (2) 'This piece of marble is larger than that, and therefore is heavier.'

65. The parts of pure space are immovable, which follows from their inseparability, motion being nothing but change of distance between any two things; but this cannot be between parts that are inseparable, which therefore must be at perpetual rest one amongst another.

66. All civilized peoples are progressive; all uncivilized peoples are superstitious; therefore some superstitious peoples are not progressive. (St. Andrews.)

67. Ignorance is no crime; and as you did not know what you were doing, you should not be punished. (St. Andrews.)

68. He could not face bullets on the field of battle, and is therefore a coward. (St. Andrews.)

69. If a man be rightfully entitled to the produce of his labour, then no one can be rightfully entitled to anything which is not the produce of his labour. (St. Andrews.)

70. In moral matters we cannot stand still; therefore he who does not go forward is sure to fall behind. (Glasgow.)

71. A man that hath no virtue in himself ever envieth virtue in others; for men's minds will either feed upon their own good or upon others' evil; and who wanteth the one will prey upon the other.

72. A miracle is incredible because it contradicts the laws of nature.

73. He who calls you a man speaks truly; he who calls you a fool calls you a man; therefore he who calls you a fool speaks truly.

74. If a body moves, it must move either in the place where it is, or in the place where it is not. But a body cannot move in the place where it is, nor yet in the place where it is not. Hence a body cannot move at all.

75. The governor of a country ought not to be blamed for using his influence to further his religious views, for every man has a right to inculcate his own opinions.

76. If a man be rightly entitled to the produce of his labour, then no one can be rightly entitled to anything which is not the produce of his labour.

77. The instance of Berkeley shows that philosophical profundity and literary style are not incompatible.

78. No honest man can advocate a change in the creed of his church; for he must either believe it or not believe it; if he believes it, he cannot honestly help to change it, while if he does not believe it, he cannot honestly belong to the church at all.

79. There can be no such thing as an omniscient mind, since all thinking is a succession of mental states. (St. Andrews.)

80. The advantages which would accrue to the working classes are not sufficient to justify Protection, neither are the advantages which it would bring to the farmers or the manufacturers, or to any other class in the community; Protection, therefore, has not enough advantages to justify it.

81. Against what fallacy does the proverb, 'All that glitters is not gold,' warn us?

82. The student of history is compelled to admit the law of progress, for he finds that society has never stood still.

83. I will not do this act because it is unjust; I know that it is unjust because my conscience tells me so, and my conscience tells me so because the act is wrong.

84. Gold and silver are wealth; therefore the diminution of the gold and silver of the country by exportation is a diminution of the wealth of the country.

85. Nations are justified in revolting when badly governed, for every people has a right to a good government. (Edinburgh.)

86. When Crœsus was about to make war upon Cyrus, King of Persia, he consulted the oracle at Delphi, and received for an answer that, if he should wage war against the Persians, he would overthrow a mighty empire.

87. England has a gold coinage, and is a very wealthy country, therefore it may be inferred that other countries having a gold coinage will be wealthy.

88. Your arguments against the philosophy of Hegel are of no value; for you uphold that of Schopenhauer, which is equally repugnant to common sense.

89. For those who are bent on cultivating their minds by diligent study, the incitement of academical honours is unnecessary; and it is ineffectual for the idle, and such as are indifferent to

mental improvement; therefore the incitement of academical honours is either unnecessary or ineffectual.

90. This person is very learned, and very sociable, hence it follows that learning increases sociability.

91. Why advocate socialism? Until men become morally perfect, it is impossible; when they have become so, it will be unnecessary. (Edinburgh.) In what ways could you reply to this?

92. The diameter of the earth is, in round numbers, forty millions of feet. Consequently the attraction of a sphere of the same mean density as the earth, but one foot in diameter, will be $\frac{1}{40000000}$ part the attraction of the earth; that is, $\frac{1}{40000000}$ of the weight of the body attracted. Consequently, if we should measure the attraction of such a sphere of lead, and find that it was just $\frac{1}{40000000}$ that of the weight of the body attracted, we would conclude that the mean density of the earth was equal to that of lead. But the attraction is actually found to be nearly twice as great as this; consequently a leaden sphere is nearly twice as dense as the average of the matter composing the earth. (Newcomb, *Popular Astronomy*.)

93. Mr. C. said that he was certain that the donors gave the property to the institution with a distinct and unanimous understanding as to its future use. The directors who acted for the institution in this transfer must necessarily have had an understanding, either the same as that of the donors, or different. If the understanding of the directors was the same as that of the donors, then they, the former, were unquestionably bound to live up to that understanding. If it was different, then the property was conveyed on a misunderstanding, and every dictate of honour and fair play would demand the return of the property.

94. There is no connection between sex and the ballot. If woman is like man, and it is right for man to vote, it must be

right for woman to do so. If woman is unlike man, he can never truly represent her, and she ought to be allowed to represent herself. (From letter to *N. Y. Times.*)

95. The right to use and kill animals for the relief and convenience of man is universally recognized throughout Christendom and, in general, throughout the civilized world. Dominion over the animate creation means, of course, the right of man to use animals for his own good; and those who kill animals for food have a poor logical ground to stand on when they object to the use of animals for the experimentation in scientific laboratories by experts who are aiming to discover remedies for the terrible diseases which attack and destroy human life. Shall a man have animals killed for his nourishment and pleasure, and object to that experimental research upon animals which has enabled scientific and medical investigators to conquer numerous diseases?

96. If pain is long continued, it is not severe; and if it is severe, it does not last long. (Stoic axiom.)

97. State formally the argument against which the following is directed, and examine from the logical point of view both argument and rejoinder: —

"Because thou must not dream, thou needst not then dispair." (M. Arnold.)

98. "Our ideas reach no farther than our experience. We have no experience of divine attributes and operations. I need not conclude my syllogism. You can draw the inference yourself." (Hume.)

99. What fallacy is charged to the defenders of Charles the Second by Macaulay in the following statements: "We charge him with having broken his coronation oath, and we are told that he kept his marriage vow! We accuse him of having given up his people to the merciless inflictions of the most hot-headed and hard-hearted of prelates, and the defense is that he

took his little son on his knee and kissed him! We censure him for having violated the articles of the Petition of Right, after having for good and valuable consideration promised to observe them, and we are informed that he was accustomed to hear prayers at six o'clock in the morning."

100. What fallacies do you find in the following argument?

Ghosts exist, because all people in all ages have had faith in their existence. We know so little of souls even when they animate our bodies and those of our friends. How, then, can we say they may not become ghosts after death? Besides, there are kindly ghosts as well as evil-minded ones. It will be an incentive to virtue if we know that the souls of our departed dear ones will, in the form of ghosts, hover about us to watch, guide, and protect us from going.

101. Analyze the following argument:

Without contentment man cannot be happy, whether rich or poor; for then the poor will try to become rich and the rich to become richer. The desire to get richer destroys peace of mind, and peace of mind is an essential requisite of happiness.

102. A cheesemonger asked Tarlton why cheese and butter were so dear, and Tarlton told him it was because wood and coal were so scarce, as people could eat butter and cheese without a fire.

103. If Columbus had never lived, America would still have been discovered; if Newton had never lived, someone else would have discovered the law of gravitation; if Wilberforce had never lived, the slave-trade could not have lasted forever. . . . Therefore the world could do without great men.

104. Certain people look on all punishment as implying deep degradation in someone, — if it is justified, the offender is little better than a brute; if it is not justified, the brutality is in the person who inflicts it. The reasoning appears to travel thus: Punishment is degrading; therefore it can work no moral

improvement.—But if punishment could work a moral improvement, it would not degrade but elevate. The humanitarian argument alternately proves that punishment can only intimidate because it is brutalizing, and that it is brutalizing because it can only intimidate.

PART II.—Inductive Methods

Chapter XIII.—*The Problem of Induction*

1. Explain the relation between Syllogistic logic and Inductive reasoning.

2. Give a statement of the general problem of Induction. What precisely is the problem in understanding how the mind reaches universal truth? Explain.

3. It has been said that Deductive logic seeks to bring ideas into harmony with each other, and Inductive logic to bring ideas into harmony with facts. Comment upon this distinction.

4. Explain the distinction between Induction by Enumeration and Induction by Analysis.

5. Explain the following terms: Induction by Simple Enumeration, Prerogative Instances, and Crucial Experiments. Why is it difficult to devise a strictly crucial experiment?

6. What rules may be given for the selection of instances in an inductive investigation?

7. It is sometimes said that Elimination is the essential principle of Induction. Discuss this statement.

8. Explain the function of Analogy and Hypothesis in Induction.

Chapter XIV

1. What is the general assumption of all Inductive thinking? Explain the relation of this assumption to the laws of Thought.

2. Explain and consider the statement that 'The Uniformity of Nature is the ultimate major premise in all induction.' Comment on Newton's maxim, '*hypotheses non fingo*.'

3. What is meant by a category of thought? Illustrate. What is the distinction between a 'dynamic' and a 'static' category?

4. Trace the essential steps for discovering universal laws through induction. Are these steps mutually exclusive? Would it be permissible to add Experimental methods as a third and independent class?

5. Explain the relation between facts and theories.

6. What is the distinction between 'empirical' and 'scientific' knowledge?

Chapter XV. — *Enumeration and Statistics*

1. What is the justification for beginning our account of the inductive methods with Enumeration?

2. Explain how it is sometimes possible to reach certain conclusions on the basis of instances. In what respect are such conclusions defective?

3. For what purpose are Statistics employed? To what classes of phenomena are they applied? Explain the statement that Statistics are valuable only when compiled intelligently.

4. What is the relation between the warning that observation and inference should not be confused, and the frequent claim that statistics are misleading?

5. State and distinguish three uses to which Statistics may be put.

6. Explain how Statistics may suggest causal laws, or confirm our expectation of them. May Statistics also be used to disprove a proposed law of causal connection? Illustrate your answer.

7. Explain what is meant by the 'average,' the 'mean,' and the 'mode,' and show how each is obtained.

8. How does the procedure of insurance companies differ from gambling?

Chapter XVI. — *Causal Determination*

1. What are the two main principles upon which the canons proposed by Mill are founded?

2. Give the canon of the method of Agreement, and illustrate its use.

3. 'I have noticed that A always precedes B; it is therefore the cause of B.' Is this good reasoning?

4. What is meant by the 'Plurality of Causes,' and by the 'Reciprocity of Causes'?

5. Under what disadvantages does the method of Agreement labour? How is it supplemented?

6. State and illustrate the canon of the method of Difference.

7. Why is this method applicable only to the spheres where experiment can be employed? Explain with illustrations why there cannot be a real social experiment.

8. How might the canons of Agreement and Difference respectively be stated negatively, as principles of Elimination? Would this statement do full justice to the inductive procedure involved?

Chapter XVII. — *Causal Determination* (*continued*)

1. Where do we employ the Joint method?

2. What precisely would it be necessary to establish in order to establish inductively that some change in the tariff laws was beneficial to the country?

3. Explain what qualifications it is necessary to introduce in interpreting Mill's statement of the Joint method.

4. 'One of the main characteristics of modern science is its *quantitative* nature.' Explain.

5. How does the law of Concomitant Variations assist us in determining causal relations?

6. In what two ways may the method of Residues be applied?

7. Mention some discoveries to which the investigation of unexplained residues has led.

Chapter XVIII. — *Analogy*

1. Why do we include Analogy among the methods of Explanation?

2. What do you mean by Analogy? What is the principle upon which it proceeds? What are the dangers of analogical reasoning?

3. How is the word used in mathematical reasoning, and in physiology?

4. Into what Figure of the Syllogism does an argument from Analogy naturally fall? Is the argument formally valid, and if not, to what syllogistic fallacy does it correspond?

5. Explain how Analogy may suggest a true law or explanatory principle.

6. Why do we speak of Analogy as Incomplete Explanation?

7. If all analogical reasoning yields only probability, is not one analogy as good as another for purposes of inference? If not, upon what does the *value* of an inference from Analogy depend?

Chapter XIX. — *The Use of Hypotheses*

1. How do you distinguish the terms 'theory' and 'hypothesis'?

2. What is an hypothesis, and how is it used?

3. Do hypotheses play any part in assisting Observation? Explain and illustrate.

4. Give some instances in which hypotheses have proved injurious, and have misled people regarding the nature of facts.

5. Trace the essential steps for discovering universal laws

through induction. Has imagination any place or value in scientific investigation?

6. Discuss the two following statements of Darwin: 'Any fool can generalize and speculate,' and 'No one can be a good observer unless he is an active theorizer.'

7. What part does Elimination play in the proof of an hypothesis? Explain the nature of the formal fallacy involved in the statement that an hypothesis is established when its results are shown to be true. How is this difficulty overcome?

8. What canons have been laid down to which a good hypothesis must conform? Why are the first and third of these rules of little value?

9. Explain why an unverifiable hypothesis is not worth discussing.

Chapter XX. — *Fallacies of Induction*

1. What is the source of fallacy? How far is it true that the study of Logic can protect us from fallacies?

2. How do you classify Inductive Fallacies?

3. Explain and illustrate the following fallacies: *Question-begging Epithet*, *Equivocation*, *Fallacies due to Figurative Language*.

4. Explain and illustrate the tendency of the mind to neglect negative cases.

5. Is it an easy matter to 'tell just what we saw and heard' at a particular time?

6. What do you mean by *post hoc ergo propter hoc?* Why may we take this as the general type of inductive fallacies?

7. What did Bacon mean by the Idols of the Cave, of the Tribe, of the Market-Place, of the Theatre?

8. 'Every age, as well as every individual, has its idols.' Explain and illustrate this statement.

Miscellaneous Examples of Inductive Arguments

Analyze the examples of inductive reasoning given below, and point out what methods are employed, indicating also whether or not the conclusion is completely established, and naming the fallacy, if any be present: —

1. Beeswax, gum-arabic, and balsam all take on the brilliant coloring of mother of pearl when they receive an impress from its surface. The only circumstance in common is the shape of the mother of pearl. The shape therefore is the cause of the color.

2. Overdriven cattle, if killed before recovery from their fatigue, become rigid and putrefy in a surprisingly short time. A similar fact has been observed in the case of animals hunted to death; cocks killed during or shortly after a fight; and soldiers slain in the field of battle. These various cases agree in no circumstance directly connected with the muscles, except that these have just been subjected to exhausting exercise.

3. In the African race there are more female than male births; since the colored children under one year of age in the United States in 1880 form a random sample of births of Africans, we may expect more females than males in this group. Suppose this expectation were not fulfilled, what would be the next step in the inductive process?

4. In Sweden the population and the smallpox mortality have both been known year by year since 1774. Before vaccination the mortality from smallpox for thirty years averaged 2045 per million. With permissive vaccination from 1802 to 1816, it was reduced to 480; during seventy-seven years of compulsory vaccination the mortality averaged 155 per million; and for ten years ending 1894 it has been down to 2 per million.

5. The great famine in Ireland began in 1845 and reached its climax in 1848. During this time agrarian crime increased very

rapidly, until, in 1848, it was more than three times as great as in 1845. After this time it decreased with the return of better crops, until, in 1851, it was only 50 per cent more than it was in 1845. It is evident from this that a close relation of cause and effect exists between famine and agrarian crime. (Hyslop.)

6. Sachs maintained, in 1862, that starch is formed by the decomposition in chlorophyl of carbon-dioxide gas under the influence of light. He found that when all other conditions were constant, and light was excluded from a plant, no starch was formed; the single circumstance of readmitting light was accompanied by renewed formation of starch. Further, he found that if certain portions of the leaves of an illuminated plant were covered with black paper, no starch was found in these portions.

7. Jupiter gives out more light than it receives from the sun. What is the obvious conclusion, and by what method is it reached?

8. What methods would you employ in order to test the truth of the proposition, *omne vivum ex vivo?*

9. It is evident that the green colour of plants holds some necessary relation to light, for the leaves of plants growing in the dark, as potatoes sprouting in a cellar, do not develop this colour. Even when leaves have developed the green colour, they lose it if deprived of light, as is shown by the process of blanching celery and by the effect on the colour if a board has lain upon it for a long time. (Coulter.)

10. Another indication that the green colour is connected with light may be obtained from the fact that it is found only in the surface region of plants. If one cuts across a living twig or into a cactus body, the green colour will be seen only in the outer part of the section. (Coulter.)

11. If an active leaf or water plant be submerged in water in a glass vessel and exposed to the light bubbles may be seen com-

ing from the leaf surface and rising through the water. The water is merely a device by which the bubbles of gas may be seen. If the leaf is very active, the bubbles are numerous. That this activity holds a definite relation to light may be proved by gradually removing the vessel containing the leaf from the light. As the light diminishes the bubbles diminish in number, and when a certain amount of darkness has been reached the bubbles will cease entirely. If now the vessel be brought back gradually into the light, the bubbles will reappear, more and more numerous as the light increases. (Coulter.)

12. War is a blessing, not an evil. Show me a nation that has ever become great without bloodletting.

13. If wages depend upon the ratio between the amount of labour-seeking employment, and the amount of capital devoted to its employment, the relative scarcity or abundance of one factor must mean the relative abundance or scarcity of the other. Thus capital must be relatively abundant where wages are high, and relatively scarce where wages are low. Now, as the capital used in paying wages must largely consist of the capital-seeking investment, the current rate of interest must be the measure of its relative abundance or scarcity. So if it be true that wages depend upon the ratio between the amount of labour-seeking employment, and the capital devoted to its employment, then high wages must be accompanied by low interest, and, reversely, low wages must be accompanied by high interest. This is not the fact but the contrary. (George.)

14. A unique phenomenon of colouration in the sky occurred in 1883. In the same year a tremendous volcanic explosion occurred in the Straits of Sunda, and that also was of unique intensity. The coincidence of the two led to the belief that the one was caused by the other.

15. Some comets have been observed to have the same orbit as certain meteoric showers. The hypothesis is suggested that

all meteoric showers may represent the débris of disintegrated comets. Biela's comet having been missing for some time, it was accordingly predicted that when next due it would be replaced by a meteoric shower. This prediction was verified by observation.

16. We have this, that, and the other hypothesis urged by different students as the cause of the great excavations of the Great Lake basins; and all of these are urged with force and with fact, urged justly to a certain degree. It is practically demonstrable that these basins were river-valleys antecedent to the glacial incursion, and that they owe their origin in part to the pre-existence of those valleys and to the blocking up of their outlets. And so this view of their origins is urged with a certain truthfulness. So, again, it is demonstrable that they were occupied by great lobes of ice, which excavated them to a marked degree, and therefore the theory of glacial formation finds support in fact. I think it is further demonstrable that the earth's crust beneath these basins was flexed downward, and that they owe a part of their origin to crust deformation. But to my judgment neither the one nor the other, nor the third, constitutes an adequate explanation of the phenomena. All these must be taken together, and possibly they must be supplemented by other agencies. (Chamberlin, "The Method of Multiple Working Hypotheses," *Science*, 1890.)

17. We should be guided by the decisions of our ancestors; for old age is wiser than youth.

18. Before it was known that light traveled in waves, it was known that sound did so. Light and sound were both capable of being reflected, and the direction of their reflection obeyed the same law, that the angle of reflection is equal to the angle of incidence. From these facts it was inferred that light, like sound, traveled in waves.

19. After Lavoisier had recognized the analogy between the

processes of combustion and respiration, the idea naturally suggested itself to his mind that the heat observed in the animal organism must have the same origin as that which is evolved during combustion; and, afterwards, in a paper published jointly with Laplace, these two illustrious philosophers distinctly stated it as their opinion, based upon experiment, that the sensible heat of the animal organism is the combustion heat of the carbon and hydrogen which, in the form of food, are burnt in the body. (Hofmann, *The Life-work of Liebig.*)

20. For many generations the people of the Isle of St. Kilda believed that the arrival of a ship in the harbor inflicted on the islanders epidemic colds in the head, and many ingenious reasons were devised why the ship should cause colds. At last it occurred to somebody that the ship might not be the cause of the cold, but that both might be effects of some other common cause, and it was then remembered that a ship could only enter the harbor when there was a strong northeast wind blowing.

21. Schwabe, observing sun-spots for many years, discovered that they reached a maximum, roughly speaking, once in every ten years. In 1851, Lamont, reviewing a series of magnetic observations carried on from 1835 to 1850, perceived with some surprise that they gave unmistakable indications of a period of $10\frac{1}{3}$ years, during which the range of the daily variation of the magnetic needle increased and diminished once. In the following winter, Sir Edward Sabine, ignorant as yet of Lamont's conclusions, undertook to examine a totally different set of observations concerning magnetic 'storms.' Once in about ten years magnetic disturbances were perceived to reach a maximum of violence and frequency. Sabine was the first to note the coincidence between this unlooked-for result and Schwabe's sun-spot period. He showed that, so far as observation had yet gone, the two cycles of change agreed perfectly both in duration and phase, maximum corresponding to maximum, minimum to

minimum. What the nature of the connection could be that bound together by a common law effects so dissimilar was, and still remains, beyond the reach of well-founded conjecture; but the fact was from the first undeniable. (Clerke, *History of Astronomy*.)

22. Professor Loeb has shown that some animals, exposed to a ray of light, turn either towards or away from the source of light; and he has applied to such behavior the term 'heliotropism'; one long used by the botanists to denote the bending of plants towards the light. Hence, without more ado, he speaks of the 'establishment of the identity of the reaction of animals and plants to light,' and reasons as follows: 'We have seen that, in the case of animals which possess nerves, the movements of orientation towards light are governed by exactly the same external conditions, and depend in the same way upon the external form of the body, as in the case of plants which possess no nerves. These heliotropic phenomena consequently cannot depend upon *specific* qualities of the central nervous system.' That is to say — having extended to certain reactions of animals the name 'tropism,' which had been used to denote certain plant reactions to which they bear a purely external and superficial resemblance, Professor Loeb holds himself justified in regarding reactions of these two classes as essentially similar or identical, although it is well known to him, as to everybody else, that they differ profoundly, if only in that a complex nervous system plays an essential part in the animal reactions, but is absent from the plants. (McDougall, "Modern Materialism," *Bedrock*.)

23. The community cannot possibly govern itself while its parts are separated. . . . Now our 'residential' sections are separated from the rest of our cities, and are often the most useless. The people in the 'residential' sections are those who dominate our big business interests. And yet they are out of

the current of civilization. There is the atrophy in the body politic. (Woodrow Wilson, quoted in the *Chicago Tribune.*)

24. The freshwater crayfish has a sense of smell, as is proved by the rapid way in which it retreats from strong odors. Investigation led to the hypothesis that this sense was located in the antennules or smaller feelers. This was verified by observing that a crayfish bereft of these appendages did not react to a strong odor, whereas in exactly the same conditions and to the same stimulus another crayfish with its antennules intact did actively respond. (J. A. Thomson, *Introduction to Science.*)

25. When the sun sets in the sea, a curious appearance, as of a bluish-green flame, is sometimes observed. This has been thought to be due to the light passing through the crests of waves. But this view is thought to be disproved by such an observation as that recently made by Professor Lang at a watering-place on the Baltic. Shortly before sunset, the disk was divided in two by a thin strip of cloud; and just as the upper part disappeared under the cloud, the blue flame was observed. Thus the cause appears to be in the air, not in the sea. It is a case of atmospheric refraction. (*Science,* 1890.)

26. When Von Siebold and his fellow-workers had convinced themselves indirectly that certain bladder-worms, *e.g.* those which occur in the pig and the ox, were the young stages of certain tapeworms which occur in man, they made the crucial . . . experiment of swallowing the bladderworms. By becoming soon afterwards infected with the tapeworms they proved the truth of their theory. (J. A. Thomson, *Introduction to Science.*)

27. Why does blood clot when out of the body, and why does it not clot while it remains within the blood-vessels? The accepted explanation until the middle of the eighteenth century was that the clotting was due to cold and rest. In 1767 William Hewson put this explanation to a crucial test. He ligatured a

vein in the neck of a dog in two places and then covered it with the skin to prevent its cooling. Opening the vein after an interval he found the blood in it coagulated. . . . Exactly what was proved by this experiment?

28. In the summer of 1840, Mayer, practising medicine in Java, was struck with the brighter red color of the venous blood of his patients. Reasoning on this he conceived it possible that the brighter color was due to less bodily oxidation being necessary to keep up the body temperature in hot climates. This drew his attention to animal heat, thence to heat production in relation to mechanical work, and, finally, to all forms of force. From extensive researches along these lines he formulated the theory that throughout the universe, both in the inorganic and the organic world, there are forces which are convertible but are not destructible. (T. B. Strong, *Lectures on the Method of Science.*)

29. When the Iliad was composed, the letter F, called Digamma from its form, was a living spoken sound, but it had practically disappeared from the spoken language when the Iliad was committed to writing in the old alphabet. The ancient scholars were not aware of the great rôle played by the initial F in Homeric verse. On account of their ignorance they treated many passages as metrically irregular or imperfect, which through the discovery of the lost element came to appear as intelligible and regular. The existence of the letter was suggested to a student of Greek on purely theoretical grounds. He observed the irregularities in scansion, and judging from the analogy of Latin words what sound was needed, framed the hypothesis of the letter Digamma. Some time afterward the letter was observed on ancient inscriptions and thus the hypothesis was verified.

30. There are frogs which change their color from time to time. In looking for the cause of this phenomenon, the first

step is to establish the occasion on which the change occurs, and the inquirer naturally restricts himself to such suggestions as might occur to a biologist. A number of these being disposed of by direct observation, according to the tests of presence and absence of the phenomena suggested to be connected, we reach the only reasonable suggestion left, namely, that which connects the change of color with the color of the surroundings. Further hypotheses as to the *modus operandi* of this connection are put forward, and one of these, otherwise not unreasonable, being excluded by the same test, it remains that the color-change involves the stimulation of the eye by light. Here two alternative interpretations of this condition are further suggested, awareness of color and a reflex mechanism. Awareness of color is excluded by observed color-change in a blind frog, and the suggestion of nervous excitation through a reflex mechanism as a circumstance common to the blind frog and the normal frog when changing color is accepted in so far as the cause and noted to be in harmony with other biological facts. (Bosanquet, from Joseph.)

31. A buttercup leaf, a blade of grass, a fern, a moss, a volvox, and a protococcus, all contain green coloring matter. I infer that all the members of the vegetable kingdom contain green coloring matter.

32. M. Arago, having suspended a magnetic needle by a silk thread, and set it in vibration, observed that it came much sooner to a state of rest when suspended over a plate of copper, than when no such plate was beneath it. Now, in both cases there were two true causes . . . why it should come to rest, *viz.*, the resistance of the air, which opposes, and at length destroys, all motions performed in it; and the want of perfect mobility in the silk thread. But the effect of these causes being exactly known by the observation made in the absence of the copper, and being thus allowed for and subducted, a residual phenome-

non appeared, in the fact that a retarding influence was exerted by the copper itself; and this fact, once ascertained, speedily led to the knowledge of an entirely new and unexpected class of relations. (Mill, *System of Logic.*)

33. Wages in the United States are higher than in England, because the former country is a republic and has a protective tariff.

34. It does not follow that an institution is good because a country has prospered under it, nor bad because a country in which it exists is not prosperous. It does not even follow that institutions to be found in all prosperous countries, and not to be found in backward countries, are therefore beneficial. For this at various times might confidently have been asserted of slavery, of polygamy, of aristocracy, of established churches; and it may still be asserted of public debts, of private property in land, of pauperism, and of the existence of distinctly vicious or criminal classes. (George.)

35. "No Body can be healthful without *Exercise*, neither Naturall Body, nor Politique: And certainly, to a Kingdome or Estate, a Just and Honourable Warre is the true *Exercise*. A Civill Warre, indeed, is like the Heat of a feaver; but a Forraine Warre, is like the Heat of *Exercise*, and serveth to keepe the Body in Health." (Bacon, *Essays.*)

36. Explain the procedure of the *reductio ad absurdum* form of argument.

37. It may be a coincidence merely; but, if so, it is remarkably strange that while the chloroform has not changed, while the constitutions of the patients have not changed, where the use of the inhaler is the rule, there are frequent deaths from chloroform; whilst in Scotland and Ireland, where the use of the inhaler is the exception, deaths are proportionally rare.

38. "Under the fallacies of hasty generalization naturally fall all local prejudices which arise from a narrow nature or habit of mind." Give illustrations.

39. "For there are only two possible *a priori* explanations of adaptations for the naturalist; namely, the transmission of functional adaptations and natural selection; but as the first of these can be excluded, only the second remains." (Weismann.)

40. The planet Mars resembles the Earth in possessing atmosphere, water, and moderate temperature, and we may therefore suppose it to be inhabited. (St. Andrews.)

41. In the last years of the eighteenth century a controversy raged between the schools of Hutton and of Werner, as to whether heat or the action of water had been the dominating influence at work in the formation of the rocks of the Earth's crust. By what agency had chalk been converted into limestone or marble? How can this have been affected by heat, said the school of Werner, since heat decomposes carbonate of lime, expelling the carbonic acid? The answer to this question was furnished by the experiments of Sir James Hall, "On the Action of Heat as modified by Pressure." Chalk was heated in a gun-barrel, the end of which was firmly closed. Under these conditions, the pressure increasing as the temperature is raised, the carbonic acid is not driven off from the carbonate of lime, the change induced being not chemical but physical, the powdery non-coherent chalk being converted into a compact crystalline mass, having all the characters of limestone, or of marble. (Cornish, *Short Studies in Physical Science.*)

42. Manufacturing countries are always rich countries; countries that produce raw material are always poor. Therefore, if we would be rich, we must have manufactures, and in order to get them, we must encourage them. . . . But I could make as good an argument to the little town of Jamaica . . . in support of a subsidy to a theatre. I could say to them: all cities have theatres, and the more theatres it has the larger the city. Look at New York! . . . Philadelphia ranks next to

New York in the number and size of its theatres, and therefore comes next to New York in wealth and population. . . . I might then drop into statistics . . . and point to the fact that when theatrical representations began in this country, its population did not amount to a million, that it was totally destitute of railroads, and without a single mile of telegraph wire. Such has been our progress since theatres were introduced that the census of 1880 showed we had 50,155,783 people, 90,907 miles of railroad, and $291,212\frac{9}{10}$ miles of telegraph wires. (George, *Protection and Free Trade.*)

43. What methods would you employ to investigate the connection between changes in the barometer and in the weather?

44. In Sir Humphry Davy's experiments upon the decomposition of water by galvanism, it was found that, besides the two components of water, oxygen and hydrogen, an acid and an alkali were developed at the two opposite poles of the machine. The insight of Davy conjectured that there might be some hidden cause of this portion of the effect: the glass containing the water might suffer partial decomposition, or some foreign matter might be mingled with the water, and the acid and alkali be disengaged from it, so that the water would have no share in their production. . . . By the substitution of gold vessels for glass, without any change in the effect, he at once determined that the glass was not the cause. Employing distilled water, he found a marked diminution of the quantity of acid and alkali evolved; yet there was enough to show that the cause, whatever it was, was still in operation. . . . He now conceived that the perspiration from the hands touching the instruments might affect the case, as it would contain common salt, and an acid and an alkali would result from its decomposition under the agency of electricity. By carefully avoiding such contact, he reduced the quantity of the products still

further until no more than slight traces of them were perceptible. What remained of the effect might be traceable to impurities of the atmosphere decomposed by contact with the electrical apparatus. An experiment determined this: the machine was put under an exhausted receiver, and when thus secured from atmospheric influence, it no longer evolved the acid and the alkali. (Gore, *The Art of Scientific Discovery.*)

45. The snake hallucinations of alcoholics are difficult to explain. Disturbances in the peripheral organs of vision seem hardly competent to account for such aggravated symptoms, although there are facts suggesting the plausibility of such an explanation. A patient in a room where the pattern of the wall-paper or the carpet abounds in geometrical figures and circles, is apt to find these endowed with gyratory movement, and as a result may come to imagine snakes about him. But the usual causes of this hallucination seem central in origin and due to pre-existing imaginative impulse. Why should this impulse assume the snake form? May not the explanation lie in the facts of nascent consciousness? We know that stimuli cannot be co-ordinated without some ganglion through which they are brought into relation. In effecting this co-ordination the ganglion must necessarily be subject to the influences of each stimulus and must undergo a succession of changes. This action and reaction implying perpetual experiences of resemblances and differences constitutes, according to psychologists, the raw material of consciousness. Therefore, as a corollary of this process, Herbert Spencer asserts that as 'consciousness is developed, some kind of instinct becomes nascent.' That there is a nascent instinctive dread of the serpent in man and monkey is obvious. There is every reason for it. The early history of our race abounds with record and tradition of that internecine strife between man and the serpent. We find the serpent permeating all his mythology, a chief feature of his legends,

inscribed on his monuments, engraved on his symbols and worshipped as his God. (*Science*, 1890.)

46. It was formerly supposed that all the nervous fibres in the body exercised both the function of conveying motor stimuli to the muscles, and sensory stimuli to the brain. This was apparently confirmed by the fact that when any nerve was severed both sensation and motion disappeared in the part to which it led. But in 1811, Sir Charles Bell published an essay to show that nerves were composed of various filaments, whose function differed according to the location of their original roots in the brain or in the spinal cord. This theory, he pointed out, would account for the extreme complexity of the structure of the brain and of the nervous system, which on the older supposition remained entirely unexplained. It was absurd, he also maintained, to suppose that one and the same nerve-fibre could conduct sensory stimuli *to* the brain and motor stimuli *from* it at the same instant; yet we are constantly moving a part at the same time that we receive sensations from it.

47. In order to experimentally test his theory, he selected two of the cerebral nerves, the portio dura and the fifth pair, the first of which has one root, while the latter has two. On cutting the portio dura in a living animal, motion only was lost in the parts with which it communicates. The fifth pair has some branches which arise from only one of its roots, and some which arise from both roots. On cutting the first set of branches, sensation only disappeared; on cutting the second, both sensation and the power of motion were destroyed.

48. The spinal nerves have two roots, an anterior and a posterior. When Bell exposed and irritated the anterior root, convulsive movements of the muscles were set up; but on irritating the posterior root, no movement followed. He felt assured, therefore, that the motor function was confined to the fibres of the anterior root; but inasmuch as the operation of

exposing the roots was intensely painful to the animal, he could not be certain that sensation was set up by the fibres of the posterior root only.

49. It was pointed out, however, that in some cases of partial paralysis of the limbs with which these nerves communicate, motion alone is lost, while the power of sensation is retained; in other cases, the reverse condition obtains. This seems to show — (What? and How?)

50. Glaciers are ice-streams, or rivers in which the moving material is frozen instead of liquid water. Like large rivers, they ordinarily have their sources in high mountains, and descend along the valleys; but the mountains are such as to take snow from the clouds instead of rain, because of their elevation. Like large rivers, many tributary streams coming from the different valleys unite to make the great stream. As with rivers, their movement is dependent on gravity, or the weight of the material; but the average rate of motion, instead of being several miles an hour, is generally in summer but 10 to 18 inches a day, or a mile in 18 to 20 years. As with rivers, the central portions move most rapidly, the sides and bottom being retarded by friction. (J. D. Dana.)

51. If I am not justified in general in inferring that *d* is a good book because *a*, *b*, and *c* are good books, why may I nevertheless conclude with some probability that *Guy Mannering* is a good book because *Waverley*, *Ivanhoe* and *Rob Roy* are? What bearing has this on the question of induction by simple enumeration and the assertion that all inference is by means of a universal?

52. Slips of flexible and tough Muntz's yellow metal instantly become rigid and brittle when dipped into a solution of pernitrate of mercury. Discuss the method by which this generalization might be reached from a single instance, and explain why in many other cases a large number of instances fail to yield a universal conclusion.

53. William Smith, a land-surveyor, whose business took him into many parts of England, profited by the peculiarly favorable conditions offered by the arrangement of our secondary strata to make a careful examination and comparison of their fossil contents at different points of the large area over which they extend. The result of his accurate and widely extended observations was to establish the important truth that each stratum contains certain fossils which are peculiar to it; and that the order in which the strata, characterised by these fossils, are superimposed one upon the other, is always the same. (Huxley, *Essays on Controverted Questions.*)

54. In order to investigate the ability of insects to find their mates, Loeb arranged the following experiment. A female butterfly was placed in a closed and otherwise empty cigar box, which was then suspended from the ceiling of a room. The windows were opened. At the time, no other butterflies of this species were visible in the neighborhood. During the course of a few hours, however, several males of this species entered the room and alighted on the box. Would you feel justified in drawing inferences from this result?

55. Properties known to exist in potassium have been predicted of and found to exist in rubidium; for instance, the carbonates of sodium and potassium are not decomposed by a red heat, neither are those of rubidium, or cæsium. Some of the statements which are true of chlorine have been found to be true, in varying degrees, of bromine and iodine. . . . After I had found the molecular change in antimony electro-deposited from its chloride, I sought for and discovered it in that deposited from its bromide and iodide; and after having found magnetic changes in iron by heat, I also found similar ones in nickel. (Gore, *The Art of Scientific Discovery.*)

56. What inductive fallacy may David be said to have committed when he said in his haste that all men are liars?

57. It has been found that linnets when shut up and educated with singing larks — the skylark, woodlark, or titlark — will adhere entirely to the songs of these larks, instead of the natural song of the linnets. We may infer, therefore, that birds learn to sing by imitation, and that their songs are no more innate than language is in man. (Hyslop.)

58. We observe very frequently that very poor handwriting characterizes the manuscripts of able men, while the best handwriting is as frequent with those who do little mental work when compared with those whose penmanship is poor. We may, therefore, infer that poor penmanship is caused by the influence of severe mental labour. (Hyslop.)

59. Galileo describes his invention of the telescope as follows: This then was my reasoning; this instrument [of which he had heard a rumour] must either consist of one glass, or of more than one; it cannot be of one alone, because its figure must be either concave or convex, or comprised within two parallel superficies, but neither of these shapes alter in the least the objects seen, although increasing or diminishing them; for it is true that the concave glass diminishes, and that the convex glass increases them; but both show them very indistinctly, and hence one glass is not sufficient to produce the effect. Passing on to two glasses, and knowing that the glass of parallel superficies has no effect at all, I concluded that the desired result could not possibly follow by adding this one to the other two. I therefore restricted my experiments to combinations of the other two glasses; and I saw how this brought me to the result I desired. (Quoted by Gore, *The Art of Scientific Discovery*.)

60. Darwin was struck by the number of insects caught by the leaves of the common sun-dew. It soon became evident to him that "Drosera was excellently adapted for the special purpose of catching insects." . . . As soon as he began to work on Drosera, and was led to believe that the leaves absorbed nu-

tritious matter from the insects, he began to reason by analogy from the well-understood digestive capacity of animals. . . . Having by analogy established the power of digestion in plants, analogy led him to seek in plants the elements that do the work of digestion in animals. Bringing together what was known of plants, he pointed out that the juices of many plants contain an acid, and so one element of a digestive fluid was at hand; and that all plants possess the power of dissolving albuminous or proteid substances, protoplasm, chlorophyl, etc., and that "this must be effected by a solvent, probably consisting of a ferment together with an acid." After writing the last-quoted sentence, he learned that a ferment which converted albuminous substances into true peptones had been extracted from the seeds of the vetch. (Cramer, *The Method of Darwin.*)

61. Strongly impressed with the belief that some "harmonic" relation must exist among the distances of the several planets from the sun, and also among the times of their revolution, Kepler passed a large part of his early life in working out a *series of guesses* at this relation, some of which now strike us as not merely most improbable, but positively ridiculous. His single-minded devotion to truth, however, led him to abandon each of these hypotheses in turn so soon as he perceived its fallacy by submitting it to the test of its conformity to observed facts. . . . But he was at last rewarded by the discovery of that relation between the times and the distances of the planetary revolutions, which with the discovery of the ellipticity of the orbits, and of the passage of the radius vector *over equal areas in equal times* has given him immortality as an astronomical discoverer. But . . . he was so far from divining the true *rationale* of the planetary revolutions that he was led to the discovery of the elliptical orbit of Mars by a series of happy accidents . . . whilst his discovery of the true relations of times and distances was the *fortunate* guess which closed a

long series of *un*fortunate ones, many of which were no less ingenious.

Now it was by a grand effort of Newton's *constructive* imagination, based on his wonderful mastery of geometrical reasoning, that, starting with the conception of two *forces*, one of them tending to produce continuous uniform motion in a straight line, the other tending to produce a uniformly accelerated motion towards a fixed point, he was able to show that if these *dynamical* assumptions were granted, Kepler's laws, being consequences of them, must be universally true. And it was his still greater glory to divine the profound truth that the fall of the moon towards the earth — that is, the deflection of her path from a tangential line to an ellipse — *is a phenomenon of the same order* as the fall of a stone to the ground. (Gore, *The Art of Scientific Discovery*.)

62. After Franklin had investigated the nature of electricity for some time, he began to consider how many of the effects of thunder and lightning were the same as those produced by electricity. Lightning travels in a zigzag line, and so does an electric spark; electricity sets things on fire, so does lightning; electricity melts metals, so does lightning. Animals can be killed by both, and both cause blindness. Pointed bodies attract the electric spark, and in the same way lightning strikes spires, and trees, and mountain tops. Is it not likely then that lightning is nothing more than electricity passing from one cloud to another, just as an electric spark passes from one substance to another? (Buckley, *A Short History of Natural Science*.)

63. How did Franklin proceed to verify the hypothesis stated in the last example?

64. When men had formed a notion of the moon as a solid body revolving about the earth, they had only further to conceive it spherical, and to suppose the sun to be beyond the

region of the moon, and they would find that they had obtained an explanation of the varying forms which the bright part of the moon assumes in the course of a month. For the convex side of the crescent-moon, and her full edge when she is gibbous, are always turned towards the sun. And this explanation, once suggested, would be confirmed the more it was examined. For instance, if there be near us a spherical stone, on which the sun is shining, and if we place ourselves so that this stone and the moon are seen in the same direction (the moon appearing just over the top of the stone), we shall find that the visible part of the stone, which is then illuminated by the sun, is exactly similar in form to the moon, at whatever period of her changes she may be. (Whewell.)

65. Not long ago the adherents of spontaneous generation urged as an argument on their side that if biogenesis be true, innumerable facts and experiments prove that the air must be thick with germs, and they regarded this as the height of absurdity. But that micro-organisms exist everywhere has since been shown beyond the shadow of a doubt.

66. To establish the fundamental law regarding the pitch of sound, Mersenne stretched a hempen rope over ninety feet in length, so that the eye could easily follow its displacements. It did not then emit any sound, but one could easily count the vibrations it made in any given time. He then shortened the cord by one half, and found it then made twice the number of vibrations in the same length of time. In reducing it to a third or a fourth of the original length, he observed that the oscillations became three and four times as rapid. He also made similar experiments, with like results, with a brass wire. He thus established the law that, all other things being equal, the number of vibrations of a cord is inversely as its length. (Zahm, *Sound and Music.*)

67. The foundations of Fort Jefferson, which is built entirely

of coral rock, were laid on the Tortugas Islands in the year 1846. A very intelligent head-workman watched the growth of certain corals that established themselves on these foundations, and recorded their rate of increase. He had shown me the rocks on which corals had been growing for some dozen years, during which they had increased at the rate of about half an inch in ten years. I have collected facts from a variety of sources and localities that confirm this testimony. A brick placed under water, in the year 1850, by Captain Woodbury of Tortugas, with the view of determining the rate of growth of corals, when taken up in 1858 had a crust of Mændrina upon it a little more than half an inch in thickness. . . . Estimating the growth of the coral reef according to these and other data of the same character, it should be about half a foot in a century; and a careful comparison which I have made of the condition of the Reef as recorded in an English survey made about a century ago with its present state would justify this conclusion. (Agassiz, *Methods of Study in Natural History.*)

68. Lord Curzon, arguing for the continued existence of a hereditary Chamber: "The hereditary principle is established in every branch and aspect of our national life. We have hereditary bankers, lawyers, and even hereditary cotton-spinners. Why should it be a blot and offence when applied to the House of Lords?"

69. The following is the cardinal passage in Harvey's famous argument for the circulation of the blood: "Let us assume either arbitrarily or from experiment, the quantity of blood which the left ventricle of the heart will contain when distended, to be, say, two ounces, three ounces, or one ounce and a half — in the dead body I have found it to hold upwards of two ounces. . . . Let us suppose as approaching the truth that the fourth, or fifth, or sixth, or even that the eighth part of its charge is thrown into the artery at each contraction; this would give

either half an ounce, or three drachms, or one drachm of blood as propelled by the heart at each pulse into the aorta; which quantity, by reason of the valves at the root of the vessel, can by no means return into the ventricle. Now, in the course of half an hour, the heart will have made more than one thousand beats, in some as many as two, three, and even four thousand. Multiplying the number of drachms propelled by the number of pulses, we shall have either one thousand half ounces, or one thousand times three drachms, or a like proportional quantity of blood, according to the amount which we assume as propelled with each stroke of the heart, sent from this organ into the artery; a larger quantity in every case than is contained in the whole body! . . . (Thus), supposing even the smallest quantity of blood to be passed through the heart and the lungs with each pulsation, a vastly greater amount would still be thrown into the arteries . . . than could by any possibility be supplied by the food consumed. It could be furnished in no other way than by making a circuit and returning." (*De motu cordis*, Ch. IX.)

70. Some thirty years ago, a student of the Germanic languages, reading over an old English poem of considerable length, called the *Genesis*, was struck by the fact that five or six hundred lines, in the heart of the poem, seemed to differ in various respects from the lines which preceded and followed. Pursuing his inquiry further, and comparing the forms of these lines with those of a kindred language, he came to the conclusion that this section, which had always been supposed to be original Old English, had in fact been translated from Old Saxon, and was therefore led to believe in the existence of an Old Saxon poem on this subject of Genesis, though he was obliged to confess that he found no other trace of its existence. Some twenty years after, another scholar, at work in the Vatican Library, which had only recently rendered its treasures more

accessible, discovered a fragment of the missing Old Saxon *Genesis*, of which probably no one had read a line for a thousand years. Yet such had been the faith of competent scholars in Sievers's processes that no one was surprised when the missing manuscript swam into sight, any more than astronomers were amazed when the telescope pointed to the quarter of the heavens indicated by Adams and Leverrier, and revealed the planet Neptune, which no human eye till then had ever seen. (Albert S. Cook, *The Higher Study of English*.)

71. During the retreat of the Ten Thousand a cutting north wind blew in the faces of the soldiers; sacrifices were offered to Boreas, and the severity of the wind immediately ceased, which seemed a proof of the god's causation. (*Anabasis*, Bk. IV.)

72. A nectary implies nectar, but Sprengel had come to the conclusion that *orchis morio* and *orchis maculata*, though furnished with nectaries, did not secrete nectar. Darwin examined the flowers of *orchis morio* for twenty-three consecutive days, looking at them after hot sunshine, after rain, and at all hours; he kept the spikes in water and examined them at midnight and early the next morning. He irritated the nectaries with bristles, and exposed them to irritating vapours. He examined flowers whose pollinia had been removed, and others which would probably have them soon removed. But the nectary was invariably dry.

He was thoroughly convinced, however, that these orchids require the visits of insects for fertilization, and that insects visit flowers for the attractions offered in the way of nectar, and yet that in these orchids the ordinary attraction was absent. In examining the orchids he was surprised at the degree to which the inner and outer membranes forming the tube or spur were separated from each other, also at the delicate nature of the inner membrane, and the quantity of fluid contained between the two membranes. He then examined other forms that do

secrete nectar in the ordinary way, and found the membranes closely united, instead of separated by a space. "I was therefore led to conclude," he says, "that insects penetrate the lax membrane of the nectaries of the above-named orchids and suck the copious fluid between the two membranes." He afterwards learned that at the Cape of Good Hope moths and butterflies penetrate peaches and plums, and in Queensland a moth penetrates the rind of the orange. These facts merely proved his anticipation less anomalous than it had seemed. (Cramer, *The Method of Darwin.*)

73. Construct an hypothesis to explain some fact of your experience, and explain how it may be either verified or overthrown.

74. When Darwin began to work on *Drosera* he was led to believe that the leaves absorbed nutritious matter from insects. He then reasoned by analogy from the well-understood digestive capacity of animals. He made preliminary 'crucial' experiments by immersing some leaves of *Drosera* in nitrogenous and others in non-nitrogenous fluids of the same density, to determine whether the former affected the leaves differently from the latter. This he found to be the case. He then experimented with solid animal matter and found that the leaves are capable of true digestion. Analogy then led him to seek in plants the elements that do the work of digestion in animals. He pointed out that the juices of many plants contain an acid, and so one element of a digestive fluid was at hand; and that all plants possess the power of dissolving albuminous or proteid substances, protoplasm, chlorophyl, and that this must be effected by a solvent consisting probably of a ferment together with an acid. Afterwards he learned that a ferment which converted albuminous substances into true peptones had been extracted from the seeds of the vetch. (Cramer, *The Method of Darwin*, pp. 95–99.)

75. In opposition to the facts stated above, Tischutkin main-

tains that the 'digestion' of insectivorous plants is not accomplished in the same way as in animals, but is due to bacteria; that the pepsin is not a secretion of the plant, but a by-product of the activity of the bacteria. Suppose that this theory is true, and Darwin's false, what would you say regarding the character of the latter's reasoning?

76. Vesalius, the founder of modern anatomy, found that the human thigh bone was straight, and not curved, as Galen, the great authority on the subject for over a thousand years, had asserted. Sylvius replied that Galen must be right; that the bone was curved in its natural condition, but that the narrow trousers worn at the time had made it artificially straight.

77. "From looking at species as only strongly-marked and well-defined varieties, I was led to anticipate that the species of the larger genera in each country would oftener present varieties than the species of the smaller genera; for wherever many closely related species (*i.e.* species of the same genus) have been formed many varieties or incipient species ought, as a general rule to be now forming. . . . To test the truth of this anticipation I have arranged the plants of twelve countries, and the coleopterous insects of two districts, into two nearly equal masses, the species of the larger genera on one side and those of the smaller genera on the other side, and it has invariably proved to be the case that a larger proportion of the species on the side of the larger genera presented varieties than on the side of the smaller genera. Moreover, the species of the large genera which present any varieties invariably present a larger average number of varieties than do the species of the small genera. Both of these results follow when another division is made, and when all the least genera with only one to four species are altogether excluded from the tables." (Darwin, *Origin of Species.*)

78. Sir Joseph Lister, the founder of aseptic surgery, states the origin of his method as follows: "When it had been shown

by the researches of Pasteur that the septic property of the atmosphere depended, not on oxygen or any gaseous constituent, but on minute organisms suspended in it, which owed their energy to their vitality, it occurred to me that decomposition in the injured part might be avoided without excluding the air, by applying as a dressing some material capable of destroying the life of the floating particles." At first he used carbolic acid for this purpose. The wards of which he had charge in the Glasgow Infirmary were especially affected by gangrene, but in a short time became the healthiest in the world; while other wards separated only by a passageway retained their infection. (Locy.)

79. The spectroscope . . . has suggested the presence of substances not known upon the earth. To one of these substances, indicated by a green line in the spectrum of the sun's corona, the name Coronium has been given provisionally. It has been suggested that this line may represent not a new substance, but known substances under the unknown conditions of the sun's temperature. However, as it exists at least 300,000 miles from the sun, it is impossible that the conditions of temperature are so entirely different from those known to us as completely to disguise known substances, and most scientists now accept the conclusion that the green line is caused by the presence of an element hitherto unknown in any other region of nature. Recently, Professor Nasini of Padua has been examining the gases of the volcanic regions of his country and has discovered coronium in them.

80. A leading expert in pathology remarks that "a chemist may, and frequently does, accept certain biological evidence as proved which we (pathologists) should reject as inconclusive, owing to the omission of certain controls and checks." What generalization regarding scientific methods and evidence does this remark suggest, and how would you go to work to verify the generalization?

81. Some remarkably interesting experiments are reported from Zurich, showing the effect of temperature on the development of species among butterflies. The experiments have been continued for fourteen years, and it is found, for instance, that the common small tortoise-shell butterfly, if subjected to warmth of 37 to 39 degrees centigrade, develops into the variety found in Sardinia, while those bred in a temperature of from 4 to 6 degrees produced the Lapland variety.

82. Gilbert formed a small sphere of a magnetite (lodestone) and observed the behavior of small pieces of iron placed on the sphere. He found that a bit of iron tended to lie along a meridian line of this spherical magnet, just as does a compass needle on the surface of the earth. From these and other experiments he came to the conclusion that the terrestrial globe is itself a magnet. (Mills, *The Realities of Modern Science.*)

83. Stenson gives a graphic description of his discovery of the parotid duct. He relates how, one morning, when he was dissecting the head of a sheep and examining the parotid gland, the style which he was using, inserted by chance, it would seem, into an opening in the duct, slipped easily down and struck with a sharp clink against the teeth; he recognized that he had discovered the duct of the gland. . . . Stenson, who had learnt from his master, Sylvius of Leyden, the distinction between conglomerate glands, such as the salivary glands and the pancreas, and conglobate glands, such as the lymphatics, laid hold of the idea that the former were secretory glands and hence must all have ducts. He soon found the duct of the sublingual gland, as well as those of the small buccal glands, and cleared up the problem of the secretion of tears by the lachrymal gland, concerning which in spite of the lead given by older anatomists there was as yet much confusion. (Foster, *Lectures on Physiology.*)

84. In the following account of the study of 'pellagra,' a

disease affecting those who eat moldy corn, what methods of investigation are indicated and how serviceable were they?

In some regions the fall of pellagra ran closely parallel with the fall of the price of corn, in others it rose; in some, pellagra fell as the harvests rose, in others it reversed. Then there were places where pellagra seemed to vary with the rainfall — this was true of Veneti, but not of the mountainous districts. Again, there were places where it seemed to vary with vine culture. In some of the smaller Italian provinces pellagra went down in proportion to the increasing number of the vineyards; on the other hand, exactly the opposite happened in Sicily, Sardinia, and Corfu. As investigation went deeper, these contradictions were turned into a chain of evidence. Heavy rainfall meant molded crops. Large harvests meant improper handling and improper storage; poor harvests, that the peasants were forced to eat corn they would at other seasons refuse. Restriction of the corn area by vine culture in a mainland province meant more money for varied food and fresh meat; on an island the same thing meant importation by the coastwise trade, and all experts pronounce corn in the least damaged by water to be highly dangerous.

85. The Feejeans are, physically, so intimately connected with the adjacent Negritos of New Caledonia that no one can doubt to what stock they belong, and yet, in the form and substance of their language, they are Polynesian. The case is as remarkable as if the Canary Islands should have been found to be inhabited by negroes speaking Arabic, or some other clearly Semitic dialect, as their mother tongue. As it happens, the physical peculiarities of the Feejeans are so striking, and the conditions under which they live are so similar to those of the Polynesians, that no one has ventured to suggest that they are merely modified Polynesians — a suggestion which would otherwise certainly have been made. But if languages may be thus

transferred from one stock to another, without any corresponding intermixture of blood, what ethnological value has philology? (T. H. Huxley, *Of the Methods and Results of Ethnology.*)

86. As evidence both of their intelligence and of their affection for their friends, it has been said by various observers that when ants have been accidentally buried, they have been very soon dug out and rescued by their companions. It seemed to me it would not be difficult to test whether the excavations made by ants under the circumstances were the result of the general habit of burrowing in loose, fresh soil and of digging out fresh galleries when their nests are disturbed, or really due to a desire to extricate their friends. I placed some honey near a nest of *Lasius niger* on a glass surrounded with water, and so arranged that in reaching it the ants passed over another glass covered with a layer of sifted earth about one-third of an inch in thickness. I then put some ants to the honey, and by degrees a considerable number collected around it. Then at 1.30 P.M. I buried an ant from the same nest under the earth, and left her there till 5 P.M., when I uncovered her. She was none the worse, but during the whole time, not one of her friends had taken the least notice of her. (Sir John Lubbock, *Scientific Lectures.*)

87. A striking characteristic of many animals, especially of certain insects, is that they resemble or mimic other animals, or even inanimate objects, in a way that protects them from the attacks of enemies, sometimes by making them inconspicuous, sometimes by making them appear dangerous or unpalatable. Four causes of such resemblances have been proposed: (1) external or environmental causes, — food, climate, etc.; (2) internal physiological causes, compelling different species to pass through similar phases; (3) sexual selection; (4) natural selection. Professor Poulton, examining the question, reasons as follows: —

(*a*) These resemblances are often to inanimate objects, — twigs, leaves, earth, etc. If we admitted the action of either internal or external causes, they might, since they would by hypothesis act alike on the different animals, make them resemble one another; but it is difficult to see why they should make them resemble lifeless things. As for sexual selection, that is exercised only at the mature stage; and these resemblances to inanimate things are very common in the immature stages of insects. Natural selection, however, explains all kinds of resemblance equally well; for resemblance to any object, animate or inanimate, which serves in any way to conceal or to protect the animal, will be a useful variation in the struggle for life.

(*b*) These resemblances, when between animals, are as often as not quite independent of any affinity between the species; *e.g.* the larva of a moth looks like a wasp. But both external and internal causes would obviously produce the closest likeness where there was most physiological similarity, *i.e.* where the species were most nearly related.

(*c*) The resemblances in question are not accompanied by any internal changes in the direction of the mimicked species except such as assist in producing a superficial likeness, which is the useful element in the result. Natural selection, by its very nature, brings about the retention and accumulation of useful changes only. Physical and internal causes would bring to pass changes of all sorts, superficial and deeply seated, indiscriminately.

(*d*) The same resemblance is often produced in very different ways, in different examples of it; for example, other insects mimic ants and wasps, sometimes by an actual likeness in form and movement, sometimes only by an outline strongly marked in contrasting colour on bodies of very different shape. But either a similar environment or like internal causes would bring

these resemblances about, if at all, in a uniform way. It makes no difference to natural selection, however, what the original causes of a resemblance are; if it is useful, any change towards it will be preserved. The differences in the way in which it is produced will be due to the original differences in the animals.

(*e*) The food and conditions of life of many of the resembling species are very different.

(*f*) These resemblances are far commoner in females than in males. Yet there is no assignable difference which would make them more responsive than the males to the action either of environmental conditions or of internal causes. In fact, the female usually varies less from the ancestral type than the male. Such resemblances are more useful to the females than to the males, however, because of their slower flight when laden with eggs, and their greater exposure to attack during egg-laying, incubation, and at other times.

(*g*) The supposed direct effect of environment implies the inheritance of acquired characters, which has never been satisfactorily proved to take place. (Poulton, *Essays on Evolution*, VIII, IX.)

88. Borelli, with all his zeal for the exact mathematical treatment of physiological problems, assumed, being led to do so by reasons of analogy without attempting to make any direct observations on the matter, that a muscle during contraction was inflated, that it suffered increase in bulk. . . . Glisson confronted [this idea] with a single experiment, the result of which deprived [it] of all solid basis. He says: . . . Take an oblong glass tube of suitable capacity and shape. Fit into the top of its side near its mouth another small tube like a funnel. Let a strong muscular man insert into the mouth of the larger tube the whole of his bared arm, and secure the mouth of the tube all round to the humerus with bandages so that no water can escape from the tube. Then pour water through the

funnel until the whole of the larger tube is completely filled, and some water rises up into the funnel. This being done, now tell the man alternately to contract powerfully and to relax the muscles of his arm. It will be seen that when the muscles are contracted the water in the tube of the funnel sinks, rising again when relaxation takes place. From which it is clear that muscles are not inflated or swollen at the time that they are contracting but on the contrary are lessened, shrunk, and subsided. (Foster, *Lectures on the History of Physiology.*)

89. Darkness, according to Goethe, had as much to do as light with the production of color. Color was really due to the commingling of both. He looked to the zenith at midnight, and found before him the blackness of space, while in daylight he saw the blue firmament overhead; and he concluded that this coloring of the sky was due to the shining of the sun upon a turbid medium with darkness behind. (Tyndall, *New Fragments.*)

90. In 1838 Schleiden, who had been studying the cellular structure of plants under the microscope, communicated his observations to Schwann. He mentioned in particular the nucleus and its relationship to the other parts of the cell. Schwann was at once struck by the fact that he had found similar nuclei in the elements of animal tissue. Schleiden also recognized these nuclei as in effect the same on being shown Schwann's sections, and the latter was thus aided to come to the conclusion that the elements in animal tissue were practically identical with those of plant tissue.

91. In 1835, before this cell theory was announced, living matter had been observed by Dujardin. In lower animal forms he noticed a semifluid, jelly-like substance, which he designated sarcode, and which he described as being endowed with all the properties of life. He observed it very carefully in different forms of the invertebrates, not only as to its structure, but also

as to its chemical properties, which distinguished it from albumen, mucus, gelatin, and other like substances. Dujardin was far from appreciating the full importance of his discovery, and for a long time his description of sarcode remained separate; but in 1846 Hugo von Mohl, a botanist, observed a similar jelly-like substance in plants, which he called plant-slime, and to which he attached the name of protoplasm. On the basis of these observations, and of his own study of the movements of the spores of one of the simplest plants, Cohn, in 1850, declared that vegetable protoplasm and animal sarcode, if not identical, were at least in the highest degree analogous substances. Finally, in 1861, Max Schultze showed that sarcode, which was supposed to be confined to the lower invertebrates, was present also in the tissues of higher animals, and there exhibits the same properties, especially those of contractility and irritability. He showed also that sarcode agreed in physiological properties with protoplasm in plants, and that the two living substances were practically identical. It was on physiological likeness, rather than on structural or chemical grounds, that he based his sweeping conclusions. He therefore defined both plant and animal cells as little masses of protoplasm surrounding a nucleus.

92. On the basis of continued microscopic study during the years intervening, Verworn, in 1895, redefined a cell as "a body consisting essentially of protoplasm in its general form, including the unmodified cytoplasm, and the specialized nucleus and centrosome; while as unessential accompaniments may be enumerated (1) the cell membrane, (2) starch grains, (3) pigment granules, and (4) chlorophyl granules."

93. Meanwhile, the cell has come to be regarded not only as the element of structure, but also as the unit of physiological activities, and the conveyor of hereditary qualities. It is seen that all life, both in plants and in animals, arises from cells; and that where sexual reproduction takes place, in the plant

and the animal alike, both the egg and its fertilizing agents are modified cells of the parents' bodies. Therefore the cell is the only possible agent of heredity. And by microscopic observation of fertilized ova, it has been determined that half of their chromosomes are derived from the male cell and half from the female, — each egg thus containing hereditary substance derived from both parents. (Locy, Chs. XI, XII.)

94. In 1620 Jean Tarde argued that because the sun is "The eye of the world," and the eye of the world cannot suffer from ophthalmia, sun-spots must be due not to actual specks or stains on the bright solar disk, but to the transits of a number of small planets across it. To this new group of heavenly bodies he gave the name of "Borbonia Sidera."

Most of those who were capable of thinking at all on such subjects adhered either to the theory that the spots were clouds, or that they were slag thrown up in solar conflagrations.

In the following century, Derham gathered from observations carried on during the years 1703–1711, "That the spots on the sun are caused by the eruption of some new volcano therein." Lalande upheld the view that the spots were rocky elevations uncovered by the casual ebbing of a luminous ocean. This view had even less to recommend it than Derham's volcanic theory. Both were, however, significant of a growing tendency to bring solar phenomena within the compass of terrestrial analogies. (Clerke, *History of Astronomy.*)

95. In November, 1769, a spot of extraordinary size engaged the attention of Alexander Wilson, Professor of Astronomy in the University of Glasgow. He watched it day by day, and as the great globe slowly revolved, carrying the spot towards its western edge, he was struck with the gradual contraction and final disappearance of the penumbra *on the side near the centre of the disk*, and when on the 6th of December the same spot re-emerged on the eastern limb, he perceived, as he had anticipated,

that the shady zone was now deficient *on the opposite side*, and resumed its original completeness as it returned to a central position. Similar perspective effects were visible in numerous other spots subsequently examined by him, and he was thus in 1774 able to prove by strict geometrical reasoning that such appearances were, as a matter of fact, produced by vast excavations in the sun's substance. In 1861 De la Rue obtained a stereoscopic view of a sun-spot which confirmed Wilson's inference as to their depressed nature. (*Ibid.*)

96. Until 1791 Herschel never doubted that all the luminous patches in the sky were actually clusters of stars, and that with the improvement in instruments these stars would more and more become visible. There seemed to him to be an unbroken series from evident clusters of stars to spots without a trace of stellar formation. He was led to the hypothesis of true nebulæ by the following analogy. He saw that natural philosophers sometimes neglected the important differences between human beings and humble members of the organic world, because they thought of all living things as forming an infinite series. Under such circumstances their minds have to be startled into a perception of the real differences within the series by bringing together two instances of living things which are very unlike. He therefore tried the effect of looking at once from a very brilliant star to a faint star-like cloud. His conclusion was that there existed real nebulous matter which never could be resolved into a cluster of stars.

97. Since the days of Cavendish, the composition of the air had been looked upon as an ascertained fact; a certain proportion had been shown to be oxygen, varying amounts of carbonic acid and aqueous vapor were known to be present, while the remainder, as the result of careful investigation, was supposed to be nitrogen. Cavendish himself knew, so accurate was his work, that any undetected residue could not exceed the 120th

part. But in the course of a long series of experiments, undertaken afresh to determine the densities of the principal gases, Lord Rayleigh detected a slight difference in the density of nitrogen as prepared from ammonia and as extracted from the air. This difference, amounting at first to about 0.1 per cent, was increased on subsequent more careful examination to nearly a half per cent. It was clear that the gases prepared by these two methods were not identical, and that some hitherto unknown body was responsible for the complication. The existence of this new body, the inert gas now known as argon, was announced by Rayleigh and Ramsay in 1894, and shortly afterwards it was isolated from its companion. (Whetham, *The Recent Development of Physical Science.*)

98. The heavenly bodies are laboratories on a vast scale in which nature has provided conditions of temperature, pressure, and electrical state which we may never hope to rival on the Earth. The spectroscope gives us data from which it may be possible to form some idea of these conditions by comparison with feeble laboratory imitations of celestial phenomena, and conversely the latter may aid in the interpretation of terrestrial phenomena.

99. What is the exact meaning of the requirement that the minds of the jury be free from prejudice? Does an 'open mind' require the absence of previous ideas on the subject, as, for example, that the juror should not have read the newspapers?

100. Point out the value in the acquisition of knowledge of (*a*) the daily paper, (*b*) the United States Census Reports, (*c*) the microscope, (*d*) the scientific laboratory.

101. After Becquerel's discovery of the photographic and electric activity of uranium, it was found that, like Röntgen rays, the rays from uranium produced electric conductivity in air and other gases through which they passed. In the year 1900,

M. and Madame Curie made a systematic search for similar properties in a great number of chemical elements and compounds, and in many natural minerals. They found that several minerals containing uranium were more active than that metal itself. Pitch-blende, for instance, a substance consisting chiefly of an oxide of uranium, but containing also traces of many other metals, was especially active. When obtained from Cornwall its activity was about equal to that of the same weight of uranium, but samples from the Austrian mines were found to be three or four times as effective. The presence of some more active constituent was thus suggested. To examine this point, the various components of pitch-blende were separated chemically from each other and their radio-activities determined. In this way three different substances, radium, polonium, and actinium, all previously unknown, appear to have been isolated by various observers. (Whetham.)

102. It was known of old that venous blood was dark and arterial bright, but the change in color was thought to be only a superficial accompaniment of profound differences between the blood in the arteries and the blood in the veins. Lower's careful quantitative determinations and calculations of the flow of blood through the heart raised doubts in his mind "as to whether there could be that great difference between venous and arterial blood which the vulgar think." He suspected that the change of color took place in the lungs as the contents of the pulmonary artery found their way into the pulmonary veins and that it was due simply to the exposure of the blood to the air in the lungs. But so long as he made observations on natural breathing he failed to satisfy himself of the correctness of his supposition. Hook's experiment on artificial respiration gave him the opportunity he desired. Examining the lungs of an animal kept alive by artificial respiration after the chest had been opened, he had no difficulty in ascertaining that the blood

in the pulmonary veins, long before it reached the heart, was florid in color. He further saw that when the artificial respiration was stopped, when no fresh air was driven into the lungs, when the animal was suffocated, the blood in the pulmonary veins and in the left side of the heart became dark and venous. He took dark venous blood from the vena cava, and injected it artificially through the lungs. He found that so long as insufflation of the lungs was kept up the blood ran out by the pulmonary veins florid in color, but ran out dark and unchanged if no fresh air was driven into the lungs. He concluded that the change in color was due simply to the blood being exposed in the lungs to the air. (Foster, *Lectures on Physiology.*)

103. Newton showed that the bodies known as comets obey the law of gravitation; but it was by no means certain that the individual of the species observed by him in 1680 formed a permanent member of the solar system. With another comet, however, which appeared in 1682, the case was different. Edmund Halley calculated the elements of its orbit on Newton's principles, and found them to resemble so closely those arrived at for comets observed by Peter Apian in 1531, and by Kepler in 1607, as almost to compel the inference that all three apparitions were of a single body. This implied its revolution in a period of about seventy-six years, and Halley accordingly fixed its return for 1758–1759. It punctually reappeared on Christmas Day, 1758, and effected its perihelion passage on the 12th of March following, thus proving beyond dispute that some at least of these erratic bodies are domesticated within our system, and strictly conform to its fundamental laws. (Clerke.)

104. Periodical comets are evidently bodies which have lived, each through a chapter of accidents; and a significant hint as to the nature of their adventures can be gathered from the fact that their aphelia are pretty closely grouped about the tracks of the major planets. Halley's, and four other comets, are thus

related to Neptune; eight connect themselves with Uranus, nine with Saturn, twenty-five at least with Jupiter. Some form of dependence is plainly indicated, and recent researches leave scarcely a doubt that the 'capture-theory' represents the essential truth in the matter. The original parabolic paths of these comets were then changed into ellipses by the backward pull of a planet, whose sphere of influence they chanced to enter when approaching the sun from outer space. Moreover, since a body thus affected should necessarily return at each revolution to the scene of encounter, the same process of retardation may, in some cases, have been repeated many times, until the more restricted cometary orbits were reduced to their present dimensions. (*Ibid.*)

105. "Science for the past is a description; for the future a belief; it does not show the necessity of any sequence of phenomena." Discuss.

106. Koch found that, while guinea-pigs, mice, and other animals were killed by inoculation with anthrax, birds were not affected. This invulnerability had very much struck Pasteur and his two assistants. What was it in the body of a fowl that enabled it thus to resist inoculations of which the most infinitesimal quantity sufficed to kill an ox? They proved by a series of experiments that the microbe of splenic fever does not develop when subjected to a temperature of 44° Centigrade. Now, the temperature of birds being between 41 and 42°, may it not be, said Pasteur, that the fowls are protected from the disease because their blood is too warm? Might not the vital resistance encountered in the living fowl suffice to bridge over the small gap between 41–42°, and 44–45°? . . . This idea conducted Pasteur and his assistants to new researches. 'If the blood of a fowl were cooled,' they asked, 'could not the splenic fever parasite live in this blood?' The experiment was made. A hen was taken, and after inoculating it with splenic

fever blood, it was placed with its feet in water at 25°. The temperature of the blood of the hen went down to 37° or 38°. At the end of twenty-four hours the hen was dead, and all its blood was filled with splenic fever bacteria. But if it was possible to render a fowl assailable by splenic fever simply by lowering its temperature, is it not also possible to restore to health a fowl so inoculated by warming it up again? A hen was inoculated, subjected, like the first, to the cold-water treatment, and when it became evident that the fever was at its height it was taken out of the water, wrapped carefully in cotton wool, and placed in an oven at a temperature of 35°. Little by little its strength returned; it shook itself, settled itself again, and in a few hours was fully restored to health. The microbe had disappeared. Hens killed after being thus saved, no longer showed the slightest trace of splenic organisms. There have been great discussions in Germany and France upon a mode of treatment in typhoid fever, which consists in cooling the body of the patient by frequently repeated baths. The possible good effects of this treatment may be understood when viewed in conjunction with the foregoing experiment on fowls. In typhoid fever the cold arrests the fermentation, which may be regarded as at once the expression and the cause of the disease, just as, by an inverse process, the heat of the body arrests the development of the splenic fever microbe in the hen. (Vallery-Radot, *Louis Pasteur*.)

107. In 1865 Pasteur undertook the investigation of the silk-worm disease which was ruining the silk industry of France. The presence of vibratory corpuscles in the blood of the diseased worms was already known, and he was prepared by his previous discoveries of the micro-organisms which cause fermentation to see in these corpuscles the cause of the disease.

By the use of the microscope, he secured a number of healthy worms, free from corpuscles. He prepared two infusions, one

by pounding up a diseased worm in water, the other by pounding up a healthy worm. These infusions were then brushed over mulberry leaves, separately, and the healthy worms were allowed to feed, some on the first bed of leaves, the others on the second. The first group of worms became diseased, the second remained healthy.

It was further established, by observation of the diseased worms, that in the first stages of the disease, when they cannot readily be distinguished from the healthy, these corpuscles are confined to the intestines. As the disease progresses and becomes obvious, they are found in the other tissues; and at death the body is full of them.

Separating, therefore, the uninfected moths from the infected, by the use of the microscope, taking care that the food should be free of infection, the progeny of the former were found to be always free from the disease, and that of the latter to be always diseased. (Vallery-Radot, *Louis Pasteur*.)

108. The first to employ the prism in the examination of various flames was a young Scotchman named Thomas Melvill. He studied the spectrum of burning spirits, into which were introduced successively sal ammonia, potash, etc., and noticed the singular predominance, under almost all circumstances, of a particular shade of yellow light, taking up a perfectly definite and invariable position in the spectrum. Fraunhofer, the great Munich optician, later rediscovered Melvill's deep yellow ray and measured its place in the colour scale. It has since become well known as the 'sodium line,' and has played a very important part in the history of spectrum analysis. Nevertheless, its ubiquity and conspicuousness long impeded progress.

It was because of this perplexing fact that Fox Talbot hesitated in 1826 to announce his theory that the presence in the spectrum of any individual ray told unerringly of the volatilization in the flame under scrutiny of some body as whose badge

or distinctive symbol that ray might be regarded. The yellow ray appeared indeed without fail where sodium *was;* but it also appeared where it might be thought only reasonable to conclude that sodium *was not.* Nor was it until thirty years later that William Swan, by pointing out the extreme delicacy of the spectral test, and the singularly wide dispersion of sodium, made it appear probable (but even then only probable) that the questionable yellow line was really due invariably to that substance. Common salt (chloride of sodium) is, in fact, the most diffusive of solids. It floats in the air; it flows with water; every grain of dust has its attendant particle; its absolute exclusion approaches the impossible. And withal, the light that it gives in burning is so intense and concentrated, that if a single grain be divided into 180 million parts, and one alone of such inconceivably minute fragments be present in a source of light, the spectroscope will show unmistakably its characteristic beam. (Clerke.)

109. In 1859 Kirchhoff and Bunsen entered on a long series of stringent and precise experiments, as a result of which they were able to state positively that certain rays in the spectrum are necessarily and invariably connected with certain kinds of matter. The assurance of their conclusion was rendered doubly sure by the discovery, through the peculiarities of their light alone, of two new metals, named, from the blue and red rays by which they were respectively distinguished, 'Cæsium' and 'Rubidium.' Both were immediately afterwards actually obtained in small quantities by evaporation of the Dürkheim mineral waters. (*Ibid.*)

Fraunhofer in 1815, by means of a slit and a telescope, made the surprising discovery that the solar spectrum is crossed, not by seven, but by thousands of obscure transverse streaks. Of these he counted some 600, and carefully mapped 324. The same system of examination applied to the rest of the heavenly

bodies showed the mild effulgence of the moon and the planets to be deficient in precisely the same rays as sunlight; while in the stars it disclosed the differences in likeness which are always an earnest of increased knowledge.

One solar line especially — that marked in his map with the letter D — proved common to several of the stars examined; and it was remarkable that it exactly coincided in position with the conspicuous yellow beam which he had already found to accompany most kinds of combustion. Moreover, both the *dark* solar and the *bright* terrestrial 'D-lines' were displayed by his refined appliances as double. In this striking correspondence was contained the very essence of solar chemistry; but its true significance did not become apparent until long afterwards. (*Ibid.*)

110. The 'fixed lines' (as they were called) of the solar spectrum took up the position of a standing problem. One view was that the atmosphere of the earth was the agent by which sunlight was deprived of its missing beams. For some of them this is actually the case. Brewster found in 1832 that certain dark lines, which were invisible when the sun stood high in the heavens, became increasingly conspicuous as he approached the horizon. These are the well-known 'atmospheric lines'; but the immense majority of their companions in the spectrum remain quite unaffected by the thickness of the stratum of air traversed by the sunlight containing them. (*Ibid.*)

111. There remained the true interpretation — absorption in the sun's atmosphere; and this, too, was extensively canvassed. But a remarkable observation made by Professor Forbes of Edinburgh on the occasion of the annular eclipse of May 15, 1836, appeared to throw discredit upon it. If the problematical dark lines were really occasioned by the stoppage of certain rays through the action of a vaporous envelope surrounding the sun, they ought, it seemed, to be strongest in light proceeding from

his edges, which, cutting that envelope obliquely, passed through a much greater depth of it. But the circle of light left by the interposing moon, and of course derived entirely from the rim of the solar disk, yielded to Forbes's examination precisely the same spectrum as light coming from its more central parts. This circumstance helped to baffle inquirers, already sufficiently perplexed. It still remains an anomaly, of which no complete explanation has been offered. (*Ibid.*)

112. Convincing evidence as to the true nature of the solar lines was however at length, in the autumn of 1859, brought forward at Heidelberg. Kirchhoff's experiment in the matter was a very simple one. He threw bright sunshine across a space occupied by vapour of sodium, and perceived with astonishment that the dark Fraunhofer line D, instead of being effaced by flame giving a *luminous* ray of the same refrangibility, was deepened and thickened by the superposition. He tried the same experiment, substituting for sunbeams light from a Drummond lamp, and with similar result. A dark furrow, corresponding in every respect to the solar D-line, was instantly seen to interrupt the otherwise unbroken radiance of its spectrum. The inference was irresistible, that the effect thus produced artificially was brought about naturally in the same way, and that sodium formed an ingredient in the glowing atmosphere of the sun.

This first discovery was quickly followed up by the identification of numerous bright rays in the spectra of other metallic bodies with others of the hitherto mysterious Fraunhofer lines. Kirchhoff was thus led to the conclusion that (besides sodium) iron, magnesium, calcium, and chromium are certainly solar constituents, and that copper, zinc, and nickel are also present, though in smaller quantities.

These memorable results were founded upon a general principle first enunciated by Kirchhoff, which may be expressed as

follows: Substances of every kind are opaque to the precise rays which they emit at the same temperature; that is to say, they stop the kinds of light or heat which they are then actually in a condition to radiate. (*Ibid.*)

113. When a tree, or a bundle of wheat or barley straw, is burnt, a certain amount of mineral matter remains in the ashes — extremely small in comparison with the bulk of the tree or of the straw, but absolutely essential to its growth. In a soil lacking, or exhausted of, the necessary mineral constituents, the tree cannot live, the crop cannot grow. Now contagia are living things, which demand certain elements of life just as inexorably as trees, or wheat, or barley; and it is not difficult to see that a crop of a given parasite may so far use up a constituent existing in small quantities in the body, but essential to the growth of the parasite, so as to render the body unfit for the production of a second crop. The soil is exhausted, and, until the lost constituent is restored, the body is protected from any further attack of the same disorder. Such an explanation of non-recurrent diseases naturally presents itself to a thorough believer in the germ theory. . . . To exhaust a soil, however, a parasite less vigorous and destructive than the really virulent one may suffice; and if, after having by means of a feebler organism exhausted the soil, without fatal result, the most highly virulent parasite be introduced into the system, it will prove powerless. This, in the language of the germ theory, is the whole secret of vaccination. (Tyndall.) Have you any remarks to make on this explanation?

PART III. — The Nature of Thought

Chapter XXI. — *Judgment the Elementary Process*

1. What objections are there to speaking of thought as 'a thing like other things'?

2. What is the general law of Evolution? Explain what is meant by a change from the homogeneous to the heterogeneous.

3. What general conclusions are reached by the application of the law of Evolution to the thought-process?

4. What do you understand by Judgment? How does the view of mind which takes judgment as the elementary process differ from that of psychology?

5. In what sense may our judgments be said to be the union of two concepts?

6. Would the doctrine that in knowing we first have Simple Apprehension, then as separate intellectual processes, Judgment and finally Inference, agree with the general evolutionary view of consciousness? Explain fully.

Chapter XXII. — *The Characteristics of Judgment*

1. What do you understand by the universality of judgments? What is the distinction between the universality of a judgment and that of a proposition?

2. How would you prove that all judgments are universal?

3. Is any judgment necessary in itself? If not, whence do judgments derive their necessity?

4. What is the argument by which it has been maintained that there must be judgments or principles which are in themselves necessary? How would you reply to this argument?

5. Explain how it is possible for a judgment to be at once both analytic and synthetic.

6. Explain what is meant by a 'system' of knowledge.

7. When judgment brings new facts into relation to what we already know, does the old body of knowledge itself undergo any modification?

Chapter XXIII. — *The Laws of Thought*

1. What do you mean by a Law of Thought? In what sense, if any, can a Law of Thought be violated?

2. Explain what is meant by the law of Identity.

3. How has this law been interpreted by Boole and Jevons?

4. What does Jevons mean by the 'substitution of similars,' and how does he propose to employ this principle?

5. What objections are there to employing the sign of equality to represent the relation between the subject and predicate of a judgment?

6. Explain how the law of Identity is related to the characteristics of judgment treated in the last chapter.

7. What is the meaning of the law of Contradiction?

8. Explain the use of the law of Excluded Middle.

9. In what general postulate of thought is the meaning of all these laws included?

Chapter XXIV. — *Types of Judgment*

1. Why do we begin with judgments of Quality?

2. Explain how we pass in the development of intelligence from Quality to Quantity.

3. In what sense is it true that judgments of Quantity never give us the real nature of things, but only their relation to something else?

4. What is meant by anthropomorphic causes? How are they distinguished from scientific causes? What is meant by Animism?

5. What new element did the discovery of the law of the Con-

servation of Energy introduce in the causal conception as employed in certain sciences?

6. Why cannot this new extension have any application in the field of the mental sciences?

7. How does the standpoint of judgments of Individuality differ from that of judgments of Causality? What is meant by an 'infinite regress'?

8. Discuss the question whether a judgment of individuality may take the form of a definition.

Chapter XXV. — *Inference*

1. How does Inference differ from Judgment? In what sense may it be said that it is an extension of the latter process?

2. Does the passage from Judgment to Inference illustrate the general law of Logical Evolution? Explain.

3. In the development of our knowledge, which usually comes first, premises or conclusion?

4. How is it possible to pass from the known to the unknown?

5. Explain under what circumstances only an Inference is possible.

6. What is the common element in both Induction and Deduction? How do they differ?

Chapter XXVI. — *The Unification of Knowledge*

1. Explain the distinction between 'Science' and 'the sciences.'

2. What part does philosophy play in the progress of knowledge toward unity?

3. Why would it be unsatisfactory to construct a philosophy simply by taking as ultimate the most general laws and principles of physical science? Can you mention any philosophers who have proceeded in this way?

4. What is meant by the abstract or hypothetical character of the special sciences? Illustrate in the case of physics and psychology.

5. Do the various sciences differ in their degree of abstractness? If so, how would you classify them in order of concreteness? Compare mathematics and biology in this respect.

6. Explain the function of philosophy as the interpretation of the results of the sciences.

7. What is meant by the statement that philosophy must find a new category or principle of synthesis? Illustrate by showing what categories might conceivably be employed by philosophy.

INDEX

A

Abstract, two Meanings of the Word, 51 ff.
Accent, the Fallacies of, 168.
Accident, the Fallacy of, 178 f.
A fortiori Arguments, 141.
Agreement, the Method of, 239; Deficiencies in the Method of, 242–243.
Amphiboly, the Fallacy of, 168.
Analogy, Explanation by Means of, 266; the Principle of, 268; Statements of Law, 269; its Function in suggesting Hypothesis, 271; its Use by Darwin, 272; its Incompleteness as a Method of Explanation, 274.
Analysis, its Relation to Synthesis, 279.
Analytic Procedure, 135 f.
Anthropomorphism, 364.
A priori Truths, 334.
Argument, Irregular Forms of, 133 ff.
Argumentum, ad rem, 184; *ad hominem*, 184 f.; *ad populum*, 185; *ad ignorantiam*, 186; *ad verecundiam*. 186; *ad misericordiam*, 185; *ad baculum*, 187.
Aristotle, Logic of, 23; List of Logical Works, 23; his Theory of the Syllogism, 23; as Founder of Modern Sciences, 23; Importance of Induction and Deduction in his Logic, 25; his Classification of Fallacies, 164; his Statement of the Law of Contradiction, 350.
Art, an, its Relation to a Science, 9 f.

B

Bacon, Logic of, 28; his Method, 28; on Induction by Simple Enumeration, 193; on the Tendency to neglect Negative Instances, 310; his Doctrine of the Four Idols, 310-14 f.
Bosanquet, his remarks on Analogy, 276.
Bradley, 13.

C

Cant Words and Phrases, 302.
Causal Connection, as Principle of Science, 235 f.; Judgments of, 362 ff.
Cause, the Fallacy of the False, 189; the Development of the Principle of, 362 ff.
Causes, the Plurality of, 244.
Chances, the Calculation of, 228.
Circle, Argument in a, 181.
Classification, Principles of, 79; Rules of, 81; of Fallacies, 165, 299; Aristotle's, of Fallacies, 164.
Composition, the Fallacy of, 174 f.
Concepts, Relation to Percepts and Judgments, 44 ff., 324 ff.
Conclusion, the Irrelevant, 182.
Concrete, two Senses of the Word, 51.
Connotation, of Terms, 58 ff.
Consequent, Fallacy of the, 187.
Conservation of Energy, the Law of, and its Influence on the Conception of Cause, 366.
Contradiction, the Law of, 38, 350.
Conversion, the, of Propositions, 105; Simple, 101; by Limitation, 106; Errors in, 167.

D

Darwin, his Power of arresting Exceptions, 263; his Use of Analogy, 272; his Employment of Hypotheses, 281.
Deduction, its Relation to Induction, 384.
Definition, the Necessity of, 64; Verbal and Real, 66; Ways of Regarding, 67; Socrates' Search for, 68; Rules of, 70; Genetic, 74 ff.
Denotation, of Terms, 58 ff.; Descartes 30.
Dialectic, Socrates' Use of, 68.
Dichotomy. 77.
Difference, Method of, 244 f.
Differentia, 70.

Dilemma, the Simple Constructive, 157; the Complex Constructive, 158; the Simple Destructive, 157; the Complex Destructive, 159; the Fallacies of, 161 ff., 179.
Division, Rules for, 81; the Fallacy of, 176.

E

Eduction, 99.
Elimination, the Part of, in Induction, 199, 289.
Enthymemes, 40, 133.
Enumeration, as the Starting-point of Induction, 194, 217; Judgments of, 359.
Episyllogisms and Prosyllogisms, 134.
Equivocation, the Fallacies of, 172.
Ethics, its Standpoint compared with that of Psychology, 372.
Euler, 92.
Evolution, the Law of, 315; the Application of the Law of, to Thought, 317 ff.
Excluded Middle, the Law of, 77, 352.
Experiment and Observation, 197; Advantages of employing, 211.
Explanation and Observation, 207 f.; the Problem of, 212.
Extension and Intension of Terms, 55.

F

Fallacies, Classification of, 165, 299; Syllogistic, 165; Inductive, 298; the Source of, 298; of Interpretation, 166; occasioned by Language, 299; of Reasoning, 170, 309; of Observation, 303; Individual, 312.
Figures of the Syllogism, 120; the Special Canons of the four, 123; Determination of the Valid Moods in, 127; the Perfect, 130; the Imperfect, 130; Reduction of, 130; the Organic Relation of, 132 note.

G

Galen, 130.
Generalization, Danger of hasty, 310 f.
Genus, its Definition, 70.
Guericke, 287.

H

Hegel, his Influence on the Development of Logic, 32.
Herschel, J., 31.
Hypothesis, as guiding Induction, 201. Reasoning from an, 278; the Employment of, to explain Common Events, 279; Darwin's Use of, 281; the Necessity for an, 282; Formation of, 282 f.; the Function of Analogy in suggesting, 200, 284; the Proof of, 285 ff.; Requirements of a Good, 293 ff.

I

Identity, the Law of, 38, 343; Jevons's Interpretation of the Law of, 344.
Ignoratio Elenchi, 182.
Imagination, its Part in the Formation of Theories, 282.
Individuality, Judgments of, 370.
Induction and Deduction, 384; the Baconian Method of, 28; Mill's Emphasis on, 31; the Problem of, 190 f.; Perfect and Imperfect, 193 f.
Inference, Mediate and Immediate, 97; the Nature of, 378; as distinguished from Judgment, 373; the Paradox of, 379; as a Development of Judgment, 378. (See also Reasoning.)
Instances, the Value of Numerous, 196 f.
Intension and Extension of Terms, 58 ff.
Interpretation, of Propositions, 97 ff.; Errors of, 166; Judgment a Process of, 322.

J

James, 9.
Jevons, his Account of Perfect Induction, 193; his Calculation of Chances, 228; his Interpretation of the Law of Identity, 344; his Principle of the Substitution of Similars, 345.
Judgment, Relation to Perception and Conception, 44 ff., 324 ff.; the Starting-point of Knowledge, 322; as a Process of Interpretation, 323; and Concept, 324; the Universality of, 329; the Necessity of, 331; *a priori*, 334; as involving both Analysis and Synthesis, 334 ff.; as constructing a System of Knowledge, 339; its Relation to Inference, 373.
Judgments, of Quality, 355; of Quantity, 358; of Enumeration, 359; of Measure, 360; of Causal Connection, 362; of Individuality, 370.

L

Language, Relation to Thought, 3 f., 46, 326; Dangers from the Careless Use of, 64, 298; Fallacies of, 299; Figurative, 302.
Law, of Identity, 38, 343; of Contradiction, 38, 350; of Excluded Middle, 77, 352; of Conservation of Energy, 367.
Laws of Thought, 38, 77, 343; on the Careless Use of Words, 64, 300.
Logic, Definition of, 1; Derivation of the Word, 3; Relation to Psychology, 5; Relation to Rhetoric, 3 f.; Comparison with Physiology, 7; as Normative Science, 6, 14; as a Science and an Art, 9; Utility of, 11; Necessity of, 13; the Materials of, 14; of the Sophists, 20; of Socrates, 20; of Aristotle, 23, 36; of the Schoolmen, 27; of Bacon, 28; of Mill, 31, 34; Development of Modern, 32; the Equational, 344.
Lyell, his Overthrow of the 'Catastrophic' Theory in Geology, 296.

M

Malthus, his Theories of Population, 183, 273.
Measure, Judgments of, 360.
Metaphors, Dangers from the Use of, 303.
Method, the Progressive or Synthetic, 135; the Regressive or Analytic, 135; the, of Agreement, 239; the, of Difference, 244; the Joint, of Agreement and Difference, 249; the, of Concomitant Variations, 255; the, of Residues, 260.
Middle Term, the Function of the, 113; Ambiguous, 171.
Mill, his Importance in the History of Logic, 31; his Experimental Methods, 237.
Mnemonic Lines, for Syllogism, 129.
Moods, of Syllogism, 121.
Morphology, compared with Psychology, 7.

N

Negative Instances, Tendency to neglect, 304.
Neptune, the Discovery of, 264.
Newton, his Care in testing Theories, 288.
Non sequitur, 187.
Normative Science, Logic as, 6, 14.

O

Observation, and Explanation, 207 ff.; and Experiment, 211; Errors of, 303.
Obversion, the, of Propositions, 103; Errors in, 167.
Opposition, the, of Propositions, 99.

P

Perception, Relation to Conception and Judgment, 44 ff.; as involving Judgment, 45, 325; Difficulty in distinguishing between Inference and, 308.
Petitio Principii, 180.
Philosophy and Science, 390; as interpretation of the sciences, 405.
Physiology compared with Logic, 7.
Plato, in the History of Logic, 23; and the Doctrine of Reminiscence, 380.
Post hoc propter hoc, 189, 310.
Predicables, the, 69.
Prejudices, Individual, 312; of an Age, 313.
Premises, Definition of, 40.
Presumption, Fallacies of, 180.
Propositions, Categorical, 85; Conditional, 85; the Nature of, 84; Quality and Quantity of, 86; Difficulties in classifying, 89; Relation of Subject and Predicate in, 90; the Opposition of, 99; Contrary and Contradictory, 100; the Obversion of, 103; the Conversion of, 105; the Contraposition of, 107; the Inversion of, 109.
Prosyllogisms, 134.
Psychology, its Relation to Logic, 5; Comparison with Morphology, 7; Comparison with Ethics, 372.

Q

Quality, of Propositions, 86; Judgments of, 355.
Quantity, of Propositions, 86; Judgments of, 358.

Quaternio Terminorum, 170.
Question, the Fallacy of the Complex, 181.
Question-Begging Epithet, 301.

R

Reasoning, the Nature of Syllogistic, 112; Mediate, 97, 113; Immediate, 97; Mistakes in, 309; Inductive and Deductive, 384; from Particulars to Particulars, 384; from Particulars to a Universal, 388. (See also Inference.)
Reduction of the Imperfect Figures, 130.
Residues, the Method of, 260.

S

Scepticism, of the Sophists, 20 ff., 330.
Schönbein, his Discovery of Ozone, 263.
Science, as related to Art, 9; as related to Philosophy, 390; as Philosophy, 395; the Assumptions of 399.
Sigwart, on the Difference between Ancient and Modern Science, 219; on the Application of Statistics, 220.
Similars, the Principle of the Substitution of, 345.
Socrates, his Place in the History of Logic, 20; his Search for Definitions, 68; his Employment of Dialectic, 68.
Sophists, the Logic of, 20; Socrates' Refutation of, 22; Plato's Criticism of their Theory of Knowledge, 23; their Scepticism, 330.
Sorites, Aristotelian, 137; Goclenian, 138.
Species, Definition, 70.
Statistics, 219.
Subject, Relation of Predicate and, 90.
Syllogism, the Aristotelian, 23, 36; the Nature of the, 36; the Principle of the, 37; the Parts of the, 39; the Rules of the, 115; the Figures of the, 120; the Hypothetical, 143; Rules for the Hypothetical, 145; Relation of Categorical and Hypothetical, 148; the Disjunctive, 154; Fallacies of the Disjunctive, 156.
Synthesis, its Relation to Analysis, 334.
Synthetic Procedure, 135 f.
System, Difference between a, and an Aggregate, 339.

T

Terms, Definition of Logical, 49; Singular or Individual, 49; General and Collective, 49; Abstract and Concrete, 51 ff.; Positive and Negative, 55; Contradictory and Contrary, 56 f.; Privative, 56; Absolute and Relative, 57; Extension and Intension of, 58 ff.
Thales, 365.
Thought, its Relation to Language, 3 f., 46, 326; the Laws of, 38, 77, 343; as a process of Transformation and Conservation, 47 f; the Nature of, 316.
Torricelli, 287.

U

Unification of Knowledge, 390.
Uniformity of Nature, 203, 236.

V

Variations, of Statistics, 225; the Method of Concomitant, 255.

W

Whewell, 16, 206.
Words, the Abuse of, 65, 288.

Printed in the United States of America.

P - M

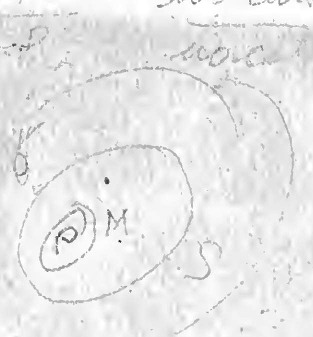
S - M

S - P

All [illegible]

no cats [illegible]

[illegible]



P - M

M - S

S - P

All P - M

[illegible]

[illegible] P